Conversely suppose B is not connected, choose $z_0, z_1 \in B$, $z_2 \notin B$ with $z_0 < z_2 < z_1$. Assume that $f(z)$ and $g(z)$ are not both equal to $1/z$. Using the notation of (8) except that we fix $z = z_2$ and allow λ to vary,

$$t(\lambda) = t(z_2, \lambda) = 1 + \lambda \sum_{n=1}^{\infty} a_n(z_0^{n+1} - z_2^{n+1}) + (1-\lambda) \sum_{n=1}^{\infty} b_n(z_1^{n+1} - z_2^{n+1}).$$

Since $t(z_2, 0) > 1$ and $t(z_2, 1) < 1$, there exists a λ_0, $0 < \lambda_0 < 1$, such that $t(z_2, \lambda_0) = 1$. Hence $h(z) \in \Sigma^{**}_{N_0}(\beta, z_2)$ for $\lambda = \lambda_0$. Since $z_2 \notin B$, applying above lemma, it follows that $h(z) \notin \Sigma^{**}_{N_0}(\beta, B)$ which is a contradiction. Hence B is connected.

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A CLASSIFICATION OF ALMOST CONTACT METRIC SUBMERSIONS BASED ON THE FIBRES

BY

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0. Introduction. It is well known that the fibres of an almost contact metric submersion of type I, in the sense of W a t s o n [17], are almost Hermitian manifolds. Looking through this type of fibration, one can obtain a large number of results such as the following:

Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is cosymplectic, quasi–Sasakian or a Kenmotsu manifold, then the fibres are Kählerian.

This result shows that, taken as a total space of an almost contact metric submersion of type I, many different almost contact metric manifolds may yield a determined class of the fibres. Indeed, since the fibres are almost Hermitian manifolds, they have to belong to one of the sixteen classes obtained by G r a y and H e r v e l l a in [7].

According to the classification of Gray and Hervella, we attempt to produce a classification of almost contact metric submersions of type I; this problem was brought up in [14, p.716]. Among the defining relations of the total space, we bring out those which are common and emerge in forcing the fibres to belong to the same class.

In Section 1, we recall some fundamental notions of almost Hermitian and almost contact metric structures which will be used in this text.

Section 2 is specifically devoted to the classification in which we generalize many results of [2, Th. 2.1], [14, Section 3] and [17, Th. 3.1].

In this text, arbitrary vector fields of $\mathcal{X}(M)$, the Lie algebra of smooth vector fields, will be denoted by D, E and G while those of the vertical distribution $V(M)$ will be U, V and W.

1. Preliminaries. An almost Hermitian manifold is a Riemannian manifold (M, g) equipped with a tensor field J of type $(1, 1)$ satisfying:

(i) $J^2D = -D$, *and* (ii) $g(JD, JE) = g(D, E)$, *for all* $D, E \in \mathcal{X}(M)$.

Any almost Hermitian manifold (M, g, J) admits a differential 2-form F, defined by $F(D, E) = g(D, JE)$ and called the *fundamental form* or the *Kähler form*. Almost Hermitian manifolds are orientable and of even dimension which we will denote by $2m$. The Lie form Θ, of an almost Hermitian manifold is defined by $\Theta(D) = \frac{1}{m-1}\delta F(JD)$, where δ denotes the coderivative. From the classification of G r a y and H e r v e l l a [7], we recall the defining relations of some classes which will be needed.

An almost Hermitian manifold (M, g, J) is said to be:

(a) **Kählerian** *if* $dF(D, E, G) = 0 = N$, *where N is the Nijenhuis tensor;*

(b) **nearly Kählerian** (*or W_1-manifold*) *if* $(\nabla_D F)(D, E) = 0$;

(c) **almost Kählerian** (*or W_2-manifold*) *if* $dF(D, E, G) = 0$;

(d) **quasi Kählerian** (*or $W_1 \oplus W_2$-manifold*) *if* $(\nabla_D F)(E, G) + (\nabla_{JD} F)(JE, G) = 0$;

(e) **W_3–manifold** *if* $(\nabla_D F)(E, G) - (\nabla_{JD} F)(JE, G) = 0 = \delta F$;

(f) **$W_1 \oplus W_3$–manifold** *if* $(\nabla_D F)(D, E) - (\nabla_{JD} F)(JD, E) = 0 = \delta F$;

(g) **G_1–manifold** *if* $(\nabla_D F)(D, E) - (\nabla_{JD} F)(JD, E) = 0$;

(h) **G_2–manifold** *if* $\sigma\{(\nabla_D F)(E, G) - (\nabla_{JD} F)(JE, G)\} = 0$, *where σ denotes the cyclic sum over D, E and G:*

(i) **locally conformal Kählerian** (*or W_4-manifold*) *if* $dF = F \cap \Theta$, $N = 0$;

(j) **locally conformal almost Kählerian** (*$W_2 \oplus W_4$-manifold*) *if* $dF = F \cap \Theta$;

(k) **$W_2 \oplus W_3$–manifold** *if* $\sigma\{(\nabla_D F)(E, G) - (\nabla_{JD} F)(JE, G)\} = 0 = \delta F$;

(l) **semi–Kählerian** (*or $W_1 \oplus W_2 \oplus W_3$-manifold*) *if* $\delta F = 0$;

(m) **$W_1 \oplus W_4$–manifold** *if* $(\nabla_D F)(D, E) = \frac{-1}{2(m-1)}\{\|D\|^2\delta F(E) - g(D, E)\delta F(D) - g(JD, E)\delta F(JD)\}$;

(n) **$W_1 \oplus W_2 \oplus W_4$–manifold** *if* $(\nabla_D F)(E, G) + (\nabla_{JD} F)(JE, G) = \frac{-1}{m-1}\{g(D, E)\delta F(G) - g(D, G)\delta F(E) + g(D, JG)\delta F(JE) - g(D, JE)\delta F(JG)\}$.

An almost contact structure on a differentiable manifold M is a triple (φ, ξ, η) where:

(i) *ξ is a distinguished vector field,*

(ii) *η is a differential 1-form such that $\eta(\xi) = 1$, and*

(iii) *φ is a tensor field of type $(1, 1)$ satisfying $\varphi^2 D = -D + \eta(D)\xi$, for all $D \in \mathcal{X}(M)$.*

If in addition, M admits a Riemannian metric g such that $g(\varphi D, \varphi E) = g(D, E) - \eta(D)\eta(E)$, then g is called a compatible metric and $(M, g, \varphi, \xi, \eta)$ is called an *almost contact metric manifold* [12]. Almost contact metric manifolds are of odd dimension. The fundamental 2-form Φ, of an almost contact metric manifold is defined by $\Phi(D, E) = g(D, \varphi E)$. If we denote by $\{E_1, \ldots, E_m, \varphi E_1, \ldots, \varphi E_m, \xi\}$ a local φ-basis of an open subset of M, then the coderivative δ is given by:

$$\delta\Phi(D) = -\sum_{i=1}^{m}\{(\nabla_{E_i}\Phi)(E_i, D) + (\nabla_{\varphi E_i}\Phi)(\varphi E_i, D)\} - (\nabla_\xi \Phi)(\xi, D); \tag{1.1}$$

$$\delta\eta = -\sum_{i=1}^{m}\{(\nabla_{E_i}\eta)E_i + (\nabla_{\varphi E_i}\eta)\varphi E_i\}. \tag{1.2}$$

In [10], J.A. O u b i ñ a has defined a 1-form ω by setting $\omega = \frac{1}{m}(\varphi^*(\delta\Phi) - (\delta\eta)\eta)$ which resembles to the Lee form.
In [13], S a s a k i and H a t a k e y a m a have defined two tensors N_1 and N_2 by:

$N_1(D, E) = [\varphi, \varphi](D, E) + 2d\eta(D, E)\xi$;

$N_2(D, E) = (\mathcal{L}_{\varphi D}\eta)E - (\mathcal{L}_{\varphi E}\eta)D$, where $\mathcal{L}$ denotes the Lie derivative.

If $N_1 = 0$, the almost contact metric structure is said to be normal and in this case, $N_2 = 0$.

Following G r a y and H e r v e l l a [7] in the classification of almost Hermitian structures, C h i n e a and G o n z a l e z [5] have obtained a classification of almost contact metric structures. Note that, among the 4096 classes of these structures, only a few number of defining relations have been identified. We recall those which will be used in this paper.

An almost contact metric manifold is said to be:

(1) **almost cosymplectic** *if* $d\Phi = 0 = d\eta$;

(2) **cosymplectic** *if* $d\Phi = 0 = d\eta = N_1$;

(3) **closely cosymplectic** *if* $(\nabla_D\varphi)D = 0 = d\eta$;

(4) **nearly cosymplectic** *if* $(\nabla_D\varphi)D = 0$;

(5) **nearly–K–cosymplectic** *if* $(\nabla_D\varphi)E + (\nabla_E\varphi)D = 0 = \nabla_D\xi$;

(6) **quasi–K–cosymplectic** *if* $(\nabla_D\varphi)D + (\nabla_{\varphi D}\varphi)\varphi E - \eta(E)(\nabla_{\varphi D}\xi) = 0$;

(7) **G_2–semi–cosymplectic** *if* $d\Phi = 0 = d\eta$ *and* $\sigma\{(\nabla_D\Phi)(E,G) - (\nabla_{\varphi D}\Phi)(\varphi E, G) - \eta(E)(\nabla_{\varphi d}\eta)G\} = 0$;

(8) **locally conformal almost cosymplectic** *if* $d\Phi = -2\Phi \wedge \omega$ *and* $d\eta = \eta \wedge \omega$;

(9) **locally conformal cosymplectic** *if* $d\Phi = -2\Phi \wedge \omega$, $d\eta = \eta \wedge \omega$ *and* $N_\varphi = 0$, *where* N_φ *denotes the Nijenhuis tensor* [6];

(10) **quasi–Sasakian** *if* $d\Phi = 0 = N_1$;

(11) **almost trans–Sasakian** *if* $d\Phi = \Phi \wedge \omega$ *and*
$$d\eta = \frac{1}{2m}\{\delta\Phi(\xi)\Phi - 2\eta \wedge \varphi^*(\delta\Phi)\};$$

(12) **G_1–Sasakian** *if* $(\nabla_D\varphi)D - (\nabla_{\varphi D}\varphi)\varphi D + \eta(D)(\nabla_{\varphi D}\xi) = 0$;

(13) **G_2–Sasakian** *if* $\sigma\{(\nabla_D\Phi)(E,G) - (\nabla_{\varphi D}\Phi)(\varphi E, G) - -\eta(E)(\nabla_{\varphi D}\eta)G\} = 0$;

(14) **almost Kenmotsu** *if* $d\Phi(D,E,G) = \frac{2}{3}\sigma\{\eta(D)\Phi(E,G)\}$ *and* $d\eta = 0$;

(15) **a Kenmotsu manifold** *if it is almost Kenmotsu and normal;*

(16) **quasi Kenmotsu** *if* $(\nabla_D\Phi)(E,G) + (\nabla_{\varphi D}\Phi)(\varphi E, G) = = \eta(E)\Phi(G,D) + 2\eta(G)\Phi(D,E)$ *and* $d\eta = 0$;

(17) **nearly Kenmotsu** *if* $(\nabla_D\varphi)D = -\eta(D)\varphi D$ *and* $d\eta = 0$;

(18) **G_1–Kenmotsu** *if* $(\nabla_D\Phi)(D,E) - (\nabla_{\varphi D}\Phi)(\varphi D, E) = \eta(D)\Phi(E,D)$ *and* $d\eta = 0$;

(19) **G_2–Kenmotsu** *if* $\sigma\{(\nabla_D\Phi)(E,G) - (\nabla_{\varphi D}\Phi)(\varphi E, G) - \eta(D)\Phi(E,G) = 0 = d\eta$;

(20) **a C_4–manifold** *if* $(\nabla_D\Phi)(E,G) = \frac{-1}{2(m-1)}\{g(\varphi D, \varphi E)\delta\Phi(G) - -g(\varphi D, \varphi G)\delta\Phi(E) + \Phi(D,G)\delta\Phi(\varphi E) - \Phi(D,E)\delta\Phi(\varphi G)\}$ *and* $\delta\Phi(\xi) = 0$.

Other classes of almost contact metric structures which are not listed here can be found in [1], [3], [11] and [16].

Looking through the defining relations of all various classes of almost contact metric structures, it appears that the differential $d\Phi$ and the covariant derivative $\nabla\Phi$ of the fundamental form can be expressed in one of the following formulas in which b denotes a real number.

(1.3) $d\Phi(D,E,G) = \frac{b}{3}\sigma\{\eta(D)C\}$;

(1.4) $d\Phi(D,E,G) = \frac{b}{3}\sigma\{\Phi(D,E)C\}$;

(1.5) $d\Phi(D,E,G) = b\Phi \wedge \omega$;

(1.6) $\{(\nabla_D\Phi)(D,E) - (\nabla_{\varphi D}\Phi)(\varphi D, E) + b\eta(D)C\} = 0$;

(1.7) $\sigma\{(\nabla_D\Phi)(E,G) - (\nabla_{\varphi D}\Phi)(\varphi E, G) + b\eta(D)C\} = 0$.

Note that in these formulas, C is a factor which is determined by the class of the manifold. For instance, if we take $b = 2$ and $C = \Phi(E,G)$ in (1.3) we obtain $d\Phi(D,E,G) = \frac{2}{3}\sigma\{\eta(D)\Phi(E,G)\}$ which is one of the defining relations of an almost Kenmotsu manifold [8]. Taking $b = 1$ and $C = (\nabla_G\eta)\varphi E + (\nabla_{\varphi G}\eta)E$ in the same formula, we get $d\Phi(D,E,G) = = \frac{1}{3}\sigma\{\eta(D)[(\nabla_G\eta)\varphi E + (\nabla_{\varphi G}\eta)E]\}$ which leads to define a C_η–manifold [5]. However, if $b = C$ in the same formula, we get one of the defining relations of almost cosymplectic, quasi-Sasakian or a C_2–manifold which is defined in [5] by $d\Phi = 0 = \nabla\eta$. Taking $b = -1$ and $C = \Phi(E,D)$ in (1.4) we obtain one of the defining relations of a G_1–Kenmotsu manifold; if $b = -1$ and $C = (\nabla_{\varphi D}\eta)E$, we get the defining relation of G_1–Sasakian structures.

Clearly (1.5) can be illustrated by G_2–almost contact metric structures such as: G_2–Sasakian, G_2–semi-cosymplectic and some others.

2. Almost contact metric submersions. A Riemannian submersion is a submersion $f : M \to M'$ between Riemannian manifold such that $f_*/(\mathrm{Ker}\, f_*)^\perp$ is a linear isometry [9]. The tangent bundle $T(M)$ of the total space M admits an orthogonal decomposition $T(M) = V(M) \oplus H(M)$. We denote the vertical and horizontal projections by $\mathcal{V}$ and $\mathcal{H}$ respectively. For any $y \in M'$, the closed embedded submanifold $f^{-1}(y)$ is called the fibre of f over y. A vector field X of the horizontal distribution $H(M)$ is called a basic vector field if it is f–related to a vector field X_* of the base space M'. Such a vector field will be denoted by $f_*X = X_*$.

Let $(M, g, \varphi, \xi, \eta)$ and $(M', g', \varphi', \xi', \eta')$ be almost contact metric manifolds. A Riemannian submersion $f : M \to M'$ is said to be an *almost contact metric submersion of type I*, [17], if it satisfies.

(i) $f_*\varphi D = \varphi' f_* D$, *for all* $D \in \mathcal{X}(M)$, *and* (ii) $f_*\xi = \xi'$.

Proposition 2.1. (Watson) *Let* $f : M \to M'$ *be an almost contact metric submersion of type I. Then:*

(a) *$U \in V(M)$ implies that $\varphi U \in V(M)$;* (b) *$\eta(U) = 0$, for all $U \in V(M)$.*

Proof. See [14], [15] and [17].

In passing, let us note that, on the fibres, $\varphi U = JU$. In the case of necessity, we will denote by a caret $\wedge$ the tensors on the fibres.

Proposition 2.2. (Watson) *Let $f : M \to M'$ be an almost contact metric submersion of type I. Then:*

(a) $\hat{\Phi}(U,V) = \Phi(U,V)$; (b) $\hat{N}(U,V) = N_1(U,V)$.

Proof. See Watson [17, Prop. 2.2].

Now, let us look towards the classes of the fibres.

Proposition 2.3. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is cosymplectic, quasi-Sasakian or a Kenmotsu manifold, then the fibres are Kählerian.*

Proof. It is clear that these manifolds have in common the following defining relations $d\Phi(D,E,G) = \frac{b}{3}\sigma\{\eta(D)\Phi(E,G)\}$and $N_1 = 0$. Let U, V and W be tangent to the fibres. The vanishing of η on the vertical distribution yields $d\hat{\Phi}(U,V,W) = 0$. Since $N_1 = 0$, we get $\hat{N} = 0$. So, the fibres are defined by $d\hat{\Phi}(U,V,W) = 0 = \hat{N}$, which are the defining relations of the Kählerian structure.

Proposition 2.4. *If the total space of an almost contact metric submersion of type I is almost cosymplectic, a C_2-manifold or an almost Kenmotsu manifold, then the fibres are almost Kählerian.*

Proof. As in the proof of the preceding proposition, all these manifolds have in common the following defining relation $d\Phi(D,E,G) = \frac{b}{3}\sigma\{\eta(D)\Phi(E,G)\}$, which yields $d\hat{\Phi}(U,V,W) = 0$. Since no more is said about N_1, the fibres are almost Kählerian.

Proposition 2.5. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is closely cosymplectic, nearly cosymplectic, nearly-K-cosymplectic or nearly Kenmotsu, then the fibres are nearly Kählerian.*

Proof. The common defining relation for all these manifolds is $(\nabla_D\Phi)(D,E) = b\eta(D)C$. The vanishing of η on vertical vector fields yields $(\hat{\nabla}_U\hat{\Phi})(U,V) = 0$, which defines the nearly Kählerian structure.

Proposition 2.6. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is quasi-K-cosymplectic or quasi Kenmotsu, then the fibres are quasi Kählerian.*

Proof. As in the proof of the preceding proposition, all these manifolds have in common the following defining relation

$$(\nabla_D\Phi)(E,G) + (\nabla_{\varphi D}\Phi)(\varphi E,G) = b\eta(D)C.$$

We get the proof by using the procedure of the preceding proposition.

Proposition 2.7. *If the total space of an almost contact metric submersion of type I is G_i-Sasakian or G_i-Kenmotsu for $i = 1,2$, then the fibres are G_i-manifolds.*

Proof. Note that for $i = 1$, the considered manifolds have in common the defining relation$(\nabla_D\Phi)(D,E) - (\nabla_{\varphi D}\Phi)(\varphi D,E) + b\eta(D)C = 0$, while for $i = 2$, we have $\sigma\{(\nabla_D\Phi)(E,G) - (\nabla_{\varphi D}\Phi)(\varphi E,G) + b\eta(D)C\} = 0$. The proof is established by applying the preceding procedure.

Now, we shall be concerned by the manifolds which are defined by the coderivative of the fundamental form Φ or the contact 1-form η.

Recall that the O'Neill configuration tensor A, of the total space of a Riemannian submersion, is defined in [9] by $A_DE = \mathcal{V}\nabla_{\mathcal{H}D}\mathcal{H}E + \mathcal{H}\nabla_{\mathcal{H}D}\mathcal{V}E$. Using this tensor, Chinea [4] has defined an associated tensor A^* on horizontal vector fields by setting $A^*(X,Y) = A_X\varphi Y - A_{\varphi X}Y$, and has established the following equations

(2.1) $\delta\Phi(U) = \delta\hat{\Phi}(U) + \frac{1}{2}g(\mathrm{tr}\,A^*, U)$;

(2.2) $\delta\Phi(X) = \delta\Phi'(X_*) + g(H, \varphi X)$;

(2.3) $\delta\eta = \delta\eta' of - g(H,\xi)$,

where H denotes the mean curvature of the fibres while $\mathrm{tr}\,A^*$ is the trace of the tensor A^*.

Concerning the Lee forms ω, ω' and Θ, we have the following

Lemma 2.8. *Let $f : M \to M'$ be an almost contact metric submersion of type I. Then we have:*

(a) *$f^*\omega' = \omega$ if and only if the fibres are minimal;*

(b) *$\Theta(U) = \omega(U)$ if and only if $\mathrm{tr}\,A^* = 0$.*

Proof. (a) Let X be a basic vector field, by calculation we have

$$(f^*\omega')(X) = (f_*X) = \frac{1}{m}(\delta\Phi'(\varphi' f_*X) - (\delta\eta')(\eta'(f_*X)))$$
$$= \frac{1}{m}(\delta\Phi'(\varphi' X_*) - (\delta\eta')(\eta'(X_*))).$$

By (2.2) it is clear that $\delta\Phi'(\varphi' X_*) = \delta\Phi(\varphi X)$ if and only if $H = 0$ which is the required conddition for the minimality of the fibres. Similarly, by (2.3), $\delta\eta = \delta\eta' of$ if and only if $H = 0$ which completes the proof.

(b) The vanishing of η on the vertical distribution leads to $\hat{\omega}(U) = = \frac{1}{m-1}\delta\hat{\Phi}(\hat{\varphi}U)$. But by (2.1), we have $\delta\hat{\Phi}(\hat{\varphi}U) = \delta\Phi(\varphi U)$ if and only if $\operatorname{tr} A^* = 0$; that is to say $\hat{\omega}(U) = \omega(U)$ if and only if $\operatorname{tr} A^* = 0$. Since the fibres of an almost contact metric submersion of type I are almost Hermitian manifolds, we have $\hat{\varphi}U = JU$ and $\delta\hat{\Phi} = \delta F$ so that $\hat{\omega}(U) = \Theta(U)$.

Proposition 2.9. *If the total space is an almost contact metric submersion of type I is trans-Sasakian, locally conformal cosymplectic or a C_4-manifold, then the fibres are locally conformal Kählerian if and only if* $\operatorname{tr} A^* = 0$.

Proof. The considered manifolds have in common the following defining relation$d\Phi(D,E,G) = \frac{b}{3m}\sigma\{\Phi(D,E)C\}$, where the factor C is defined by the coderivative of the fundamental 2-form. Clearly, on the fibres, we have $d\hat{\Phi}(U,V,W) = \frac{b}{3(m-1)}\sigma\{\hat{\Phi}(U,V)\hat{C}\}$, in which $\hat{C}$ is defined by the coderivative if and only if $\operatorname{tr} A^* = 0$.

Proposition 2.10. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is almost trans-Sasakian or a locally conformal almost cosymplectic manifold, then the fibres are locally conformal almost Kählerian if and only if* $\operatorname{tr} A^* = 0$.

Proof. The common defining relation of these manifolds is $d\Phi = b\Phi\wedge\omega$. We have to show that $d\hat{\Phi} = b\hat{\Phi}\wedge\Theta$ if and only if $\operatorname{tr} A^* = 0$. By the preceding lemma, we have seen that $\hat{\omega}(W) = \omega(W)$ if and only if $\operatorname{tr} A^* = 0$. Since $\hat{\Phi}(U,V) = \Phi(U,V)$ we deduce that $d\hat{\Phi} = \hat{\Phi}\wedge\hat{\omega}$ if and only if $\operatorname{tr} A^* = 0$. The proof follows from the fact that, in this case, $\hat{\omega}(W) = \Theta(W)$.

Proposition 2.11. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is nearly-Sasakian or quasitrans-Sasakian, then the fibres are respectively $W_1 \oplus W_4$ or $W_1 \oplus W_2 \oplus W_4$-manifolds if and only if* $\operatorname{tr} A^* = 0$.

Proof. Analogue to the preceding.

Proposition 2.12. *Let $f : M \to M'$ be an almost contact metric submersion of type I. If the total space is semi-cosymplectic or semi-Sasakian, then the fibres are semi-Kählerian if and only if* $\operatorname{tr} A^* = 0$.

Proof. See C h i n e a [4].

Proposition 2.13. *If the total space of an almost contact metric submersion of type I is semi-cosymplectic normal, semi-Sasakian normal or semi-Kenmotsu normal, then the fibres are W_3-manifolds if and only if* $\operatorname{tr} A^* = 0$.

Proof. Note that the considered manifolds have $\delta\Phi = 0 = N_1$ as the common defining relations. Hence it is clear that $\delta\hat{\Phi} = 0$ if and only if $\operatorname{tr} A^* = 0$. As in Proposition 2.3, $\hat{N} = 0$ is obvious.

Proposition 2.14. *If the total space of an almost contact metric submersion of type I is G_i-semi-cosymplectic, G_i-Sasakian or G_i-semi-Kenmotsu, then the fibres are $W_i \oplus W_3$-manifolds if and only if* $\operatorname{tr} A^* = 0$.

Proof. We proceed as in the proof of Propositions 2.7 and 2.9 by considering $\delta\Phi = 0$.

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SEMI-PSEUDO SYMMETRIC MANIFOLD

BY

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Introduction. In a previous paper [1] M.C. C h a k i introduced a pseudo-symmetric manifold. In this paper, we consider a nonflat Riemannian manifold (M^n, g) $(n > 3)$ whose curvature tensor R satisfies the condition

$$(\nabla_X R)(Y,Z)W = 2\pi(X)R(Y,Z)W + \pi(Y)R(X,Z)W + \pi(Z)R(Y,X)W + \pi(W)R(Y,Z)X \tag{1}$$

where π is a non-zero 1-form

$$g(X,\rho) = \pi(X), \text{ for every vector field X} \tag{2}$$

and ∇ denotes the operator of covariant differentiation with respect to the metric g. Such a manifold shall be called a Semi-Pseudo-Symmetric manifold and the 1-form π shall be called its associated 1-form. An n-dimensional semi-pseudo-symmetric manifold shall be denoted by $(SPS)_n$.

In this paper, it is shown that if its scalar curvature r is constant, then $r = 0$, while if $r \neq 0$ then its associated 1-form is closed. It has been shown that Einstein $(SPS)_n$ does not exist. $(SPS)_n$ with Codazzi type of Ricci tensor and conformally flat $(SPS)_n$ are studied. It has also been shown that, in a conformally flat $(SPS)_n$ $(n \geq 3)$ with non-zero constant scalar curvature, the Ricci tensor has two distinct eigenvalues, namely $\frac{r}{n-1}$ and 0 of which the former is of multiplicity $n-1$ and the latter is simple. Finally, a necessary and sufficient condition has been proved for Ricci tensor to be of Codazzi type in a conformally flat $(SPS)_n$.

1. Preliminaries. Let S and r denote respectively the Ricci tensor of type $(0,2)$ and the scalar curvature and L denote the symmetric endomorphism of the tangent space at each point, corresponding to the Ricci