by convolution [1], we obtain

$$(3.11)\quad \begin{aligned}\psi_0(x_1,x_2) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\psi^*(u)f(x_1-u,x_2)du = \\ &= \frac{1}{2\sqrt{\pi x_2}}\int_{-\infty}^{+\infty}\psi^*(u)e^{\frac{(x_1-u)^2}{4x_2}}du.\end{aligned}$$

Having ψ_0 we can find $\psi_{0,\bar{t}}$ at L and then, from (3.5) and (3.6), one obtains $\Omega(z)$ and $\omega'(z)$.With these functions and (3.11) we obtain the stress state throughout the plate by means of (2.6).

We note that in (3.11) appears only the Fourier image of $\psi_0(t)$ which is supposed to exist.

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A CHARACTERIZATION OF REGULAR SEMIRINGS

BY

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1. Introduction. A semiring is a non-empty set R equipped with two binary operations, called addition, +, and multiplication (denoted by juxtaposition), such that R is multiplicatively a semigroup and additively a commutative semigroup and that the multiplication is distributed accross the addition both from the left and from the right. An element denoted by 0 is called the zero of R if $a+0=a$ and $a0=0a=0$ for all $a\in R$. A semiring is said to be regular in the sense of V o n N e u m a n n (cf. [1]) if for every element $a\in R$ there exist some $x,y\in R$ such that $a+axa=aya$. A k–ideal I of a semiring R is an ideal such that, if $a\in I$ and $x\in R$ and $a+x\in I$ then $x\in I$.

In this paper our aim is to establish some necessary and sufficient conditions for a semiring to be regular, in terms of its k–ideals.

2. Regular semirings.

Definition 2.1. *Let I be an ideal of a semiring R. Then $\overline{I}=\{a\in R|\ a+x\in I\}$ for some $x\in I$ is called a k–closure of I.*

It is easy to check that $\overline{I}$ is a k–ideal. In fact, it is the smallest k–ideal containing I and $I=\overline{I}$ iff I is a k–ideal. Also, it is known that, for any two ideals A,B with $A\subseteq B$ implies that $\overline{A}\subseteq\overline{B}$. We now prove the following necessary and sufficient condition for regularity of a semiring.

Theorem 2.2. *A semiring R is regular if and only if $A\cap B=\overline{AB}$ holds for every right k–ideal A and left k–ideal B of R.*

Proof. Let R be a regular semiring, that is, for any $a\in R$ there exist $x,y\in R$ such that, $a+axa=aya$.

To prove tht $A\cap B=\overline{AB}$ with A,B as given in the statement of theorem we proceed as follows.

Obviously, $AB\subseteq A$ and $AB\subseteq B$, so that $\overline{AB}\subseteq\overline{A}=A$ and $\overline{AB}\subseteq\overline{B}=B$. Conversely, let $a\in A\cap B$, therefore, $a+axa=aya$, for some $x,y\in R$. Now, $aya\in AB$ ($\because a\in A$ implies $ay\in A$ and $a\in B$ implies $aya\in AB$). Similarly, $axa\in AB$ also. Hence, by definition of k-closure we get, $a\in\overline{AB}$ that is, $A\cap B\subseteq\overline{AB}$. Consequently, $A\cap B=\overline{AB}$.

To prove the converse part, let us assume that R is a semiring such that $A\cap B=\overline{AB}$ holds for any left k-ideal B and right k-ideal A of R. We are to prove that R is regular.

Let $0\neq a\in R$ be arbitrary. We are to prove that there exist $x,y\in R$ such that, $a+axa=aya$ holds. Now right k-ideal generated by a is $\overline{aR+Z^+a}$ and similarly left k-ideal generated by a is $\overline{Ra+Z^+a}$. We see that,

$$(\alpha)\qquad \overline{aR+Z^+a}\cap R=\overline{\overline{(aR+Z^+a)}R}\subseteq\overline{aR}$$

In fact, we see that, $\overline{aR+Z^+a}R\subseteq\overline{aR}$, so that (α) follows immediately. Indeed, let $b\in\overline{(aR+Z^+a)}R\subseteq\overline{aR+Z^+a}$. Therefore, $b+t_1r=t_2s$ for some $r,s\in R$. That is $b+(ar_1+na)r=(ar_2+ma)s$ where

$t_1=ar_1+na\in aR+Z^+a$ and

$t_2=ar_2+ma\in aR+Z^+a$, for some $r_1,r_2\in R$ and some $m,n\in Z^+$,

or, $b+ar_3+nar=ar_4+mas$ where, $r_3=r_1r$ (say), $r_4=r_2s$

or, $b+ar_5=ar_6$ $\qquad r_5=r_3+nr$ (say), $r_6=r_4+ms$ (say)

and $r_3,r_5,r_6\in R$.

Therefore, $b\in\overline{aR}$.

Hence, $\overline{\overline{(aR+Z^+a)}R}\subseteq\overline{aR}$ [$\because\overline{A}$ is the smallest k-ideal containing A]. Therefore we get $a\in\overline{aR}$. In a similar way we can prove that $a\in\overline{Ra}$.

Combining these two we get,

$$(\beta)\qquad a\in\overline{aR}\cap\overline{Ra}=\overline{\overline{aR}\,\overline{Ra}}$$

So,

$$(\gamma)\qquad a+t_1t_2=s_1s_2 \text{ for some } t_1,s_1\in\overline{aR} \text{ and some } t_2,s_2\in\overline{Ra}.$$

Now, since $t_1,s_1\in\overline{aR}$, we get

$$(1)\qquad t_1+ax_1=ay_1 \quad\text{for some}\quad x_1,y_1\in R$$

$$(2)\qquad s_1+ax_2=ay_2 \quad\text{for some}\quad x_2,y_2\in R$$

and again $t_2,s_2\in\overline{Ra}$, we get

$$(3)\qquad t_2+x_3a=y_3a \quad\text{for some}\quad x_3,y_3\in R$$

$$(4)\qquad s_2+x_4a=y_4a \quad\text{for some}\quad x_4,y_4\in R$$

Now, from (1) we get $t_1t_2+ax_1t_2=ay_1t_2$. Hence, from (γ), we write

$a+t_1t_2+ax_1t_2=s_1s_2+ax_1t_2$

or $a+ay_1t_2=s_1s_2+ax_1t_2$

or $a+ay_1t_2+ay_1x_3a=s_1s_2+ax_1t_2+ay_1x_3a$

or $a+ay_1y_3a=s_1s_2+ax_1t_2+ay_1x_3a$

[$\because$ from (3) $ay_1t_2+ay_1x_3a=ay_1y_3a$]

or $a+ay_1y_3a+ax_1x_3a=s_1s_2+ax_1t_2+ax_1x_3a+ay_1x_3a$

or $a+ay_1y_3a+ax_1x_3a=s_1s_2+ax_1y_3a+ay_1x_3a$

[$\because$ from (3) $ax_1t_2+ax_1x_3a=ax_1y_3a$]

or $a+ay_1y_3a+ax_1x_3a+s_1x_4a=s_1s_2+s_1x_4a+ax_1y_3a+ay_1x_3a$

or $a+a(y_1y_3+x_1x_3)a+s_1x_4a=s_1y_4a+a(x_1y_3+y_1x_3)a$

[$\because$ from (4) $s_1s_2+s_1x_4a=s_1y_4a$]

or $a+a(y_1y_3+x_1x_3)a+s_1x_4a+ax_2x_4a=$

$=s_1y_4a+a(x_1y_3+y_1x_3)a+ax_2x_4a$

[$\because$ from (2) $s_1x_4a+ax_2x_4a=ay_2x_4a$]

or $a+a(y_1y_3+x_1x_3)a+ay_2x_4a=s_1y_4a+a(x_1y_3+y_1x_3+x_2x_4)a$

or $a+a(y_1y_3+x_1x_3+y_2x_4)a+ax_2y_4a=$

$=s_1y_4a+ax_2y_4a+a(x_1y_3+y_1x_3+x_2x_4)a$

or $a+a(y_1y_3+x_1x_3+y_2x_4+x_2y_4)a=ay_2y_4a+a(x_1y_3+y_1x_3+x_2x_4)a$

or $a+a(y_1y_3+x_1x_3+y_2x_4+x_2y_4)a=a(y_2y_4+x_1y_3+y_1x_3+x_2x_4)a$,

[$\because$ from (2) $s_1y_4a+ax_2y_4a=ay_2y_4a$].

Hence the regularity of a follows. But as a was choosen arbitrarily we conclude that R is regular.

Now let R be a semiring and $\mathcal{F}(R)$ be the set of all k-ideals of R. In $\mathcal{F}(R)$ we define the following operations of "addition" denoted by $\oplus$ and "multiplication" denoted by $\cdot$, for any $I,J\in\mathcal{F}(R)$, $I\oplus J=\overline{I+J}$, $I\cdot J=\overline{IJ}$, where IJ is the set consisting of all finite sums of the form $\sum_{I=1}^{n}a_ib_i$, $n\in I\!N$, with $a_i\in I$ and $b_i\in J$. Through a lengthy but routine calculation it can be checked that $(\mathcal{F}(R),\oplus,\cdot)$ is a semiring. If R will be taken as multiplicatively commutative semiring then so will be $\mathcal{F}(R)$.

We now prove the following theorem:

Theorem 2.3. *A multiplicatively commutative semiring with zero, $0 \neq 1$ is regular if and only if $\mathcal{F}(R)$ as defined above is regular.*

Proof. Let R be a multiplicatively commutative regular semiring. Then for any two k–ideals I and J we can write from Theorem 2.2 that $I \cap J = \overline{IJ}$.

Hence $I \cdot J = \overline{IJ} = I \cap J$.

To prove that $\mathcal{F}(R)$ is regular, let $(0) \neq I \in \mathcal{F}(R)$, we see that,

$$I \oplus I \cdot R \cdot I = I \cdot R \cdot I \quad \text{holds.}$$

In fact, $I \oplus I \cdot R \cdot I = I \oplus I \cap R \cap I = I \oplus I = \overline{I + I} = \overline{I} = I$ and $I \cdot R \cdot I = I \cap R \cap I = I$, proving the regularity of $\mathcal{F}(R)$. Conversely, let $\mathcal{F}(R)$ be regular. We want to prove that R is so.

Let $0 \neq a \in R$. We consider the principal k–ideal generated by a i.e., $\overline{Ra}$ (since R is commutative and $1 \in R$).

By regularity of $\mathcal{F}(R)$, we get that there exist $A, B \in \mathcal{F}(R)$ such that,

$$\overline{Ra} \oplus \overline{Ra} \cdot A \cdot \overline{Ra} = \overline{Ra} \cdot B \cdot \overline{Ra}$$

we see that $a \in \overline{Ra} \oplus \overline{Ra} \cdot A \cdot \overline{Ra}$. Hence $a \in \overline{Ra} \cdot B \cdot \overline{Ra}$, i.e., $a \in \overline{\overline{Ra}\; B\; \overline{Ra}}$.

$$\text{Now,} \quad \overline{\overline{Ra}\; B\; \overline{Ra}} \subseteq \overline{\overline{Ra}\;\overline{Ra}} \quad \left[\because \overline{Ra}\; B \subseteq \overline{\overline{Ra}} = \overline{Ra}\right]$$

$$= \overline{aR\; aR} \quad [\text{by commutativity}].$$

Hence, $a \in \overline{aR\; aR}$, so that $a + \sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{m} p_i q_i$ where $x_i, p_i \in \overline{aR}$ and $y_i, q_i \in \overline{Ra}$ for all $i = 1, 2, \ldots, n$ or m, as the case may be. We have,

$$(a) \qquad \begin{array}{ll} x_i + at_i = as_i & y_i + r_i a = r'_i a \\ p_i + at'_i = as'_i & q_i + \bar{r}_i a = \hat{r}_i a \end{array}$$

for some $t_i, s_1, t'_i, s'_i, r_i, r'_i, \bar{r}_i, \hat{r}_i \in R$ with $i = 1, 2, \ldots, n$ or m as the case may be.

From the above relations (a) we may have,

$$(1) \quad \sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{n} at_i y_i = \sum_{i=1}^{n} as_i y_i$$

$$(2) \quad \sum_{i=1}^{m} p_i q_i + \sum_{i=1}^{m} at'_i q_i = \sum_{i=1}^{m} as'_i q_i$$

$$(3) \quad \sum_{i=1}^{n} as_i y_i + \sum_{i=1}^{n} as_i r_i a = \sum_{i=1}^{n} as_i r'_i a$$

$$(4) \quad \sum_{i=1}^{m} at'_i q_i + \sum_{i=1}^{m} at'_i \bar{r}_i a = \sum_{i=1}^{m} at'_i \hat{r}_i a$$

$$(5) \quad \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{m} as'_i \bar{r}_i a = \sum_{i=1}^{m} as'_i \hat{r}_i a$$

$$(6) \quad \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} at_i r_i a = \sum_{i=1}^{n} at_i r'_i a$$

Now, we see that, $a + \sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{n} at_i y_i = \sum_{i=1}^{m} p_i q_i + \sum_{i=1}^{n} at_i y_i$

or $a + \sum_{i=1}^{n} as_i y_i + \sum_{i=1}^{m} at'_i q_i = \sum_{i=1}^{m} p_i q_i + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{m} at'_i q_i$ [by(1)]

or $a + \sum_{i=1}^{n} as_i y_i + \sum_{i=1}^{m} at'_i q_i = \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{n} at_i y_i$ [by(2)]

or $a + \sum_{i=1}^{n} as_i y_i + \sum_{i=1}^{m} at'_i q_i + \sum_{i=1}^{n} as_i r_i a = \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} as_i r_i a$

or $a + \sum_{i=1}^{n} as_i r'_i a + \sum_{i=1}^{m} at'_i q_i = \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} as_i r_i a$ [by (3)]

or $a + \sum_{i=1}^{n} as_i r'_i a + \sum_{i=1}^{m} at'_i q_i + \sum_{i=1}^{m} at'_i \bar{r}_i a =$

$= \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} as_i r_i a + \sum_{i=1}^{m} at'_i \bar{r}_i a$

or $a + \sum_{i=1}^{n} as_i r'_i a + \sum_{i=1}^{m} at'_i \hat{r}_i a =$

$= \sum_{i=1}^{m} as'_i q_i + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} as_i r_i a + \sum_{i=1}^{m} at'_i \bar{r}_i a$ [by(4)]

$$\text{or } a + \sum_{i=1}^{n} as_i r'_i a + \sum_{i=1}^{m} at'_i \hat{r}_i a + \sum_{i=1}^{m} as'_i \bar{r}_i a =$$

$$= \sum_{i=1}^{m} as'_i \hat{r}_i a + \sum_{i=1}^{n} at_i y_i + \sum_{i=1}^{n} as_i r_i a + \sum_{i=1}^{m} at'_i \bar{r}_i a \text{ [by(5)]}$$

$$\text{or } a + \sum_{i=1}^{n} as_i r'_i a + \sum_{i=1}^{m} at'_i \hat{r}_i a + \sum_{i=1}^{m} as'_i \bar{r}_i a + \sum_{i=1}^{n} at_i r_i a =$$

$$= \sum_{i=1}^{m} as'_i \hat{r}_i a + \sum_{i=1}^{n} at_i r'_i a + \sum_{i=1}^{n} as_i r_i a + \sum_{i=1}^{m} at_i \bar{r}_i a \text{ [by (6)]}$$

or $a + axa = aya$, (say)
where,

$$x = \sum_{i=1}^{n} s_i r'_i + \sum_{i=1}^{m} t'_i \hat{r}_i + \sum_{i=1}^{m} s'_i \bar{r}_i + \sum_{i=1}^{n} t_i r_i \in R \quad \text{and}$$

$$y = \sum_{i=1}^{m} s'_i \hat{r}_i + \sum_{i=1}^{n} t_i r'_i + \sum_{i=1}^{n} s_i r_i + \sum_{i=1}^{m} t_i \bar{r}_i \in R.$$

Hence R is regular.

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FORTIFIED JOIN HYPERGROUPS AND JOIN HYPERRINGS

BY

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1. Introduction. The *Fortified Join Hypergroup* (FJH) is a hypergroup which derived from the introduction of the Theory of Hypercompositional Structures into the Theory of Automata and Languages [4], [5]. The Join Hypergroup, as it is known, [9], [2], is a commutative hypergroup $(H, +)$ in which the join axiom is also valid:

$$(J) \qquad (a:b) \cap (c:d) \neq \emptyset \ \Rightarrow\ a+d \cap b+c \neq \emptyset$$

where ":" is the induced hypercomposition, i.e. $a : b = \{x \in H \mid a \in b + x\}$.

In the theory of languages the expression $x+y$ is used, with x, y being words over an alphabet A, to state "either x or y". Thus, beginning with the fact that $x+y$ is basically a biset, the set of the words A^* over an alphabet A, endowed with the hypercomposition $w_1 + w_2 = \{w_1, w_2\}$ becomes a join hypergroup [4], [5]. The special join hypergroup which derives in this way from the theory of languages, is named B-hypergroup. Furthermore, A^* is a semigroup with regard to the concatenation of the words and beyond that it has been proved [5] that this operation is bilaterally distributive as for the hyperoperation defined in the set of the words. Thus in a natural way, there derived the multiplicative–hyperadditive structure which was named *hyperringoid*.

A *hyperringoid* is a non void set Y with an operation "$\cdot$" and a hyperoperation "+" that satisfy the axioms:

i. $(Y, +)$ *is a hypergroup*

ii. $(Y, \cdot)$ *is a semigroup*

iii. *the operation is bilaterally distributive to the hyperoperation.*