From (2.9), we have

$$\nabla^{\perp}_{\phi X}\phi N_1 = \sum_{i=1}^{s} \alpha_i N_1 + \sum_{k=1}^{q} \beta_k E_k. \tag{2.14}$$

Since $\nabla^{\perp}_{\phi X}\phi N_1$ is normal to $M$, we have $\beta_k = 0$, $k = 1, \dots, q$.

Also, from (2.13), we have

$$\nabla^{\perp}_{\phi X}\phi N_1 \perp N_i.$$

Making use of it in (2.14), we get $\alpha_i = 0$, $i = 1, \dots, s$. Hence $\nabla^{\perp}_{\phi X}\phi N_1 = 0$, i.e. $\nabla^{\perp}_{X}\phi N_1 = 0$, which completes the required proof.

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# ABOUT $\mathcal{M}$ - TANGENT CONES IN LOCALLY CONVEX SPACES

BY

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**1. Introduction.** Let $C$ be a closed subset of a Banach space $E$ and $x \in C$. The *Clarke tangent cone* , $T(C,x)$ is the set of all $y \in E$ such that

$$\begin{cases} \forall\, Y \in \mathcal{V}(y)\ ,\ \exists X \in \mathcal{V}(x), \lambda > 0 \text{ such that} \\ \forall\, x' \in X \cap C\ ,\ \mu \in (0,\lambda) : (x' + \mu Y) \cap C \neq \emptyset, \end{cases} \tag{1.1}$$

where $\mathcal{V}(x)$ is the collection of all neighborhoods of $x$, respectively $y$. The *contingent cone* $K(C,x)$, is the set of all $y$ such that

$$\forall\ Y \in \mathcal{V}(y),\ \lambda > 0 : [x + (0,\lambda)Y] \cap C \neq \emptyset. \tag{1.2}$$

J.S.T r e i m a n [8] gave a new chracaterization of the tangent cone, derived using the method of B i s h o p and P h e l p s [2]. Also is proved the following inclusion:

$$\liminf_{x' \to x} K(C,x') \subset T(C,x). \tag{1.3}$$

where $B(y,\varepsilon)$ is the open $\varepsilon$ - ball around $y$.

To obtain similar results and their consequences in the case of a locally convex spaces, we used a new concept of limit for the multifunction. This concept (for a locally convex space E) uses a class of bounded subsets of E, and tries to substitute the fact that the neighborhoods of a point are not always bounded.

**2. Limit concept.** Throught this paper, the topology of the linear space $E$ is assumed to be locally convex and separated (Hausdorff). Let $S$

be a topological space and let $\mathcal{M}$ be a class of subsets of $E$. We denote by $\mathcal{V}(x), \mathcal{V}(s)$ the collection of all neighbourhoods of $x \in E$, $s \in S$ respectively.

**Definition 1.** *Let $F$ be any multifunction from $S$ to $E$. The set*

$$\Delta(s) = M = \liminf_{s' \to s} F(s')$$

*consists of all $y \in E$ such that:*

$$(2.1) \qquad \begin{cases} \forall Y \in \mathcal{V}(y),\ \exists M \in \mathcal{M},\ M \subset Y,\ U \in \mathcal{V}(0) \text{ such that} \\ \forall s \in U,\ M \cap F(s') \neq \emptyset. \end{cases}$$

**Remark 1.** $\Delta(s)$ will not be affected if $\mathcal{V}(y), \mathcal{V}(s)$ are replaced by a fundamental system of neighbourhoods $\mathcal{N}(y), \mathcal{N}(s)$ respectively, i.e. the condition (2.1) is a topological property of element $y$.

**Remark 2.** If $\mathcal{M}_1, \mathcal{M}_2$ are the classes of subsets of $E$, $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\Delta_i(s) = \mathcal{M}_i - \liminf_{s' \to s} F(s'), i = 1, 2$, then

$$\Delta_1(s) \subset \Delta_2(s).$$

**Proposition 1.** *Let $\mathcal{M}_0, \mathcal{M}$ be the classes of subsets of $E$ such that $\mathcal{M}_0 \subset \mathcal{M}$ and for all $y \in E$ exists a fundamental system of neighbourhoods $\mathcal{N}(y)$ such that*

$$(2.2) \qquad \forall Y \in \mathcal{N}(y),\ M \in \mathcal{M},\ M \subset Y,\ \exists M_0 \in \mathcal{M}_0;\ M \subset M_0 \subset Y.$$

*Then*

$$\mathcal{M} - \liminf_{s' \to s} F(s') = \mathcal{M}_0 - \liminf_{s' \to s} F(s').$$

**Proof.** It is obvious that $\Delta_0(s) \subset \Delta(s)$ if $\mathcal{M}_0 \subset \mathcal{M}$, where

$$\Delta_0(s) = \mathcal{M}_0 - \liminf_{s' \to s} F(s').$$

In view of condition (2.2) we also obtain that the reverse inclusion holds and so $\Delta(s) = \Delta_0(s)$.

**Remark 3.** In the special case when the family $\mathcal{M}$ is the class of all bounded subsets of locally convex space $E$, then $\mathcal{M}$ can be replaced by a fundamental system of bounded sets. In particular, we can consider only the class $\mathcal{M}_0$ of all bounded, convex, closed sets of $E$.

**Remark 4.** For any multifunction $F$ from $S$ to $E$ and for any class $\mathcal{M}$, according to Definition 1 it follows that $\mathcal{M} - \liminf_{s' \to s} F(s')$ is always a closed set.



**Remark 5.** If $\tau$ is the class of all open sets of $E$ then it is easy to verify that

$$\liminf_{s' \to s} F(s') = \tau - \liminf_{s' \to s} F(s')$$

where $\liminf_{s' \to s} F(s')$ is defined (see [7]) by definition as the set of all $y \in E$ such that

$$(2.4) \qquad \mathcal{M} - \liminf_{s' \to s} F(s') \subset \liminf_{s' \to s} F(s').$$

If $E$ is normed linear space and $\mathcal{M}$ is the class of all bounded subsets of $E$, then (2.4) is an equality since for every $Y \in \mathcal{V}(y)$ exists a set $M \in \mathcal{M}$ such that $M \subset Y$ and $M \in \mathcal{V}(s)$.

In general case the inclusion (2.4) can be strict.

For example, let be $S = E = R$ and $F(x) = [0,1] \cap (R \setminus Q)$, $\forall x \in R$, $M \in \{[a,b] \cap Q | a, b \in R\}$. We have

$$\mathcal{M} - \liminf_{x' \to x} F(x') = \emptyset$$

and

$$\liminf_{x' \to x} F(x) = [0,1], \forall x \in R.$$

**3. $\mathcal{M}$–tangent cones.** Let $F$ be a locally convex space and let $\mathcal{M}$ be a class of subsets of $E$ having the following properties:

$(M_1)$ $\forall V \in \mathcal{V}(0)$, $\exists M \in \mathcal{M}$ such that $0 \in M \subset V$.

$(M_2)$ $M \in \mathcal{M}$, $\alpha > 0$, $\alpha M \in \mathcal{M}$.

A typical exemple we have taxing $\mathcal{M} = \{\alpha M_0;\ \alpha > 0\}$ where $\mathcal{M}_0$ is a balanced bounded set in $E$.

**Definition.** *For any set $C \subset E$ and $x \in C$, the $\mathcal{M}$-tangent cone to $C$ at $x$ is the set defined as follows*

$$(3.1) \qquad \mathcal{M} - T(C, x) = \mathcal{M} - \liminf_{x' \to x} t^{-1}(C - x').$$

*More precisely, if $S$ is the toplogical space $C \times [0, \infty)$ and $F : S \to E$ is the multifunction defined by*

$$F(x', t) = \begin{cases} t^{-1}(C - x'), & \text{if } t > 0 \\ E, & \text{if } t = 0 \end{cases}$$

*then* $\mathcal{M} - T(C,x) = \mathcal{M} - \liminf_{x' \to x} F(x',t)$.

*For* $M = \tau$ *- the class of all open sets of* $E$, *we have*

$$\tau - T(C,x) = \liminf_{x' \to x} t^{-1}(C - x') = T(C,x),$$

*where* $y \in T(C,x)$ *if and only if* (1.1) *holds (see* [3]*). By Definition* 1 *(see the relation* (2.1)*) and Remark* 1 *we obtain*

(3.2) $$y \in \mathcal{M} - (C,x) \Longleftrightarrow$$

$$\begin{cases} \forall Y \in \mathcal{N}(y),\ \exists M \in \mathcal{M},\ M \subset Y,\ X \in \mathcal{N}(\alpha) \text{ such that} \\ \forall x' \in X \cap C,\ t \in (0,\lambda) : (x' + tM) \cap C \neq \emptyset, \end{cases}$$

*where* $\mathcal{N}(x)$ *is the system of all convex neighbourhoods of* $x \in E$.

**Theorem 1.** *Let* $\mathcal{M}$ *be a class of bounded subsets of* $E$ *having the properties* $(M_1), (M_2)$ *and the following property*

$(M_3)$ $\forall M_1, M_2 \in \mathcal{M},\ \alpha \in [0,1];\ \alpha M_1 + (1-\alpha)M_2 \in \mathcal{M}$.

*Then, for any set* $C \subset E$ *and any* $x \in C$ *the* $\mathcal{M}$*-tangent cone to* $C$ *at* $x$ *is a closed convex cone in* $E$ *containing the origin.*

**Proof.** It is clear that $0 \in \mathcal{M} - T(C,x)$ according to properties $(M_1)$ and (3.2). Also $\mathcal{M} - T(C,x)$ is closed by Remark 4. That $\mathcal{M} - T(C,x)$ is a cone it follows by $(M_2)$ and (3.2). We show now that $\mathcal{M} - T(C,x)$ is convex. Let be $y_1, y_2 \in \mathcal{M} - T(C,x), \alpha \in [0,1], y = \alpha y_1 + (1-\alpha)y_2$ and $Y \in \mathcal{N}(y)$. By (3.2) we have

$$\exists\, M_i \in \mathcal{M},\ M_i \subset Y,\ X_i \in \mathcal{N}(n),\ \lambda_i > 0,\ i = 1,2,$$

such that

(3.3) $$\forall x' \in X_i \cap C,\ t \in (0,\lambda_i) : (x' + tM_i) \cap C \neq \emptyset, i = 1,2.$$

If $\lambda_0 = \min\{\lambda_1, \lambda_2\}$ we choose $\lambda \in (0,\lambda_0), X \in \mathcal{N}(x)$ such that

(3.4) $$x' + \lambda M_i \subset X_1 \cap X_2,\ \forall x' \in X,\ i = 1,2.$$

For any $x' \in X \cap C,\ t \in (0,\lambda)$ we have by (3.3)

$$(x' + \alpha t M_1) \cap C \neq \emptyset.$$



Choose $x'' \in (x' + \alpha t M_1) \cap C$. Since (3.4) holds and $X_1 \cap X_2$ is convex , it follows $x'' \in X_2 \cap C$. Thus by (3.3)

$$[x'' + (1-\alpha)tM_2] \cap C \neq \emptyset$$

and

$$\{x' + t[\alpha M_1 + (1-\alpha)M_2]\} \cap C \neq \emptyset.$$

In view of $(M_3)$ and convexity of $Y$, we obtain

$$\alpha M_1 + (1-\alpha)M_2 \in \mathcal{M}$$

and

$$\alpha M_1 + (1-\alpha)M_2 \subset Y,$$

and therefore $y \in \mathcal{M} - T(C,x)$.

**4. A special characteristic of $\mathcal{M}$ tangent cone.** Let $\mathcal{M}$ be a class of closed convex, bounded subsets of $E$ having the properties $(M_1)$, $(M_2)$, $(M_3)$. For any closed $C \in E$ and $x \in C$, $M - T(C,x)$ is a closed convex cone into containing. Taking the contrapositive of the (3.2) we see that $y$ is in complement of $\mathcal{M} - T(C,x)$ if and only if there is a neighbourhood $Y \in \mathcal{N}(y)$ such that:

(4.1) $$\begin{cases} \forall\ M \in M, M \in Y, X \in \mathcal{N}(x), \lambda > 0 \\ \exists\ x' \in X \cap C, \mu \in (0,\lambda) : (x' + \mu M) \cap C) = \emptyset, \end{cases}$$

where $\mathcal{N}(x)$ is the system of all convex neighbourhood of $x \in E$. We show that this condition in the case of sequential complete locally convex spaces stronger one.

**Lemma 1.** *Let* $E$ *be sequentially complete locally convex space and let* $P(E)$ *be the set of all continuous seminorms of* $E$. *In order for* $y$ *to be in the complement of* $M - T(C,x)$ *it is necessary and sufficient that there be* $Y \in \mathcal{N}(y)$ *such that*

(4.2) $$\begin{cases} \forall\ M \in \mathcal{M},\ M \subset Y,\ X \in \mathcal{N}(x), \\ \exists\ x' \in X \cap C,\ \mu \in (0,\mu) : [x' + (0,\mu)M] \cap C) = \emptyset. \end{cases}$$

**Proof.** Since (4.2) implies (4.1), we need only demonstrate the necessity of the condition. Suppose that (4.1) holds for $Y \in N(y)$ and $M \in \mathcal{M}$, $M \in Y$, $M \neq \emptyset$, $X \in \mathcal{N}(x)$. We can find $x' \in X \cap C$, $\mu > 0$ such that

(4.3) $$x' + \mu M \subset X,$$

(4.4) $$(x'+\mu M)\cap C=\emptyset.$$

Now we order the points in $A=(x'+[0,\mu]M)\cap C$ as follows: for any $x_1,x_2\in A, x_1\ge x_2$ if and only if $x_2\in x_1+\eta M$ for some $\eta\ge 0$. Since $M$ is convex the relation " $\ge$ " is reflexive and transitive. Using this ordering we define a sequence $(x_m)\subset A, n\ge 1$, inductively.

Let $x_1=x'$. We have obtained $x_n\in A$. Since $0\in M-T(C,x)$ always, we can sypposе that $y\ne 0$ and $0\notin Y$, therefore $0\notin M$. Consequently

$$\alpha_m=\sup\{\alpha\ge 0|(x_m+\alpha M)\cap A\ne\emptyset\}$$

in infinte and nonnegative. Choose $x_{m+1}\in A$ such that $x_{m+1}\in x_m+\beta_m M$ were $\beta_m\ge 0$ and $0\le\alpha_m-\beta_m\le 2^{-(m+2)}$.

It is clearly that $x_{m+1}\ge x_m$, moreover for any $z\in A, z\ge x_m (m\ge 2)$. We have:

(4.5) $$z\in x_m+\beta M,\quad where\ \beta\le 2^{-(m+1)}$$

Indeed, if $z\in x_m+\beta M$ for a $\beta\ge 2^{-(m+1)}$ then, $x_m\in(x_{m-1}+\beta_{m-1}\mu)\cap A$ and $\alpha_{m-1}-\beta_{m-1}\le 2^{-(m+1)}$, it results $z\in[x_m,+(\beta+\beta_{m-1}M]$, where $\beta+\beta_{m+1}>\alpha_{m-1}$, in contradiction to the definition of $\alpha_{m-1}$.

Now, let $p\in P(E)$ and $d_p=\sup\{p(x)|x\in M\}$. In view of (4.5) we have

(4.6) $$p(z-x_m)\le d_p2^{-(m+1)}\ ,\ for\ all\ z\ge z_m, z\in A.$$

Because $x_{m+1}\ge x_m,\forall m\ge 1$, we have (by (4.6)) $x_k\ge x_m$ for all $k\ge m$ and so the sequence $(x_m)$ is a Cauchy sequence in seminorm $p$. An $p$ is arbitrary in P(E) and E is a sequential complet space it results that the sequence $(x_m)$ has a limit point $x''\in A$. So that, $x''\in x'+\beta H$ for a $\beta\in[0,\mu]$. Moreover $\beta<\mu$ since $(x'+\mu M)\cap C=\emptyset$.

Let show now that $x''$ is maximal under " $\ge$ ". If we suppose that there exists $z\in A$ such that $z\ge x'$, then $z\ge x_m$ for all $m\ge 1$ (because $x_{m+k}\in x_m+\beta_m M,\forall k\ge 0$ and for $k\to\infty$ it results $x''\in x_m+\beta_m M$, hence $x''\ge x_m$.

For every $p\in P(E)$, using (4.6) we have

$$p(z-x'')\le p(z-x_m)+p(x_m-x'')\le d_p2^{-m}\ \forall\ m\ge 1.$$

Hence $p(z-x'')=0$, $\forall\ p\in P(E)$ and $z=x''$.

Therefore $x''$ is maximal under " $\ge$ " and so, for some $\eta>0$

$$[x''+(0,\eta)M]\cap C=\emptyset.$$



As a direct corollary of Lemma 1 we obtain our chracterization of $\mathcal{M}-T(C,x)$ cone.

**Theorem 2.** *Let $E$ be sequentially complete locally convex space and let $C$ be a closed subset of $E$. $x\in C$. Then $y\in\mathcal{M}-T(C,x)$ if and only if*

(4.7) $$\begin{cases}\forall Y\in\mathcal{N}(y),\ \exists M\in\mathcal{M},\ M\subset Y,\ X\in\mathcal{N}(x)\ \text{such that,}\\ \forall x'\in X\cap C,\ \lambda>0:[x'+(\lambda,0)M]\cap C)=\emptyset.\end{cases}$$

**5. $\mathcal{M}$–contingent cones. Relationship between the cones.** Let $\mathcal{M}$ be a class of subsets of locally convex space $E$, having the properties $(M_1),(M_2)$. For any set $C=E$ and $x\in C$ the *$\mathcal{M}$-contingent to $C$ at $x$* is defined to be the set of all $y\in E$ such that

(5.1) $$\begin{cases}\forall Y\in\mathcal{N}(y),\ \exists M\in\mathcal{M},\ M\subset Y,\ \text{such that,}\\ \forall\lambda>0:[x+(\lambda,0)M]\cap C)=\emptyset,\end{cases}$$

For $\mathcal{M}=\tau$ we have $\tau-K(C,x)=K(C,x)$ where $y\in K(C,x)$ if and only if (1.2) is true.

**Remark 6.** $\mathcal{M}-K(C,x)$ is a closed cone in $E$ containing 0. From (3.2), (5.1) and (2.1) we have

(5.2) $$\mathcal{M}-T(C,x)\subset\mathcal{M}-K(C,x)\subset K(C,x)$$

If $E$ is a normed linear space and $M$ is the class of all bounded subsets of $E$ then,

$$M-T(C,x)=T(C,x),$$
$$M-K(C,x)=K(C,x).$$

(see Remark 5).

For any set $C\subset E$ and $x\in C$ the cone $K_0(C,x)$ is defined by

$$K_0(C,x)=\{y\in E|\forall\mu>0:[x+(0,\mu)y]\cap C\ne\emptyset\}.$$

If $\mathcal{M}=\mathcal{F}=\{F\subset E|F$ is finite set $\}$ then the relationship between $K_0(C,x)$ and $\mathcal{F}-K(C,x)$ is as follows.

**Proposition 2.** *For any set $C\subset E$ and $x\in C$, the following equality*

(5.4) $$cl\ K_0(C,x)=\mathcal{F}-K(C,x)$$

*holds.*

**Proof.** Obvious $K_0(C,x) \subset \mathcal{F} - K(C,x)$. Since $\mathcal{F} - K(C,x)$ is closed we have $cl\ K_0(C,x) \subset \mathcal{F} - K(C,x)$. Let be $y \in \mathcal{F} - K(C,x)$ and $Y \in \mathcal{N}(y)$. By property (5.1) there exists a set $F \in \mathcal{F}, F \subset Y$, such that

$$\forall\ \lambda > 0 : [x + (0,\lambda)F] \cap C \neq \emptyset.$$

Since $F$ is a finite set we find $y_0 \in F \subset Y$ such that

$$[x + (0,\lambda)y_0] \cap C \neq \emptyset\ ,\ \forall\ \lambda > 0,$$

and so $y_0 \in K_0(C,x) \cap Y$. Thus shows that $y \in cl\ K_0(C,x)$ as claimed.

**Remark 7.** $K_0(C,x)$ is a cone in $E$ containing 0. From (5.3) and (2.1) we have

$$K_0(C,x) \subset K(C,x). \tag{5.5}$$

We cam use Theorem 2 to show the relationship between the $\mathcal{M}$-tangent cone and the cone $K_0$.

**Theorem 3.** *Let $E$ be a sequentially complete locally convex space and let $C$ a closed subset of $E$ and $x \in C$. Then*

$$\mathcal{M} - \liminf_{x' \to x} K_0(C,x') \subset \mathcal{M} - T(C,x) \tag{5.6}$$

*for every class $\mathcal{M}$ of bounded, closed, convex subsets of $E$ having the properties $(M_1) - (M_3)$.*

**Proof.** We will show that if $y \notin \mathcal{M} - T(C,x)$, then $y \notin \mathcal{M} - \liminf\limits_{x' \to x} K(C,x')$.

Indeed, if $y \notin \mathcal{M} - T(C,x)$, by Lemma 1, there exists $Y \in \mathcal{N}(y)$ such that (4.2) is true. This implies

$$\begin{cases} \exists\ Y \in \mathcal{N}(y)\ \ \textit{such that}\ \forall\ M \in \mathcal{M},\ M \subset Y,\ X \in \mathcal{N}(x), \\ \exists x' \in X \cap C : M \cap K_0(C,x') = \emptyset. \end{cases} \tag{5.7}$$

Thus $y \in \mathcal{M} - \liminf\limits_{x' \to x} K(C,x')$.

The results of this paper can be used to the study of generalized directional derivatives and to the study of generalized gradients of lower semicontinuous functions on locally convex spaces obtaining similar results of those in [7], [8], [9].



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