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ON THE BENDING OF CLAMPED SEMI–INFINITE PLATE WITH VOIDS

BY

A. MANOLACHI

1. Introduction. In [3] we are concerned with a mathematical model describing the small deflections of a thin plate with voids and formulate the fundamental boundary value problems.

In what follows we consider the problem of bending of a semi–infinite plate whose median section occupies the domain S^+ $(-\infty < x_1 < +\infty, x_2 > 0)$ which is clamped along the edge L $(x_2 = 0)$ and loaded upon by a normal distributed force. To obtain the solution we use the method of complex functions and Fourier transform.

2. Statement of the problem. The field equations characterizing the equilibrium of a plate with voids are

$$\begin{aligned} &D\Delta\Delta w + \beta_2 \Delta\psi = p(x_1, x_2) \\ &\alpha\Delta\psi - \xi_1\psi - \beta_1\Delta w = 0 \end{aligned} \tag{2.1}$$

where $w(x_1, x_2)$ denotes the transverse displacement, $\psi(x_1, x_2)$ stands for the function of change in volume fraction [2], $p(x_1, x_2)$ is the normal load, D – the rigidity and $\alpha, \beta_1, \beta_2, \xi_1$ some constants.

The general solution of the system (2.1) can be represented under the form

$$\begin{aligned} &w(z,\bar{z}) = -x_1\psi_0 + z\bar{\Omega}(\bar{z}) + \bar{z}\Omega(z) + \omega(z) + \bar{\omega}(\bar{z}) + W^*, \\ &\psi(z,\bar{z}) = \psi_0(z,\bar{z}) - 4x_2[\Omega'(z) + \bar{\Omega}'(\bar{z})] + \psi^* \end{aligned} \tag{2.2}$$

where $\Omega(z), \omega(x)$ are arbitrary analytical functions of variable $z = x_1 + ix+2$ $(\bar{z} = x_1 - ix_2)$, W^* and ψ^* are some particular solutions and $\psi_0(z,\bar{z})$ is the general solution of the equation

$$4\frac{\partial^2\psi_0}{\partial z\partial\bar{z}} - m^2\psi_0 = 0,\ m^2 = \text{constant} \tag{2.3}$$

To the system (2.1) we adjoin the boundary conditions at L which, in the case of clamped edge, have the form

$$w_{|L} = 0,\ w_{,n|L} = 9,\ \psi_{|L} = 0. \tag{2.4}$$

where $w_{,n}$ denotes the normal derivative of w.

In complex formulation the problem (2.1),(2.4) comes to determine the potentials Ω, ω and ψ_0 which must satisfy at L, the boundary conditions

$$\Omega(t) + t\bar{\Omega}'(\bar{t}) + \bar{\omega}'(\bar{t}) - x_1\psi_{0,\bar{t}}(t,\bar{t}) = a(t),\ \psi_{0,\bar{t}} = \frac{\partial\psi_0}{\partial\bar{t}}; \tag{2.5$_1$}$$

$$-4x_2[\Omega'(t) + \bar{\Omega}'(\bar{t})] + \psi_0(t,\bar{t}) = b(t),\ t \in L. \tag{2.5$_2$}$$

where

$$a(t) = -\lim_{z\to t}(-W^*_{,z}),\ b(t) = \lim_{z\to t}(-\psi^*(z,\bar{z})),\ z \in S_+.$$

If Ω, ω and ψ_0 are known the stress state throughout the plate is determined by the formula [3]

$$\begin{aligned}
\Lambda &= -4D_1[\Omega'(z) + \bar{\Omega}'(\bar{z})] - \beta_2(1-\nu) + \Lambda^*,\\
\Gamma &= 4D(1-\nu)[\bar{z}\Omega''(z) + \omega''(z) - x_1\psi_{0,zz}] + \Gamma^*,\\
\Psi &= -2D[4\bar{\Omega}''(\bar{z}) - x_1 m^2\psi_{0,\bar{z}}] + \Psi^*,\\
\chi &= 2\alpha_1[4x_2\bar{\Omega}''(\bar{z}) - \psi_{0,\bar{z}}] + \chi^*,
\end{aligned} \tag{2.6}$$

where $\Lambda, \Gamma, \psi, \chi$ are expressed in terms of moments and forces as follows

$$\Lambda = M_{11} + M_{22},\qquad \Gamma = M_{22} - M_{11} + 2iM_{12},$$
$$\psi = Q_1 + iQ_2,\qquad \chi = H_1 + iH_2.$$

In (2.6) $\Lambda^*, \Gamma^*, \psi^*, \chi^*$ corresponds to the particular solution (W^*, ψ^*) and $D_1, \alpha_1, x_1, m^2, \nu, x_2$ are given constants.

In what follows we assume the particular solution be choosen in such a way to assign

(i) $a(t) = o(t^{-1}),\ b(t) = o(t^{-1}),\ t \in L.$

(ii) *$a'(t), b(t)$ satisfy a Hölder condition in the points of finite distance at L and, $ta(t), tb(t)$ and $t^2a'(t)$ satisfy the same condition at $t = \infty$.*

3. Solution of the problem. To obtain Ω, ω and ψ_0 we shall use some properties of analytical functions holomorphic in S_+ and continous in $S_+ + L$ [4]. These properties are

$$\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\Omega'(t)dt}{t-z} = \Omega'(z) \tag{3.1}$$

$$\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\bar{\Omega}'(\bar{t})dt}{t-z} = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{t\bar{\Omega}'(\bar{t})dt}{t-z} = 0,\quad z \in S_+. \tag{3.2}$$

If the function $f(t)$ defined at L satisfies a Hölder condition including the point at infinity [4], one has

$$\lim_{z\to t_0}\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{f(t)}{t-z} = \frac{1}{2}f(t_0) + \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{f(t)}{t-t_0},\ t_0 \in L. \tag{3.3}$$

In order to find the solution of our problem we begin with the relation(2.5)$_2$. Multiplying it by $\dfrac{1}{2\pi i}\dfrac{dt}{t-z}$ and integrating the result on the real axis, with the aid of (3.2), we obtain

$$\Omega'(z) = H(z) + \frac{1}{8\pi i x_2}\int_{-\infty}^{+\infty},\ H(z) = \frac{-1}{2\pi i}\int_{-\infty}^{+\infty}\frac{f(t)dt}{t-z}. \tag{3.4}$$

In the same manner, from the complex conjugate of (2.5)$_1$, by using (3.1) where Ω' is substituted by ω' and (3.2), we get

$$\begin{aligned}
\omega'(z) &= M(z) - \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\bar{t}\Omega'(t)dt}{t-z} + \frac{x_1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\psi_{0,t}dt}{t-z},\\
M(z) &= \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\bar{a}(\bar{t})dt}{t-z},\ z \in S_+.
\end{aligned} \tag{3.5}$$

From (3.4) and (3.50 we obtain $\Omega(z)$ and $\omega'(z)$ when the values of $\psi_0(t)$ at L are known.

Now, we focus our attention to find $psi_0(t)$. In this respect, we use $(2.5)_1$ and (3.2), to obtain

$$\Omega(z) = H_1(z) + \frac{x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_{0,\bar{t}}dt}{t-z};\ H_1(z) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{a(t)dt}{t-z}. \tag{3.6}$$

From this relation we deduce

$$\Omega'(z) = H_1'(z) - \frac{x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_{0,\bar{t}}dt}{(t-z)^2}. \tag{3.6$'$}$$

By integrating the term containing the integral, by pats, one gets

$$-\frac{x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_{0,\bar{t}}dt}{(t-z)^2} = \frac{x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_{0,t\bar{t}}dt}{t-z},\ \psi_{0,t\bar{t}} = \frac{\partial^2\psi_0}{\partial t\partial\bar{t}}.$$

As the function ψ_0 satisfies (2.30 and must be continous up to the frontiere of S_+, we can transform the right side term of the above relation, to obtain

$$\frac{x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_{0,t\bar{t}}dt}{t-z} = \frac{m^2 x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_0(t)dt}{t-z}.$$

By using this relation, (3.6) becomes

$$\Omega'(z) = H_1'(z) + \frac{m^2 x_1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_0(t)dt}{t-z}. \tag{3.7}$$

When $\Omega'(z)$ is eliminated among (3.4) and (3.6)$'$ we find

$$\frac{k}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_0(t)dt}{t-z} = F(z) \tag{3.8}$$

where

$$k = \frac{1}{4}\left(\frac{1}{x_2} - x_2 m^2\right),\quad F(z) = H_1'(z) - H(z).$$

Passing to the limit values for $z \to t_0 \in L$ into (3.8) and using (3.3) we obtain

$$\frac{k}{2}\psi_0(t_0) + \frac{k}{2\pi i}\int_{-\infty}^{+\infty} \frac{\psi_0(t)dt}{t-t_0} = F(t_0),\ F(t_0) = \lim_{z\to t_0} F(z),\ z \in S_+. \tag{3.9}$$

This singular integral equation which gives us the values of ψ_0 at L can be solved among other procedures by Fourier integral transform.

Let ψ_0^* be the Fourier image of ψ_0

$$\psi_0^*(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \psi_0(t)e^{-itu}dt.$$

To obtain ψ_0 in S_+ we apply the Fourier transform on the equation (2.3). Using some properties of this transform we obtain

$$\frac{d^2\psi_0^{**}}{dx_2^2} - (u^2+m^2)\psi_0^{**} = 0 \tag{3.10}$$

where ψ_0^{**} is the fourier image of ψ_0 in S_+

$$\psi_0^{**}(u,x_2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \psi_0(x_1,x_2)e^{-iux_1}dx_1.$$

The general solution of (3.10) is given by

$$\psi_0^{**}(u,x_2) = C_1(u)e^{-(u^2+m^2)x_2} + C_2(u)e^{(u^2+m^2)x_2}.$$

Since the function ψ_0 must be bounded to infinity, it results

$$\psi_0^{**}(u,x_2) = C_1(u)e^{-(u^2+m^2)x_2}.$$

Bearing in mind on the frontiere $x_2 = 0$, one has

$$\psi_0^{**}(u,0) = \psi^*(u)$$

we obtain

$$C_1(u) = \psi^*(u).$$

In view of this relation, the function ψ_0 will be given by

$$\psi(x_1,x_2) = \frac{e^{-m^2x_2}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \psi^*(u)e^{-u^2x_2}e^{iux_1}du,\ x_2 > 0.$$

As for $x_2 > 0$ the Fourier transform of $e^{-u^2x_2}$ is

$$f(x_1,x_2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-u^2x_2}e^{iux_1}du = \frac{1}{\sqrt{2x_2}}e^{-\frac{x_1^2}{4x_2}},$$

by convolution [1], we obtain

$$(3.11)\quad \begin{aligned}\psi_0(x_1,x_2) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\psi^*(u)f(x_1-u,x_2)du = \\ &= \frac{1}{2\sqrt{\pi x_2}}\int_{-\infty}^{+\infty}\psi^*(u)e^{\frac{(x_1-u)^2}{4x_2}}du.\end{aligned}$$

Having ψ_0 we can find $\psi_{0,\bar{t}}$ at L and then, from (3.5) and (3.6), one obtains $\Omega(z)$ and $\omega'(z)$.With these functions and (3.11) we obtain the stress state throughout the plate by means of (2.6).

We note that in (3.11) appears only the Fourier image of $\psi_0(t)$ which is supposed to exist.

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University of Iaşi
Faculty of Mathematics
6600 Iaşi
ROMANIA

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A CHARACTERIZATION OF REGULAR SEMIRINGS

BY

P. MUKHOPADHYAY

1. Introduction. A semiring is a non-empty set R equipped with two binary operations, called addition, +, and multiplication (denoted by juxtaposition), such that R is multiplicatively a semigroup and additively a commutative semigroup and that the multiplication is distributed accross the addition both from the left and from the right. An element denoted by 0 is called the zero of R if $a+0=a$ and $a0=0a=0$ for all $a\in R$. A semiring is said to be regular in the sense of V o n N e u m a n n (cf. [1]) if for every element $a\in R$ there exist some $x,y\in R$ such that $a+axa=aya$. A k-ideal I of a semiring R is an ideal such that, if $a\in I$ and $x\in R$ and $a+x\in I$ then $x\in I$.

In this paper our aim is to establish some necessary and sufficient conditions for a semiring to be regular, in terms of its k-ideals.

2. Regular semirings.

Definition 2.1. *Let I be an ideal of a semiring R. Then $\overline{I}=\{a\in R|\ a+x\in I\}$ for some $x\in I$ is called a k-closure of I.*

It is easy to check that $\overline{I}$ is a k-ideal. In fact, it is the smallest k-ideal containing I and $I=\overline{I}$ iff I is a k-ideal. Also, it is known that, for any two ideals A,B with $A\subseteq B$ implies that $\overline{A}\subseteq\overline{B}$. We now prove the following necessary and sufficient condition for regularity of a semiring.

Theorem 2.2. *A semiring R is regular if and only if $A\cap B=\overline{AB}$ holds for every right k-ideal A and left k-ideal B of R.*

Proof. Let R be a regular semiring, that is, for any $a\in R$ there exist $x,y\in R$ such that, $a+axa=aya$.