If $C$ is a linear connection, the components of $C, C'$ and $C''$ have the local forms:

$$N_j^\beta(x^i, y^\alpha, y^u) = \Gamma_{j\gamma}^\beta(x^i)y^\gamma + \Gamma_{jw}^\beta(x^i)y^w,$$
$$N_j^v(x^i, y^\alpha, y^u) = \Gamma_{j\gamma}^v(x^i)y^\gamma + \Gamma_{jw}^v(x^i)y^w$$
$$\widetilde{N}_j^\beta(x^i, y^\alpha) = \Gamma_{j\gamma}^\beta(x^i)y^\gamma, \widetilde{N}_j^v(x^i, y^u) = \Gamma_{jw}^v(x^i)y^w$$

and it follows that $C'$ and $C''$ are linear connections, too.

In [2] there is proved that if $D$ is a linear connection on $\xi$, then the formulas

$$D'_X u' = P'(D_X u'), D''_X u'' = P''(D_X u'')$$

for $X \in \mathcal{X}(M), u' \in S(\xi'), u'' \in S(\xi'')$, define linear connections $D'$ and $D''$ on $\xi'$ and $\xi''$ respectively.

In the adapted coordinates considered in our paper; if $D$ has the local components

$$\{\Gamma_{i\alpha}^\beta, \Gamma_{i\alpha}^v, \Gamma_{iu}^\beta, \Gamma_{iu}^v\}$$

then $D'$ and $D''$ have as local components $\{\Gamma_{i\alpha}^\beta\}$ and $\{\Gamma_{iu}^v\}$ respectively. Therefore, for a linear connection, the connections induced by *Proposition 4* are the same as the linear connections induced in [2].

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# ON JORDAN PLANE CURVES WHICH ARE ISOPTICS OF AN OVAL

BY

STANISLAW GÓŻDŹ

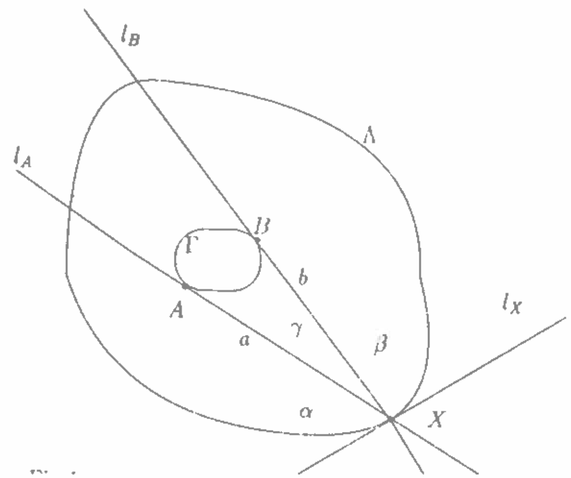
Let $\Gamma$ be an oval [3] and let $\Lambda$ be a regular Jordan plane curve. In the paper we give the sufficient condition for $\Lambda$ to be an isoptic of the oval $\Gamma$.

Let $\sigma$ be a region bounded by $\Lambda$. We assume that:

$$(1°) \qquad \Gamma \subset \sigma,$$

$$(2°) \qquad \Gamma \cap \Lambda = \emptyset.$$

Let $X$ be an arbitrary point of $\Lambda$. We denote the straight line tangent to $\Lambda$ at the point $X$ by $l_X$. Let $l_A$ and $l_B$ tangents to $\Gamma$ at points $A$ and $B$ and passing by $X$, see Fig.1.



We denote the angles:

$$\alpha = \angle(l_X, l_A), \text{ and } \beta = \angle(l_X, l_B).$$

Moreover, we denote the lenghts of the sectors $XA$ and $XB$ by $a$ and $b$, respectively, i.e.:

$$a = \mid XA \mid, b = \mid XB \mid .$$

Keeping the above-mentioned notions we give the following result of the paper.

**Theorem 1.** *If the oval $\Gamma$ and the regular Jordan plane curve $\Lambda$ satisfy the equality*

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta},$$

*then $\Lambda$ is the isoptic of the oval $\Gamma$.*

**Remark 1.** The equality

$$\frac{a}{sin\alpha} = \frac{b}{sin\beta},$$

is always satisfied on an isoptic of an oval, see [1].

**Proof.** Let $\gamma$ denote the angle between straight lines $l_A$ and $l_B$, i.e.: $\gamma = \angle(l_A, l_B)$. We prove that

$$\gamma = \text{ const.}$$

We assume that the oval $\Gamma$ is represented in the arclength parametrization $t \longrightarrow z(t) = (x_1(t), x_2(t))$ and the curve $\Lambda$ is represented by equation

$$t \longrightarrow y(t).$$

We denote by $s \longrightarrow p(s)$ and $s \longrightarrow q(s)$ functions such that

$$z(p(s)) \text{ and } z(q(s))$$

are coordinates of the points $A$ and $B$, respectively. Obviously there exist the derivatives $p'$ and $q'$. Functions $p$ and $q$ are defined as the implicite functions:

$$[z(t) - y(s), T_t] = 0,$$



where $[z(t) - y(s), T_t]$ denotes the determinant of vectors $z(t) - y(s), T_t$ and $T_t$ is the tangent vector to $\Gamma$ at point $z(t)$. We define two following vector-functions:

$$w(s) = z(p(s)) - y(s) \text{ and } v(s) = z(q(s)) - y(s).$$

Now we consider the curvature type functions of $w$ and $v$ with respect to the curve $\Lambda$, see [2]:

$$k_w = \frac{[w, w']}{|y'||w|^2}$$

and

$$k_v = \frac{[v, v']}{|y'|v|^2}.$$

We have

$$k_w = \frac{[z(p(s)) - y(s), T_{p(s)}p'(s) - y'(s)]}{|y'(s)||zp(s)) - y)s)|^2} =$$

$$= -\frac{[z(p(s)) - y(s), y'(s)]}{|y'(s)||zp(s)) - y)s)|^2}$$

and we insert

$$[z(p(s)) - y(s), y'(s)] = -|z(p(s)) - y(s)||y'(s)| \sin \angle(z(p(s)) - y(s), y'(s)) =$$
$$= |z(p(s)) - y(s)||y'(s)| \sin \alpha$$

into $k_w$. We obtain

$$k_w = -\frac{\sin \alpha}{a}.$$

Similarly we compute that

$$k_v = -\frac{\sin \beta}{b}.$$

Now by virtue of Th.1.[2] the following equality

$$k_w = k_v \pm \frac{1}{|y'|}\gamma'$$

holds, where $\gamma = \gamma(s) = \angle(w(s), v(s))$. By the assumption of the theorem we have

$$k_w = k_v.$$

This means that $\gamma' \equiv 0$ and $\gamma =$ const . Therefore the curve $\Lambda$ is the isoptic of the oval $\Gamma$.

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# ON GENERALIZED PSEUDO RICCI SYMMETRIC SASAKIAN MANIFOLDS

BY

U.C. DE and J.C. GHOSH

**0. Introduction.** The notions of Pseudo Ricci symmetric and generalized Pseudo Ricci symmetric manifolds were introduced by M.C. C h a k i in 1988 [1] and M.C.C h a k i and S.K o l e y in 1994 [2] respectively.

A non–flat Riemannian manifold $(M^n, g)$ $(n > 2)$ is called Pseudo Ricci symmetric if its Ricci tensor $S$ of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X) \tag{1}$$

where $A$ is a non–zero 1–form,

$$g(X,P) = A(X), \tag{2}$$

for every vector field $X$ and $\nabla$ denotes the operator of covariant differentiation with respect to metric tensor $g$. Such an $n$–dimensional manifold was denoted by $(PRS)_n$.

A type of non–flat Riemannian manifold $(M^n, g)$ $(n > 2)$ is called generalized Pseudo Ricci symmetric if its Ricci tensor $S$ of type (0,2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(Y,X) \tag{3}$$

where $A, B, C$ are three non–zero 1–forms,

$$g(X,P) = A(X), \quad g(X,Q) = B(X), \quad g(X,\lambda) = C(X) \tag{4}$$