We find the value $r_0 = r_0(\alpha, p_i, \beta)$ and the corresponding integer $n_0(r_0)$ so that

$$(4.9)\quad \sum_{i=1}^{k} \frac{i(i+2-\beta)2\alpha p_i}{i+1+i\alpha-\alpha} r_0^{i+1} + \frac{n_0(n_0+2-\beta)2\alpha\left(1-\sum_{i=1}^{k} p_i\right)}{n_0+1+n_0\alpha-\alpha} r_0^{n_0+1} = 1-\beta.$$

Then this value r_0 is the radius of meromorphically convex of order β for functions f belonging to the class $\Sigma_{p_k}[\alpha]$.

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SOME CHARACTERISATIONS OF FINITE π–SOLVABLE AND p–SOLVABLE GROUPS WITH THE HELP OF PRIMITIVE GROUP

BY

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1. Introduction. One of the main goals in finite group theory is to study the maximal subgroups because the knowledge of the maximal subgroups of the finite group often gives us a good information about the group itself. M u k h e r j e e and B h a t t a c h a r y a [14] introduced the concept of θ–pairs for a maximal subgroup of a finite group, which is motivated by the interesting concept of the Index Complex defined in D e s k i n s [6]–[7]. In [11] , we have shown how some conditions imposed on maximal θ–pairs for a maximal subgroup M of a finite group G, characterize the solvability of the group. We introduced a characteristic subgroup $S_P(G)$ in [9] and studied its influences on π–solvable and supersolvable group in [10]. We also introduced another characteristic subgroup $B_p(G)$ in [8]. B a l l e s t e r–B o l i n c h e s [4] studied the structure of a group by using its primitive permutation representation and the normal index (introduced by D e s k i n s in [6]) of its maximal subgroups. The purpose of this paper is to obtain some results about π–solvable and p-solvable group by applying the primitive group theory. All groups considered here are finite and we use standard notation as found in G o r e n s t e i n [12]. Sometimes, we denote $M < G$ to indicate that M is a maximal subgroup of G.

2. Preliminaries. In this section, we collect some well–known definitons and results (whose proofs we omit) that are needed in the sequel.

Definition. *For a group G,* Soc(G), *the socle of G is defined to be the product of all the minimal normal subgroups of G. if $G \neq 1$; and* Soc$(G) = 1$ *if $G = 1$. A primitive group is a group G such that for some* *maximal subgroup M of G, $M_G = 1$ (where M_G is the intersection of all G-conjugates of M, i.e., the unique largest normal subgroup of G contained in M).*

A primitive group is one of the following types.

(1) Soc(G) *the socle of G, is an abelian minimal normal subgroup of G, complemented by M.*

(2) Soc(G) *is a non-abelian minimal normal subgroup of G.*

(3) Soc(G) *is the direct product of the two minimal normal subgroups of G which are both non-abelian and complemented by M.*

A primitive group of type 2 with a core-free maximal subgroup complementing the socle is called a primitive group with small maximal subgroups. The proofs of some of our results depend on a lemma of L a f u e n t e [13], which we state below.

(2.1) *Suppose that G is a primitive group with small maximal subgroups. If M is a core-free maximal subgroup of G compementing the socle of G then M is a primitive group of type 2 and the simple component of* Soc(G) *is isomorphic to a section of the simple component of* Soc(M).

Definition. *Given a maximal subgroup M of a group G, let*

$$\theta(M) = \{(C,D) : C \leq G, D \lhd G, D \subset C, < M, C >= G, < M, D >= M$$
$$\text{and } C/D \text{ contains properly no normal subgroup of } G/D\}.$$

We call any pair (C, D) in $\theta(M)$ a θ-pair. A partial order $\leq$ may be defined on $\theta(M)$ as follows:

$$(C, D) \leq (C', D') \quad if \quad C \subseteq C',$$

no condition is placed on the second component of the pairs. (One notices that using the definiton of $\theta(M)$, it follows that $D \subset D'$. Also, $C = C'$ implies that $D = D'$). Obviously $\theta(M)$ will contain maximal elements with respect to this ordering. We shall call a maximal element a maximal θ-pair.

(2.2) [14, Lemma 2.1] *If (C, D) is a maximal θ-pair in $\theta(M)$ and $N \Delta G$, $N \subset D$, then $(C/N, D/N)$ is a maximal θ-pair in $\theta(M/N)$. Conversely, if $(C/N, D/N)$ is a maximal θ-pair in $\theta(M/N)$, then (C, D) is a maximal θ-pair in $\theta(M)$.*

Definition. *Let H and K be two normal subgroups of a group G with $K \subset H$. Then the factor group H/K is called a chief factor of G, if there is no normal subgroup N of G such that $K \subset N \subset H$ with proper inclusion. Let M be a maximal subgroup of G. H is said to be a normal supplement of M in G if $MH = G$. The normal index of M in G is defined as the order of a chief factor H/K, where H is minimal in the set of all normal supplements of M in G and is denoted by $\eta(G : M)$.*

(2.3) [1, Lemma 2] *If N is a normal subgroup of a group G and M is a maximal subgroup of G such that $N \subseteq M$, then $\eta(G/N : M/N) = \eta(G : M)$.*

Definition. *Let G be any group and p be any prime. Define three characteristic subgroups of G as follows:*

$$\begin{aligned} B_p(G) &= \cap\{M : M \in \beta_p(G)\} \\ \phi_p(G) &= \cap\{M : M \in \gamma_p(G)\} \\ S_p(G) &= \cap\{M : M \in \Sigma_p(G)\} \end{aligned}$$

$$\begin{aligned} \textit{where} \quad & B_p(G) = \{M < G : [G : M]_p = 1 \quad \textit{and } \eta(G : M) \textit{ is composite}\} \\ & \gamma_p(G) = \{M < G : [G : M]_p = 1\} \\ \textit{and} \quad & \Sigma_p(G) = \{M < G : [G : M]_p = 1 \quad \textit{and } [G : M] \textit{ is composite}\}. \end{aligned}$$

In case $\beta_p(G)$ is empty then we define $B_p(G) = G$ and the same thing is done for the other subgroups. Note that $\phi_p(G) \subseteq B_p(G)$.

(2.4) [8, Theorem 3.6] *$B_p(G)$ is solvable and so $\phi_p(G)$ is solvable.*

Definition. *A group G is p-closed if it has a normal Sylow p-subgroup, where p is any prime dividing $|G|$.*

3. Some results on π-solvable and p-solvable groups.

Theorem 3.1. *If a group G has a π-solvable maximal subgroup M such that the index of each maximal pair (C, D) in $\theta(M)$ is equal to $[G : M]$ then G is π-solvable.*

Proof. We use induction on $|G|$. If G is simple then the index of the maximal pair $(G, < 1 >)$ in $\theta(M)$ is equal to $[G : M]$ and consequently $M =< 1 >$. Hence G is cyclic and so it is π-solvable.

We now assume that G is not simple. Let $M_G \neq < 1 >$. Let N be a minimal normal subgroup of G such that $N \subseteq M_G$. Then, by induction, G/N is π-solvable, and π-solvability of M implies that N is π-solvable. Hence G is a π-solvable group and the theorem is proved. Thus we can assume that $M_G = 1$. Then G is a primitive group. Suppose that G is a primitive group of type 3. Then Soc(G) is a direct product of two minimal normal subgroups N_1 and N_2 of G, which are both non-abelian and complemented M. So $G/N_i \cong M$ $(i = 1,2)$ and hence both G/N_1 and G/N_2 are π-solvable groups. Consequently, G is a π-solvable group. Next we assume that G is a primitive group of type 2. Then $\mathrm{Soc}(G)$ is a non abelian minimal normal subgroup of G. Clearly $\mathrm{Soc}(G) \not\subset M$ and so $G = M\,\mathrm{Soc}(G)$. It can be easily verified that $(\mathrm{Soc}(G), <1>)$ is maximal pair in $\theta(M)$. By hypothesis $[G : M] = |\mathrm{Soc}(G)|$ and so $M \cap \mathrm{Soc}(G) = <1>$. Thus G is a primitive group of type 2 with a core-free maximal subgroup M complementing the socle and so it is a primitive group with small maximal subgroup M. Then, by (2.1), M is a primitive group of type 2 and the simple component of $\mathrm{Soc}(G)$ is isomorphic to a section of the simple component of $\mathrm{Soc}(M)$. Since M is π-solvable, $\mathrm{Soc}(M)$ is a π'-group and therefore $\mathrm{Soc}(G)$ is a π'-group. Since $G/\mathrm{Soc}(G) \cong M$, $G/\mathrm{Soc}(G)$ is a π-solvable group and cnsequently G is a π-solvable group. Lastly we assume that G is a primitive group of type 1. Then $\mathrm{Soc}(G)$ is an abelian minimal normal sugroup of G, complemented by M. So $G/\mathrm{Soc}(G) \cong M$ and consequently $G/\mathrm{Soc}(G)$ is a π-solvable group. Hence G is a π-solvable group. This completes the proof.

Theorem 3.2. *If a group G has two distinct π-solvable maximal subgroups M_1 and M_2 such that*

$$[C : D] = [G : M_1] = [G : M_2]$$

for each common maximal pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$ then G is π-solvable.

The proof of the Theorem 3.2 is same as that of Theorem 3.1 and so we omit it.

Theorem 3.3. *Let G be a group with two distinct maximal subgroups M_1 and M_2. Then G is π-solvable if and only if M_1 and M_2 are π-solvable and C/D is π-solvable for each common maximal θ-pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$.*

Proof. Let G satisfy the hypothesis of the theorem. We have to show that G is π-solvable. If G is simple, then $(G, <1>)$ is a common maximal pair in $\theta(M_1)$ and $\theta(M_2)$. So by hypothesis $G = G/<1>$ is π-solvable. We now assume that G is not simple. Let $L_i = \mathrm{core}_G(M_i)$ $(i = 1,2)$. First we suppose that $L_1 = <1>$ but $L_2 \neq <1>$. Since $L_2 \subset M_2$ and M_2 is π-solvable, L_2 is π-solvable. Also $L_2 \not\subset M_1$ as $\mathrm{core}_G(M_1) = <1>$. So $G = M_1 L_2$ and consequently $G/L_2 \cong M_1/M_1 \cap L_2$. Since M_1 is π-solvable, so is G/L_2. Hence G is π-solvable. If $L_1 \neq <1>$ but $L_2 = <1>$ then as above we can show that G is π-solvable. Next we suppose that $L_1 \neq <1>$ and $L_2 \neq <1>$. If $L_1 \cap L_2 = <1>$, then let N be a minimal normal subgroup of G such that $N \subseteq L_1$. So $N \not\subset M_2$ and consequently $G = M_2 N$. This implies that $G/N \cong M_2/M_2 \cap N$ and since M_2 is π-solvable, it follows that G/N is π-solvable. Also N is π-solvable. Hence G is π-solvable. If $L_1 \cap L_2 \neq <1>$ then let N be a minimal normal subgroup of G such that $N \subseteq L_1 \cap L_2$. It can be easily verified that G/N satisfies the hypothesis of the theorem. By induction G/N is π-solvable. Also it follows that N is π-solvable. Hence G is π-solvable. Lastly we suppose that $L_1 = <1>$ and $L_2 = <1>$. Then G is a primitive group. If G is a primitive group of type 3 then $\mathrm{Soc}(G)$ is the direct product of two minimal normal subgroup N_1 and N_2 of G. Clearly both G/N_1 and G/N_2 are π-solvable and hence so is G. If G is a primitive group of type 2 then $\mathrm{Soc}(G)$ is a non-abelian minimal normal subgroup of G. As $\mathrm{Soc}(G) \not\subset M_i$ $(i = 1,2)$, $G = M_i\,\mathrm{Soc}(G)$, and so $G/\mathrm{Soc}(G) \cong M_i/M_i \cap \mathrm{Soc}(G)$. This implies that $G/\mathrm{Soc}(G)$ is π-solvable. It can be checked that $(\mathrm{Soc}(G), <1>)$ is a common maximal pair in $\theta(M_1)$ and $\theta(M_2)$. By hypothesis, $\mathrm{Soc}(G) = \mathrm{Soc}(G)/<1>$ is π-solvable. Hence G is π-solvable. If G is a primitive group of type 1 then $\mathrm{Soc}(G)$ is an abelian minimal normal subgroup of G, complemented by M_1. Since M_1 is π-solvable it follows that $G/\mathrm{Soc}(G)$ is π-solvable and hence G is π-solvable. The converse follows trivially.

Theorem 3.4. *Let G be a group with two distinct π-solvable maximal subgroups M_1 and M_2. Then any one of the following conditions implies the π-solvability of G.*

(i) *$B_p(G/D) \neq <1>$ for each common maximal θ-pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$.*

(ii) *$L(G/D) \neq <1>$ for each common maximal θ-pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$.*

(iii) *$C_{G/D}(C/D) \neq <1>$ for each common normal maximal θ-pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$.*

Proof. (i) We use induction on $|G|$. If G is simple then $(G, <1>)$ is a common maximal θ-pair in $\theta(M_1)$ and $\theta(M_2)$ and so by hypothesis $B_p(G) \neq <1>$ and consquently $B_p(G) = G$. This shows that G is π-solvable. We now assume that G is not simple. Let $L_i = \mathrm{core}_G(M_i)$ $(i = 1,2)$. If either $L_1 = <1>$ or $L_2 = <1>$ but not both, then as in the proof of Theorem 3.3, we can show that G is π-solvable. We now assume that $L_1 \neq <1>$ and $L_2 \neq <1>$. If $L_1 \cap L_2 = <1>$ then as in the proof of Theorem 3.3, we have G is π-solvable. So let $L_1 \cap L_2 \neq <1>$ and N be a minimal normal subgroup of G such that

$$N \subseteq L_1 \cap L_2.$$

It can be verified that G/N satisfies the hypothesis of the theorem. By induction, G/N is π-solvable. Since $N \subseteq M_1$ and M_1 is π-solvable, N is π-solvable. Hence G is π-solvable. Next assume that $L_1 =< 1 >$ and $L_2 =< 1 >$. Then G is a primitive group. If G is a primitive group of type 1 or type 3 as in the proof of Theorem 3.3, we can show that G is π-solvable. Lastly we suppose that G is a primitive group of type 2. Then $\mathrm{Soc}(G)$ is a non-abelian minimal normal subgroup of G. Also $\mathrm{Soc}(G) \not\subseteq M_i$ $(i = 1, 2)$ and so $G = M_1 \mathrm{Soc}(G)$. Clearly, $(\mathrm{Soc}(G), < 1 >)$ is a common maximal pair in $\theta(M_1)$ and $\theta(M_2)$. By hypothesis $B_p(G) \neq < 1 >$. Since $\mathrm{Soc}(G)$ is the unique minimal normal subgroup of G, so it follows that $\mathrm{Soc}(G) \subseteq B_p(G)$ and hence by (2.4), $\mathrm{Soc}(G)$ is π-solvable. Since $G/\mathrm{Soc}(G) \cong M_1/M_1 \cap \mathrm{Soc}(G)$ and M_1 is π-solvable, $G/\mathrm{Soc}(G)$ is π-solvable. Hence G is π-solvable. This completes the proof of (i).

The proofs of the remaining parts of the theorem are similar to that of (i) and so we omit them.

Theorem 3.5. *Let G be a group which is not simple. Then the following conditions are equivalent.*

(i) *G is π-solvable.*

(ii) *G has a π-solvable maximal subgroup M such that for each maximal θ-pair (C, D) in $\theta(M)$, $\mathrm{Soc}(G/D)$ is a π-solvable subgroup of G/D.*

(iii) *$\mathrm{Soc}(G/D)$ is a π-solvable subgroup of G/D for each maximal θ-pair (C, D) in $\theta(M)$ and for each M in $\beta_p(G)$.*

(iv) *$\mathrm{Soc}(G/D)$ is a π-solvable subgroup of G/D for each common maximal θ-pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$ where M_1 and M_2 are two distinct maximal subgroups of G.*

Proof. We shall prove that (i) and (ii) are equivalent. (i) implies (ii) holds trivially. We now assume that (ii) holds. Let $L = \mathrm{core}_G(M) \neq < 1 >$. Then L is π-solvable. It can be checked that G/L satisfies the hypothesis of the theorem. By induction, G/L is π-solvable. Hence G is π-solvable. So $L = \mathrm{core}_G(M) =< 1 >$ and consequently G is a primitive group. If G is a primitive group of type 1 or type 3 then as in the proof of Theorem 3.1 we can show that G is π-solvable. So we assume that G is a primitive group of type 2. Then $\mathrm{Soc}(G)$ is a non-abelian minimal normal subgroup of G and $\mathrm{Soc}(G) \not\subseteq M$. So $G = M\,\mathrm{Soc}(G)$ and this implies that $G/\mathrm{Soc}(G) \cong M/M \cap \mathrm{Soc}(G)$ and consequently $G/\mathrm{Soc}(G)$ is π-solvable. It can be verified that $(\mathrm{Soc}(G), < 1 >)$ is a maximal pair in $\theta(M)$. By hypothesis, $\mathrm{Soc}(G/ < 1 >) = \mathrm{Soc}(G)$ is a π-solvable subgroup of G. Hence G is π-solvable. This completes the proof of the equivalence of (i) and (ii).

Similarly, we can prove that each of the conditions (iii) and (iv) is equivalent to (i).

The proof of the following theorems (Theorem 3.6 and Theorem 3.7) are similar to the proof of Theorem 3.5 and so we omit them.

Theorem 3.6. *Let G be a p-solvable group and not simple. Then G is π-solvable if and only if $\mathrm{Soc}(G/D)$ is a π-solvable subgroup of G/D for each common maximal pair in $\theta(M_1)$ and $\theta(M_2)$ where M_1 and M_2 are two distinct maximal subgroups of G, belonging to $\beta_p(G)$.*

Theorem 3.7. *Let G be not a simple group with two distinct maximal subgroups M_1 and M_2. Then G is π-solvable if and only if M_1 and M_2 are π-solvable and $\mathrm{Soc}(G/D)$ is a π-solvable subgroup of G/D for each common maximal pair (C, D) in $\theta(M_1)$ and $\theta(M_2)$.*

Theorem 3.8. *A group G is a p-solvable if the p-part of the index of each maximal pair (C, D) in $\theta(M)$ for every M in $\beta_p(G)$ is equal to $[G : M]_p$.*

Proof. Let G be a non-p-solvable group of minimal order among those satisfying the conditions of the theorem. If $\beta_p(G)$ is empty then $G = B_p(G)$ and so G is solvable by (2.4) and hence it is p-solvable, a contradiction. So we assume that $\beta_p(G)$ is non-empty. If G is simple then $(G, < 1 >)$ is a maximal pair in $\theta(M)$ for each M in $\beta_p(G)$ and so by hypothess $|G|_p = 1$ and consequently G is a p-solvable group, which is a contradiction. So let G be not simple and N be a minimal normal subgroup of G. It can be verified that G/N satisfies the hypothesis of the theorem and so by the minimality of G, G/N is p-solvable. Without loss of generality, we may assume that N is the unique minimal normal subgroup of G. If $N \subseteq B_p(G)$ then N is solvable by (2.4) and hence it is p-solvable and consequently G is p-solvable, a contradiction. If $N \not\subseteq B_p(G)$ then there exists a maximal subgroup M in $\beta_p(G)$ such that $N \not\subseteq M$ and so $\mathrm{core}_G(M) =< 1 >$. Thus G is a primitive group and $\mathrm{Soc}(G)$ is the unique minimal normal sugbroup of G and consequently $N = \mathrm{Soc}(G)$. Then G is either a primitive group of type 1 or of type 2. If G is a primitive group of type 1 then since $\mathrm{Soc}(G)$ is abelian and $G/\mathrm{Soc}(G)$ is a p-solvable group, G is a p-solvable group, a contradiction. We now assume that G is a primitive group of type 2. Then $(\mathrm{Soc}(G), < 1 >)$ is a maximal pair in $\theta(M)$ and so by hypothesis $|\mathrm{Soc}(G)|_p = 1$, that is, $\mathrm{Soc}(G)$ is a p'-group and consequently G is p-solvable, a contradiction. This completes the proof of the theorem.

The proof of the following theorem is analogous to the proof of Theorem 3.8 and so we omit it.

Theorem 3.9. *Let p be the largest prime dividing the order of a group. Then G is p-solvable if the p-part of the index of each maximal pair (C, D) in $\theta(M)$ for every M in $\Sigma_p(G)$ is equal to $[G : M]_p$.*

Theorem 3.10. *Suppose that G has a core-free maximal subgroup M such that M is p-closed, where p is a prime dividing $|M|$. Further suppose that $[C : D]_p = [G : M]_p$ for each maximal pair (C, D) in $\theta(M)$. Then G is p-solvable.*

Proof. Assume that the theorem is false and let G be a counter example of minimal order. If G is simple then $(G, < 1 >)$ is a maximal θ-pair in $\theta(M)$ and so by hypothesis $|G|_p = [G : M]_p$. Since M is p-closed, it has a normal Sylow p-subgroup P. It can be verified that P is a Sylow p-subgroup of G. Hence $[G : M]_p = 1$. This shows that G is a p'-group and consequently G is p-solvable, a contradiction. Hence G is not simple. Since G has a core-free maximal subgroup M, it is a primitive group. Suppose G is a primitive group of type 1. Then $\mathrm{Soc}(G)$ is an abelian minimal normal subgroup of G complemented by M and so $G/\mathrm{Soc}(G) \cong M$. This implies that $G/\mathrm{Soc}(G)$ is p-solvable and consequently so is G, a contradiction. Next, we suppose that G is a primitive group of type 3. Then $\mathrm{Soc}(G)$ is the direct product of two minimal normal subgroups A and B of G which are both non-abelian and complemented by M. So $G/A \cong M$ and $G/B \cong M$. This shows that both G/A ad G/B are p-solvable. Since $G = G/A \cap B$ is isomorphic to a subgroup of the p-solvable group $G/A \times G/B$, G is p-solvable, a contradiction. Lastly, we suppose that G is a primitive group of type 2. Then $\mathrm{Soc}(G)$ is a non-abelian minimal normal subgroup of G. Since M is core-free, $\mathrm{Soc}(G) \not\subset M$ and hence $G = M\,\mathrm{Soc}(G)$. This implies that $G/\mathrm{Soc}(G) \cong M/M \cap \mathrm{Soc}(G)$ and consequently $G/\mathrm{Soc}(G)$ is p-solvable. It can be checked that $(\mathrm{Soc}(G), < 1 >)$ is a maximal θ-pair in $\theta(M)$ and so by hypothesis $|\mathrm{Soc}(G)|_p = [G : M]_p$. Since M is p-closed, it has a normal Sylow p-subgroup P. Again since M is core-free, it follows that P is a Sylow p-subgroup of G and so $[G : M]_p = 1$. Consequently, $\mathrm{Soc}(G)$ is a p'-group. Hence G is p-solvable, a contradiction. This completes the proof of the theorem.

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