REFERENCES

1. B e n k o, K., C i e ś l a k, W., G ó ź d ź, S. and M o z g a w a, W. – *On Isoptic Curves*, Analele Ştiinţifice ale Universităţii "Al.I.Cuza" Iaşi (36)(1990) p.47-54.

2. G ó ź d ź, S. – *Curvature Type Functions for Plane Curves*, Analele Ştiinţifice ale Universităţii "Al.I.Cuza" Iaşi (39) 1993, f.3, p.295-303.

3. L a u g w i t z, D. – *Differential and Riemannian Geometry*, Academic Press, New York, London, 1965.

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ON GENERALIZED PSEUDO RICCI SYMMETRIC SASAKIAN MANIFOLDS

BY

U.C. DE and J.C. GHOSH

0. Introduction. The notions of Pseudo Ricci symmetric and generalized Pseudo Ricci symmetric manifolds were introduced by M.C. C h a k i in 1988 [1] and M.C.C h a k i and S.K o l e y in 1994 [2] respectively.

A non-flat Riemannian manifold (M^n, g) $(n > 2)$ is called Pseudo Ricci symmetric if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(1)\qquad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + A(Y)S(X,Z) + A(Z)S(Y,X)$$

where A is a non-zero 1-form,

$$(2)\qquad g(X,P) = A(X),$$

for every vector field X and ∇ denotes the operator of covariant differentiation with respect to metric tensor g. Such an n-dimensional manifold was denoted by $(PRS)_n$.

A type of non-flat Riemannian manifold (M^n, g) $(n > 2)$ is called generalized Pseudo Ricci symmetric if its Ricci tensor S of type (0,2) is not identically zero and satisfies the condition

$$(3)\qquad (\nabla_X S)(Y,Z) = 2A(X)S(Y,Z) + B(Y)S(X,Z) + C(Z)S(Y,X)$$

where A, B, C are three non-zero 1-forms,

$$(4)\qquad g(X,P) = A(X),\quad g(X,Q) = B(X),\quad g(X,\lambda) = C(X)$$

and ∇ has the meaning already mentioned. Such an n-dimensional manifold was denoted by $G(PRS)_n$. If, in particular $A = B = C$, then (3) will take the form (1).

Let $M^n, g)$ be an n-dimensional $(n = 2m+1,\ m > 1)$ differentiable manifold with a contact form η, the associated vector field ξ and an (1,1) tensor field ϕ. If ξ is a killing vector field, then M^n is called a K-contact Riemannian manifold. A K-contact Riemannian manifold is Sasakian [3] if and only if

$$(5) \qquad (\nabla_X\phi)(Y) = g(X,Y)\xi - \eta(Y)X$$

where ∇ denotes the Riemannian connection g.

In this paper generalized Pseudo Ricci symmetric Sasakian manifolds have been studied. At first it is shown that in a Sasakian $G(PRS)_n$ $(n > 2)$ any two of the vector fields P, Q, λ defined by (4) are orthogonal to ξ, then the other will be orthogonal to ξ. In general, it is known that the scalar curvature of a $G(PRS))_n$ is not constant [2]. Here it is shown that the scalar curvature r of a Sasakian $G(PRS)_n$ $(n > 2)$ is constant provided $Q \neq \lambda$.

Finally, considering a conformally flat Sasakian $G(PRS)_n$ $(n > 3)$, it is shown that if in such a manifold $T \neq 0, Q \neq \lambda$ where $T(X) = 2A(X) - C(X)$ then the manifold is of almost constant curvature [8].

1. Preliminaries. Let R, S, r denote respectively the curvature tensor of type (1,3), the Ricci tensor of type (0,2) and the scalar curvature of M^n. It is known that in a contact manifold M^n the Riemannian metric may be so chosen that the following relations hold [3], [4], [5].

$$(1.1) \qquad \phi(\xi) = 0$$
$$(1.2) \qquad \eta(\xi) = 1$$
$$(1.3) \qquad \phi^2(X) = -X + \eta(X)\xi$$
$$(1.4) \qquad g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$$
$$(1.5) \qquad g(X,\xi) = \eta(X)$$
$$(1.6) \qquad \nabla_X\xi = -\phi X$$
$$(1.7) \qquad S(X,\xi) = (n-1)\eta(X)$$
$$(1.8) \qquad g(R(\xi,X)Y,\xi) = g(X,Y) - \eta(X)\eta(Y)$$
$$(1.9) \qquad R(\xi,X)\xi = -X + \eta(X)\xi$$
$$(1.10) \qquad (\nabla_X\phi)(Y) = R(\xi,X)Y$$
$$(1.11) \qquad g(\phi X, Y) + g(X, \phi Y) = 0$$

for any vector fields X, Y. The above results will be used in the next sections.

2. Generalized Pseudo Ricci symmetric Sasakian manifold $G(PRS)_n$ $(n > 2)$. Now

$$(\nabla_X S)(Y,\xi) = \nabla_X S(Y,\xi) - S(\nabla_X Y,\xi) - S(Y,\nabla_X\xi).$$

Using (1.6) and (1.7) the above equation can be written as

$$(2.1) \qquad (\nabla_X S)(Y,\xi) = (n-1)(\nabla_X\eta)(Y) + S(Y,\phi X).$$

Putting $Z = \xi$ in (3) we get

$$(2.2) \quad (\nabla_X S)(Y,\xi) = 2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(Y,X).$$

From (2.1) and (2.2) we have

$$(2.3) \qquad 2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(Y,X) = \\ = (n-1)(\nabla_X\eta)(Y) + S(Y,\phi X).$$

Again putting $X = \xi$ in (2.3) and using (1.1) we get

$$(2.4) \qquad 2(n-1)\eta(P)\eta(Y) + (n-1)B(Y) + (n-1)\eta(Y)\eta(\lambda) = \\ = (n-1)(\nabla_\xi\eta)(Y).$$

Now

$$(\nabla_X g)(Z,\xi) = Xg(Z,\xi) - g(\nabla_X Z,\xi) - g(Z,\nabla_Z\xi) = \\ = X\eta(Z) - \eta(\nabla_X Z) - g(Z,\nabla_X\xi) = \\ = (\nabla_X\eta)(Z) - g(Z,\nabla_X\xi).$$

Therefore,
$(\nabla_X\eta)(Z) = g(Z,\nabla_X\xi)$, since $\nabla g = 0$.
Hence

$$(2.5) \qquad (\nabla_\xi\eta)(Z) = 0 \quad [by (1.1) and (1.6)].$$

From (2.4) and (2.5) we get

$$(2.6) \qquad 2\eta(P)\eta(Y) + B(Y) + \eta(Y)\eta(\lambda) = 0 \quad (\because n > 2).$$

Again, putting $Y = \xi$ in (2.6) we get

$$(2.7) \qquad 2\eta(P) + \eta(Q) + \eta(\lambda) = 0.$$

Hence we can state the following theorem :

Theorem 1. *If any two of the vector fields P, Q, λ are orthogonal to ξ, then the other will be orthogonal to ξ.*

Considering a vector field X on M, the tensor field $\nabla_X S$ is symmetric in its two variables. Hence it follows from (3) that

$$[B(Y) - C(Y)]L(Z) + [C(Z) - B(Z)]L(Y) = 0 \tag{2.8}$$

holds for any vector fields Y and Z on M. If in addition M is Sasakian, then regarding the case $Y = \xi$, the above equality presents

$$[\eta(Q) - \eta(\lambda)]\, L(Z) + (n-1)\, [g(\lambda, Z) - g(Q, Z)]\xi = 0. \tag{2.9}$$

Contracting (2.9) over Z it follows that

$$\eta(Q - \lambda)[r - (n-1)] = 0.$$

Then either $Q = \lambda$ or $r = (n-1)$. Thus we have the following :

Theorem 2. *In a Sasakian $G(PRS)_n$ $(n > 2)$, the scalar curvature is constant provided $Q \neq \lambda$.*

3. Conformally flat Sasakian $G(PRS)_n$ $(n > 3)$. In this section we consider the effect of conformal flatness of a Sasakian $G(PRS)_n$ on the nature of its curvature.

It is known [6] that in a conformally flat Riemannian manifold

$$(\nabla_X S)(Y, Z) - (\nabla_Z S)(Y, X) = \frac{1}{2(n-1)}[dr(X)g(Y, Z) - dr(Z)g(X, Y)]. \tag{3.1}$$

From Theorem 2 we get in a Sasakian manifold the scalar curvature of $G(PRS)_n$ $(n > 2)$ is constant, provided $Q \neq \lambda$. That is, $dr(X) = 0$. Therefore, from (3.1) it follows that

$$(\nabla_X S)(Y, Z) - (\nabla_Z S)(Y, X) = 0 \tag{3.2}$$

That is, the Ricci tensor is a Codazzi tensor. Hence the Ricci tensor $S(X, Y)$ can be written in the form according to C h a k i and K o l e y [2]

$$S(X, Y) = \sigma T(X)T(Y) \tag{3.3}$$

where $T(X) = 2A(X) - C(X)$, $\sigma = \frac{r}{T(\nu)}$ and $g(X, \nu) = T(X)$. It is known [7] that in a conformally flat Riemannian manifold (M^n, g) $(n > 3)$

$$\begin{aligned} 'R(X, Y, Z, W) = \frac{1}{n-2}[&S(Y, Z)g(X, W) - S(X, Z)g(Y, W) + \\ &+ S(X, W)g(Y, Z) - S(Y, W)g(X, Z)] + \\ &\frac{+r}{(n-1)(n-2)}[g(X, Z)g(Y, W) - g(Y, Z)g(X, W)] \end{aligned} \tag{3.4}$$

where $'R(X, Y, Z, W) = g(R(X, Y)Z, W)$. In virtue of (3.3), the equation (3.4) reduces to

$$\begin{aligned} 'R(X, Y, Z, W) &= \frac{\sigma}{n-2}[T(Y)T(Z)g(X, W) - T(X)T(Z)g(Y, W) \\ &+ T(X)T(W)g(Y, Z) - T(Y)T(W)g(X, Z)] \\ &+ \frac{r}{(n-1)(n-2)}[g(X, Z)g(Y, W) - g(Y, Z)g(X, W)] \\ &= a[g(X, Z)g(Y, W) - g(Y, Z)g(X, W)] \\ &+ b[g(X, Z)T(Y)T(W) - g(X, W)T(Y)T(Z) \\ &- g(Y, Z)T(X)T(W) + g(Y, W)T(X)T(Z)], \end{aligned} \tag{3.5}$$

where $a = \frac{r}{(n-1)(n-2)}$ and $b = \frac{-\sigma}{(n-2)}$. According to D. S m a r a n d a [8] a Riemannian manifold whose curvature tensor $'R$ is of the form (3.5) is said to be of almost constant curvature. In view of this we can state the following theorem:

Theorem 3. *If in a conformally flat Sasakian $G(PRS)_n$ $(n > 3), Q \neq \lambda$, $T \neq 0$ then the manifold is of almost constant curvature.*

The above theorem has been proved by C h a k i and K o l e y in their paper assuming constant scalar curvature.

REFERENCES

1. C h a k i, M.C. – *On Pseudo Ricci symmetric manifolds*, Bulg., J.Phys., 15(1988), 526–531.

2. C h a k i, M.C. and K o l e y, S.– *On generalized Pseudo Ricci symmetric manifolds*, Period.Math.Hung., Vol.28(2), (1994), pp.123–129.
3. B l a i r, D.E. – *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics no.509, Springer Verlag, 1976.
4. S a s a k i, S. – *Lecture Note on Almost Contact Manifolds*, Part I, Tohoku University, 1965.
5. S a s a k i, S. – *Lecture Note on Almost Contact Manifolds*, Part II, Tohoku University, Tohoku, 1967.
6. E i s e n h a r t, L.P. – *Riemannian Geometry*, Princeton University Press, 1926, p.92.
7. Y a n o, K. and K o n, M. – *Structures on Manifolds*, World Scientifics, 1989, 40.
8. S m a r a n d a, D. – *Pseudo Riemannian recurrent manifolds with almost constant curvature*. The XVIII Int. Conf. on Geometry and Topology (Oradea, 1989) 175–180, Preprint 88–2 Univ."Babeş Bolyai", Cluj–Napoca, 1988.

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THEORY OF FINSLER SPACES WITH m–th ROOT METRIC III: STRONGLY NON-RIEMANNIAN SPACES

BY

MAKOTO MATSUMOTO

Theory of Finsler spaces with $m-th$ root metric has been recently made rapid progress ([4], [7]) on the basis of two papers ([6], [8]).

The m–th root metric $L(x,y)$ which is given by

$$L^m = a_{i_1\dots i_m}(x)y^{i_1}\dots y^{i_m}$$

is seemingly a direct generalization of a Riemannian metric, because the case $m=2$ is nothing but a Riemannian metric. It may be, however, said that all the attempts to establish the Riemannian geometry–like of spaces with m–th root metric had come to nothing.

Nevertheless, to show that the m–th metrics are not so much removed from Riemannian metrics in the sense that was introduced by R.M i r o n and the present author about eighteen years ago [5], is the purpose of the present paper. The main results are stated as Theorems 3 and 4.

1. Derived F–tensors. We consider an n–dimensional Finsler space $F^n = (M^n, L(x,y))$ with the fundamental function $L(x,y)$ which is defined on the tangent bundle TM^n over a differentiable n–manifold M^n. We put $F(x,y) = L^2/2$ and define

$$F_{ij\dots k}(x,y) = \dot\partial_i\dot\partial_j\cdots\dot\partial_k F,$$

called the *derived* F-*tensors*, where $\dot\partial_i = \partial/\partial y^i$. We have first

(1.1) $$F_i = Ll_i, \quad F_{ij} = g_{ij}, \quad F_{ijk} = 2C_{ijk},$$