The suboptimal feedback control is

$$u^{\varepsilon}(t) = -\operatorname{sgn}(\varphi^{\varepsilon}_{x}(t,x)), \ x \in [(i-1)\varepsilon, i\varepsilon], \ i = \overline{1,n}, \ n\varepsilon = T.$$

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ON GENERALIZED PSEUDO PROJECTIVE RICCI SYMMETRIC MANIFOLDS

BY

M.C. CHAKI and S.KOLEY

0. Introduction. In a recent paper [1] the authors introduced and studied a type of non–flat Riemannian manifold (M^n, g) $(n > 2)$ whose Ricci tensor S of type $(0,2)$ is not identically zero and satisfies the condition

$$(1) \qquad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(Y,X)$$

where A, B, C are non–zero–1–forms and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such a manifold was called a generalized pseudo Ricci symmetric manifold, the 1–forms A, B, C were called its associated 1–forms and an n–dimensional manifold of this kind was denoted by $G(PRS)_n$. If, in particular, $B = C = A$, then (1) takes the form

$$(2) \qquad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X)$$

which reduces to a pseudo Ricci symmetric manifold introduced by one of the authors (C h a k i) [2] who denoted such an n–dimensional manifold by $(PRS)_n$. This justifies the name generalized pseudo Ricci symmetric manifold for the manifold defined by (1) and the use of the symbol $G(PRS)_n$ for it.

It is known [3] that in a Riemannian manifold the projective curvature tensor W is defined by

$$(3) \qquad W(X,Y,Z) = R(X,Y,Z) - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]$$

where R and S are respectively the curvature tensor and the Ricci tensor. If

$$(4) \qquad {}'W(X,Y,Z,U) = g[W(X,Y,Z),U]$$

then from (3) we get

$$(5)\quad 'W(X,Y,Z,U) = 'R(X,Y,Z,U) - \frac{1}{n-1}[g(X,U)S(Y,Z) - g(Y,U)S(X,Z)] \quad \text{where}$$

$$(6)\quad 'R(X,Y,Z,U) = g[R(X,Y,Z),U].$$

Let

$$(7)\quad P(X,U) = 'W(X,e_i,e_i,U)$$

where $\{e_i; i = 1,2,\ldots,n\}$ is an orthonormal basis of the tangent space at each point and e_i in (7) is summed for $1 \leq i \leq n$. Then, using (5), we get

$$(8)\quad P(X,U) = \frac{n}{n-1}S(X,U) - \frac{r}{n-1}g(X,U)$$

where r is the scalar curvature. The tensor P shall be called the projective Ricci tensor which is clearly a symmetric tensor of type $(0,2)$.

The present paper deals with a type of non-flat Riemannian manifold (M^n,g) $(n > 2)$ whose projective Ricci tensor is not identically zero and satisfies the condition

$$(9)\quad (\nabla_X P)(Y,Z) = 2A(X)P(Y,Z) + B(Y)P(X,Z) + C(Z)P(Y,X)$$

where A, B, C are non-zero 1-forms and ∇ has the meaning already stated. Such a manifold shall be called a generalized projective Ricci symmetric manifold, A, B, C shall be called its associated 1-forms and an n-dimensional manifold of this kind shall be denoted by $G(PWRS)_n$.

Let ℓ and L be the symmetric endomorphisms of the tangent space at each point of $G(PWRS)_n$ corresponding to to the projective Ricci tensor P and the Ricci tensor S, respectively. Then

$$(10)\quad g(\ell X,Y) = P(X,Y) \quad \text{and}$$

$$(11)\quad g(LX,Y) = S(X,Y).$$

Further, let

$$(12)\quad g(X,\rho) = A(X)$$

$$(13)\quad g(X,\lambda) = B(X)$$

$$(14)\quad g(X,\mu) = C(X)$$

for every vector field X.

Then ρ,λ,μ shall be called the basic vector fields corresponding to the respective associated 1-forms A, B and C. From (8) it follows that if in a $G(PWRS)_n$, $r = 0$, then $P = \frac{n}{n-1}S$. We shall exclude this possibility. Hence in this paper r shall be different from zero.

In this paper it is shown that in a $G(PWRS)_n$, $P(X,\lambda)+P(X,\mu) = 0$ for every vector field X. Since the scalar curvature of a $G(PWRS)_n$ is not necessarily a constant, a necessary and sufficient condition for constant scalar curvature of the manifold is obtained. Considering the forms $A(\ell X)$, $B(LX)$, $C(LX)$, denoting them by $\widetilde{A}(X), \widetilde{B}(X), \widetilde{C}(X)$ respectively and calling them auxiliary associated 1-forms, it is shown that if any one of these forms is closed, then the remaining two are so, while if any one of them is not closed, then the same is true of the other two. The question whether in $G(PWRS)_n$ the tensor P can be a Codazzi tensor or a cyclic tensor has been answered negatively. Further, it is shown that in a $G(PWRS)_n$ the curvature transformation $R(X,Y)$ and the projective Ricci transformation ℓ do not in general commute. In view of this, a sufficient condition for commutativity of such transformations is obtained. Moreover, it is shown that if a $G(PWRS)_n$ admits a unit parallel vector field V, then V cannot be orthogonal to any of the basic vector fields ρ, λ or μ.

1. Preliminaries. Contracting (9) we get

$$(1.1)\quad 0 = P(X,\lambda) + P(X,\mu) \ \text{ or } \ B(\ell X) + C(\ell X) = 0 \ \text{ or } \ \widetilde{B}(X) + \widetilde{C}(X) = 0$$

Again from (8) we get

$$(1.2)\quad \ell X = \frac{n}{n-1}LX - \frac{r}{n-1}X$$

[by (10) and (11)]. Differentiating (1.2) covariantly, we have

$$(1.3)\quad (\nabla_Y \ell)(X) = \frac{n}{n-1}(\nabla_Y L)(X) - \frac{dr(Y)}{n-1}X.$$

Again, from (9) and (10), we get

$$(1.4)\quad (\nabla_X \ell)(Y) = 2A(X)\ell Y + B(Y)\ell X + P(X,Y)\mu.$$

Now, contracting (1.3), we have

$$(1.5)\qquad (\operatorname{div} \ell)(X) = \frac{n-2}{2(n-1)} \,\mathrm{dr}\,(X).$$

Next, contracting (1.4), we get

$$(1.6)\qquad (\operatorname{div} \ell)(X) = 2P(X,\rho) + P(X,\mu) = 2\widetilde{A}(X) + \widetilde{C}(X).$$

From (1.5) and (1.6), it follows that

$$(1.7)\qquad \frac{n-2}{2(n-1)} \,\mathrm{dr}\,(X) = 2\widetilde{A}(X) + \widetilde{C}(X).$$

Since $\mathrm{ddr}\,(X) = 0$, from (1.7) we get

$$(1.8)\qquad 2\mathrm{d}\widetilde{A}(X,Y) + \mathrm{d}\widetilde{C}(X,Y) = 0.$$

Again, from (1.1), we have

$$(1.9)\qquad \mathrm{d}\widetilde{B}(X,Y) = \mathrm{d}\widetilde{C}(X,Y) = 0.$$

Combining (1.8) and (1.9), we can write as follows:

$$(1.10)\qquad 2\mathrm{d}\widetilde{A}(X,Y) = \mathrm{d}\widetilde{B}(X,Y) = -\mathrm{d}\widetilde{C}(X,Y).$$

These formulas will be used in the sequel.

2. Auxiliary 1–forms of the associated 1–forms. From (1.1) we find that

$$(2.1)\qquad \widetilde{B}(X) + \widetilde{C}(X) = 0.$$

Again from (1.10) we see that if any one of $\widetilde{A}, \widetilde{B}, \widetilde{C}$ is closed then the remaining two are so. On the other hand, if any one of $\widetilde{A}, \widetilde{B}, \widetilde{C}$ is not closed, then the same is true of the remaining two.

We can therefore state the following theorem:

Theorem 1. *In a $G(PWRS)_n$ the auxiliary 1-forms $\widetilde{B}$ and $\widetilde{C}$ are connected by the relation (2.1). Futher, if any one of the auxiliary 1–forms is closed, then the other two are so, while if any one of the auxiliary 1–forms is not closed, the same is true of the other two.*

3. Scalar curvature of a $G(PWRS)_n$. We can write (2.1) as follows:

$$(3.1)\qquad P(X,\lambda) + P(X,\mu) = 0.$$

Since $P(X,Y) = \frac{n}{n-1}S(X,Y) - \frac{r}{n-1}g(X,Y)$, we have

$$(3.2)\qquad P(X,\lambda) = \frac{n}{n-1}S(X,\lambda) - \frac{r}{n-1}g(X,\lambda) \quad \text{and}$$

$$(3.3)\qquad P(X,\mu) = \frac{n}{n-1}S(X,\mu) - \frac{r}{n-1}g(X,\mu).$$

Adding (3.2) and (3.3), we get

$$P(X,\lambda) + P(X,\mu) = \frac{n}{n-1}[S(X,\lambda) + S(X,\mu)] - \frac{r}{n-1}[g(X,\lambda) + g(X,\mu)]$$

$$\text{or}\qquad 0 = \frac{n}{n-1}[S(X,\lambda) + S(X,\mu)] - \frac{r}{n-1}[B(X) + C(X)]$$

[by (3.1),(13) and (14)], or

$$(3.4)\qquad r[B(X) + C(X)] = n[S(X,\lambda) + S(X,\mu)].$$

From (3.4), we get

$$(3.5)\qquad r = \frac{n[S(X,\lambda) + S(X,\mu)]}{B(X) + C(X)}\,,\ [B(X) + C(X) \neq 0].$$

This is the expression for the scalar curvature.

From (3.5) it follows that r is not necessarily a constant. Now, from (1.7) we have

$$\frac{n-2}{2(n-1)} \,\mathrm{dr}\,(X) = 2\widetilde{A}(X) + \widetilde{C}(X) \quad \text{or}$$

$$(3.6)\qquad \frac{n-2}{2(n-1)} \,\mathrm{dr}\,(X) = 2P(X,\rho) + P(X,\mu).$$

From (3.6) we see that if r is constant, then $2P(X,\rho) + P(X,\mu) = 0$.

Again, if $2P(X,\rho) + P(X,\mu) = 0$, then r is constant, because $n > 2$. We can therefore state the following theorem:

Theorem 2. *In a $G(PWRS)_n$ the scalar curvature is constant if and only if*

$$2P(X,\rho) + P(X,\mu) = 0.$$

Note: Since in a $G(PWRS)_n$, $P(X,\lambda)+P(X,\rho)=0$ [by (3.1)], an alternative form of necessary and sufficient condition for constant scalar curvature is as follows:

(3.7) $$2P(X,\rho)-P(X,\lambda)=0.$$

4. $G(PWRS)_n$ with its projective Ricci tensor as a Codazzi or a cyclic tensor. We first suppose that the tensor P is a Codazzi tensor [4]. Then

(4.1) $$(\nabla_X P)(Y,Z)-(\nabla_Z P)(Y,X)=0.$$

In virtue of (9) we get

(4.2) $$(\nabla_X P)(Y,Z)-(\nabla_Z P)(Y,X)=[2A(X)-C(X)]P(Y,Z)-\\-[2A(Z)-C(Z)]P(Y,X)=T(X)P(Y,Z)-T(Z)P(Y,X)$$ where

(4.3) $$T(X)=2A(X)-C(X)\neq 0.$$

Let

(4.3a) $$g(X,\nu)=T(X)$$

for every vector field X. Then $g(\nu,\nu)=T(\nu)\neq 0$.
In virtue of (4.1) from (4.2) we get

(4.4) $$T(X)P(Y,Z)=T(Z)P(Y,X).$$

From (4.4), it follows that $T(X)P(e_i,e_i)=T(e_i)P(e_i,X)=g(e_i,\nu)P(e_i,X)$ or $0=P(\nu,X)$ or

(4.5) $$P(X,\nu)=0.$$

From (4.4) we get $T(X)P(Y,\nu)=T(\nu)P(Y,X)$ or $0=T(\nu)P(Y,X)$ [by (4.5)].

Since $T(\nu)\neq 0$, it follows that $P(Y,X)=0$; but this is inadmissible. Hence P cannot be a Codazzi tensor.

Next, we suppose that P is a cyclic tensor, i.e.

(4.6) $$(\nabla_X P)(Y,Z)+(\nabla_Y P)(Z,X)+(\nabla_Z P)(X,Y)=0.$$

In virtue of (9) this can be written as $[2A(X)+B(X)+C(X)]P(Y,Z)+$ $+[2A(Y)+B(Y)+C(Y)]P(Z,X)+[2A(Z)+B(Z)+C(Z)]P(Y,X)=0$ or

(4.7) $$Q(X)P(Y,Z)+Q(Y)P(Z,X)+Q(Z)P(Y,X)=0$$ where

(4.8) $$Q(X)=2A(X)+B(X)+C(X)\neq 0.$$

Let $g(X,\xi)=Q(X)$ for every vector field X.
Then $g(\xi,\xi)=Q(\xi)$. So $Q(\xi)\neq 0$.
From (4.7) we get $Q(X)P(e_i,e_i)+Q(e_i)P(e_i,X)+Q(e_i)P(e_i,X)=0$ or

(4.9) $$P(X,\xi)=0.$$

Now from (4.7) we have $Q(X)P(Y,\xi)+Q(Y)P(\xi,X)+Q(\xi)P(Y,X)=0$ or

(4.10) $$Q(\xi)P(Y,X)=0.$$

Since $Q(\xi)\neq 0$, we get $P(Y,X)=0$ which is not admissible. Hence P cannot be a cyclic tensor. This leads to the following theorem:

Theorem 3. *In a $G(PWRS)_n$ the projective Ricci tensor can neither be a Codazzi tensor or be a cyclic tensor if $T(X)$ and $Q(X)$ given by (4.3) and (4.8) are not zero.*

From (8) we can write $P(Y,Z)=\dfrac{n}{n-1}S(Y,Z)-\dfrac{r}{n-1}g(Y,Z)$. Hence

(4.11) $$(\nabla_X P)(Y,Z)-(\nabla_Z P)(Y,X)=\frac{n}{n-1}[(\nabla_X S)(Y,Z)-(\nabla_Z S)(Y,X)]-\\-\frac{1}{n-1}[\mathrm{d}r(X)g(Y,Z)-\mathrm{d}r(Z)g(Y,X)].$$

If the Ricci tensor S is a Codazzi tensor, i.e.,

(4.12) $$(\nabla_X S)(Y,Z)-(\nabla_Z S)(Y,X)=0$$

then, from (4.12), it follows that $\mathrm{d}r(X)=0$. Therefore (4.11) asssumes the form

(4.13) $$(\nabla_X P)(Y,Z)-(\nabla_Z P)(Y,X)=0$$

i.e. the projective Ricci tensor is a Codazzi tensor. But by Theorem 3 it is inadmissible. This leads to the following theorem:

Theorem 4. *In a $G(PWRS)_n$ the Ricci tensor cannot be a Codazzi tensor.*

5. $G(PWRS)_n$ with commuting curvature and projective Ricci transformations. In this section we enquire whether the curvature transformation $R(X,Y)$ and the projective Ricci transformation ℓ commute in a $G(PWRS)_n$. If not, we obtain a sufficient condition for their commutativity.

In fact, since $g(\ell X,Y) = P(X,Y)$ we have to determine whether $R(X,Y)\cdot P = 0$ in a $G(PWRS)_n$.

Now

$$(5.1)\quad \begin{aligned}[R(X,Y)\cdot P](Z,W) &= 2[(\nabla_X A)(Y) - (\nabla_Y A)(X)]P(Z,W) - \\ &-[(\nabla_Y A)(Z) - A(Y)A(Z)]P(X,W)+ \\ &+[(\nabla_X B)(Z) - B(X)B(Z)]P(Y,W)+ \\ &+[(\nabla_X C)(W) - C(X)C(W)]P(Z,Y)- \\ &-[(\nabla_Y C)(W) - C(Y)C(W)]P(Z,X).\end{aligned}$$

From (5.1) it follows that in a $G(PWRS)_n$, $R(X,Y)\cdot P$ is not necessarily zero.

However, if $(\nabla_X A)(Y) = A(X)A(Y)$, $(\nabla_X B)(Y) = B(X)B(Y)$ and $(\nabla_X C)(Y) = C(X)C(Y)$ it follows from (5.1) that $[R(X,Y)\cdot P](Z,W) = 0$ i.e. $R(X,Y)\cdot P = 0$.

Hence we can state the following theorem:

Theorem 5. *If the associated 1-forms A,B,C of a $G(PWRS)_n$ are recurrent with A,B,C as their respective 1-forms of recurrence, then the $G(PWRS)_n$ will have commuting curvature and projective Ricci transformations.*

6. $G(PWRS)_n$ admitting a parallel vector field. A vector field V in a Riemannian manifold (M^n, g) is said to be parallel [5], [6] if

$$(6.1)\quad \nabla_X V = 0$$

for every vector field X.

We now suppose that a $G(PWRS)_n$ admits a parallel vector field V. Applying Ricci identity to (6.1) we get

$$(6.2)\quad R(X,Y,V) = 0.$$

From (6.2) it follows that

$$(6.3)\quad R(X,Y,Z,V) = 0.$$

From (6.3) we get

$$(6.4)\quad S(X,V) = 0.$$

Therefore from (8) it follows that

$$(6.5)\quad P(X,V) = -\frac{r}{n-1}g(X,V).$$

From (6.5) we see that $-\frac{r}{n-1}$ is an eigenvalue of the projective Ricci tensor P and V is an eigenvector corresponding to this eigenvalue. This leads to the following theorem:

Theorem 6. *If a $G(PWRS)_n$ admits a parallel vector field V, then $-\frac{r}{n-1}$ is an eigenvalue of the tensor P and V is an eigenvector corresponding to this eigenvalue.*

If V is a unit vector field, the question arises whether any of the basic vector fieds may be orthogonal to V. The answer will be obtained from the following discussion.

From (9) we get

$$(6.6)\quad (\nabla_X P)(Y,V) = 2A(X)P(Y,V) + B(Y)P(X,V) + C(V)P(Y,X).$$

Again from (8) we have

$$(6.7)\quad P(X,V) = \frac{n}{n-1}S(X,V) - \frac{r}{n-1}g(X,V) = -\frac{r}{n-1}g(X,V)$$

[by (6.4)]. From (6.6) and (6.7) it follows that

$$(6.8)\quad \begin{aligned}(\nabla_X P)(Y,V) &= -2A(X)[\frac{r}{n-1}g(Y,V)]- \\ &-B(Y)[\frac{r}{n-1}g(X,V)] + C(V)P(Y,X).\end{aligned}$$

Again

$$(6.9)\quad \begin{aligned}(\nabla_X P)(Y,V) &= \nabla_X P(Y,V) - P(\nabla_X Y,V) - P(Y,\nabla_X V) = \\ &= -\frac{1}{n-1}\,\mathrm{dr}\,(X)g(Y,V)\end{aligned}$$

[by (6.8) and (6.1)]. From (6.8) and (6.9) we get $\frac{1}{n-1}\,\mathrm{dr}\,(X)g(Y,V) = = 2A(X)\frac{r}{n-1}g(Y,V) + B(Y)\frac{r}{n-1}g(X,V) - C(V)P(Y,X)$ or

$$(6.10)\quad g(Y,V)[2rA(X) - \mathrm{dr}(X)] + B(X)rg(X,V) = (n-1)C(V)P(Y,X).$$

Putting $X = V$ in (6.10) we get

$$\begin{aligned}g(Y,V)[2rA(V) - \,\mathrm{dr}\,(V)] + B(Y)rg(V,V) &= (n-1)C(V)P(Y,V) = \\ &= (n-1)C(V)[-\frac{r}{n-1}g(Y,V)] = -rC(V)g(Y,V).\end{aligned}$$

Or

(6.11) $g(Y,V)[2rA(V) - \mathrm{dr}(V) + rC(V)] = -rB(Y)$ $[g(V,V) = 1]$

Since $r \neq 0$ and $B(Y)$ is assumed to be no-where zero, it follows from (6.11) that neither $g(Y,V) = 0$ or $2rA(V) - \mathrm{dr}(V) + rC(V) = 0$. Thus we see that V cannot be orthogonal to ρ, λ or μ for otherwise $g(\rho, V) = 0$ or $g(\lambda, V) = 0$ or $g(\mu, V) = 0$. This leads to the following theorem.

Theorem 7. *If a $G(PWRS)_n$ admits a unit parallel vector field V, then V cannot be orthogonal to any of the basic vector fields ρ, λ or μ. Further, the scalar $2rA(V) - \mathrm{dr}\,V + rC(V)$ cannot be zero.*

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SOME RESULTS ON CR–SUBMANIFOLDS OF A NEARLY COSYMPLECTIC MANIFOLD

BY

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0. Introduction. CR-submanifolds of a Kaehler manifold has been defined and studied by A. B e j a n c u [1] and since then many papers have appeared on this topic. On the other hand, CR-submanifolds of an almost contact manifold were studied by M. K o b a y a s h i [6], K. Y a n o and M. K o n [8], and the present authors [8]. In [4], D. B l a i r defined a nearly cosymplectic structure on an odd dimensional differentiable manifold and CR-submanifold of such manifolds were studied by I a n u s [5]. The aim of this note is to continue the study of CR-submanifolds of a nearly cosymplectic manifold.

1. Preliminaries. Let $\overline{M}$ be an n-dimensional almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g) satisfying

(1.1) $$\phi^2 X = -X + \eta(X)\xi,\ \eta(\xi) = 1,\ \phi \circ \xi = 0$$

(1.2) $$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y),\ g(X,\xi) = \eta(X)$$

where X and Y are vector fields on $\overline{M}$. Further, $\overline{M}$ is said to be a nearly cosymplectic manifold if

(1.3) $$(\overline{\nabla}_X \phi)(Y) + (\overline{\nabla}_Y \phi)(X) = 0$$

for any vector fields X and Y on $\overline{M}$, where $\overline{\nabla}$ denotes the Riemannian connection of g on $\overline{M}$.