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THE NULL CONTROLLABILITY OF THE POPULATION DYNAMICS WITH DIFFUSION AND INHOSPITABLE BOUNDARY

BY

S. ANIŢA

1. Introduction and problem formulation. Consider a biological population which is free to move in a bounded and open subset of $\boldsymbol{R}^N$ ($N \geq 1$), with sufficiently smooth boundary $\partial\Omega$. Suppose that the boundary of the environment is extremely inhospitable. If we denote by $y(a,x,t)$ the density of population of age $a \in [0,A]$, in the position $x \in \Omega$ and at the moment $t \in \boldsymbol{R}^+$, then the dynamics of the population is described by the following system:

$$(1.1)\quad \begin{cases} y_t + y_a + \mu(a)y - \Delta_x y = u(a,x,t) & \text{on } (0,A)\times\Omega\times(0,\infty) \\ y(a,x,t) = 0 & \text{on } (0,A)\times\partial\Omega\times(0,\infty) \\ y(0,x,t) = \int_0^A b(a)y(a,x,t)da & \text{on } \Omega\times(0,\infty) \\ y(a,x,0) = y_0(a,x) & \text{on } (0,A)\times\Omega,\ \ A\in(0,\infty), \end{cases}$$

The biological significances of the terms in (1.1) are as follows:
i) μ is the rate of mortality;
ii) b is the rate of fertility;
iii) y_0 is the initial density of population;
iv) u is a possible infusion or harvest of population in Ω.

Equation $(1.1)_2$ is the mathematical formulation of the completely inhospitability of the boundary and $(1.1)_3$ (the so called "renewal equation") describes the birth.

For a more detailed description of the biological significance of the model we refer to [1], [6] and [7].

An important problem is to determine the biological population to have a certain profile $\tilde{y}(a,x)$ (independent of t). If $\tilde{y}$ is a stationary solution of (1.1) with $u=0$, then the problem is equivalent to the null controllability problem (see also [3]).

For the large time behaviour of y we refer to [7].

The null controllability of the population is realistic from biological point of view only if the control $u(a,.,t)$ is bounded (in a certain space).

Consider $r \in \boldsymbol{R}_+^*$. The problem we are going to study is now: *"to find $T \in \boldsymbol{R}_+^*$ and $u \in L^\infty((0,A)\times(0,T); L^1(\Omega))$, with $\|u(a,.,t)\|_{L^1(\Omega)} \le r$, a.e. $(a,t) \in (0,A)\times(0,T)$ such that the solution of (1.1) becames zero for $t := T$ (i.e. $y(a,x,T) = 0$ a.e. $(a,x) \in (0,A)\times\Omega$)".*

In fact we shall indicate u in a feedback form.

2. Null controllability in the heat problem. Consider the following problem:

$$(2.1)\qquad \begin{cases} z_t(t,x) - \Delta_x z(t,x) \in -r\ sgn|_{L^1(\Omega)} z(t,x) & \text{on } (0,\infty)\times\Omega \\ z(t,x) = 0 & \text{on } (0,\infty)\times\partial\Omega \\ z(0,x) = z_0(x) & \text{on } \Omega \end{cases}$$

Here we have denoted by

$$sgn|_{L^1(\Omega)} z(t,\cdot) = \begin{cases} \frac{z(t,x)}{\|z(t,.)\|_{L^1(\Omega)}}, & \text{for } \|z(t,.)\|_{L^1(\Omega)} \neq 0 \\ \{h \in L^1(\Omega);\ \|h\|_{L^1(\Omega)} \le 1\}, & \text{for } \|z(t,.)\|_{L^1(\Omega)} = 0 \end{cases}$$

The main result of this section is the following:

Theorem 2.1. *For every $z_0 \in L^1(\Omega)$, problem (2.1) has an unique weak solution, i.e., $z \in C([0,\infty); L^1(\Omega))$ and there exists $f \in L^\infty(0,\infty; L^1(\Omega))$, $f(t,x) \in sgn|_{L^1(\Omega)} z(t,x)$ a.e. $t \in (0,\infty)$ and a.e. $x \in \Omega$ such that*

$$\int_\Omega z(T,x)\zeta(T,x)dx - \int_\Omega z_0(x)\zeta(0,x)dx - \int_0^T\int_\Omega z(t,x)(\zeta_t + \Delta_x\zeta)(t,x)dxdt$$

$$= -r\int_0^T\int_\Omega f(t,x)\zeta(t,x)dxdt, \quad \forall T \in (0,\infty),$$

$$(2.2)\qquad \forall \zeta \in W = \{h \in C^{1,2}([0,\infty)\times\overline{\Omega});\ h(t,x) = 0 \text{ on } [0,\infty)\times\partial\Omega\}$$

and there exists $\tilde{T} \in (0,\infty)$ such that $\|z(t,.)\|_{L^1(\Omega)} = 0, \quad \forall t \ge \tilde{T}$.

Proof. Denote by

$$D(\mathcal{A}) = W^{2,1}(\Omega)\cap W_0^{1,1}(\Omega),$$

$$\mathcal{A}z = \Delta_x z, \quad \forall z \in D(\mathcal{A})$$

It is well known that $\mathcal{A}$ is m-dissipative and is the generator of a compact semigroup on $L^1(\Omega)$ (see [5]). Since the mapping $z \mapsto -r\ \|z\|_{L^1(\Omega)}^{\varepsilon-1} z$ ($\varepsilon > 0$) is dissipative and continuous on $L^1(\Omega)$ we may conclude that $z \mapsto \mathcal{A}z - r\ \|z\|_{L^1(\Omega)}^{\varepsilon-1} z$ is m-dissipative in $L^1(\Omega)$.

We may conclude now that the following problem:

$$(2.3)\qquad \begin{cases} z_t(t,x) - \Delta_x z(t,x) = -r\ \|z(t,.)\|_{L^1(\Omega)}^{\varepsilon-1} z(t,x) & \text{on } (0,\infty)\times\Omega \\ z(t,x) = 0 \text{ on } (0,\infty)\times\partial\Omega \\ z(0,x) = z_0(x) & \text{on } \Omega\ , \end{cases}$$

or equivalentely

$$(2.4)\qquad \begin{cases} z'(t) - \mathcal{A}z(t) = -r\ \|z(t)\|_{L^1(\Omega)}^{\varepsilon-1} z(t), & t \in (0,\infty) \\ z(0) = z_0, \end{cases}$$

(in the space $L^1(\Omega)$) has an unique mild solution, which is also an integral solution and a weak solution.

Denote by z_ε the unique mild solution of (2.3) (or equivalentely (2.4)). It follows immediately that $\|z_\varepsilon(t)\|_{L^1(\Omega)} \le \|z_0\|_{L^1(\Omega)}$ $\forall t \in [0,\infty)$, $\forall 0 < \varepsilon < 1$ (because $z \mapsto \mathcal{A}z - r\ \|z\|_{L^1(\Omega)}^{\varepsilon-1} z$ is m-dissipative in $L^1(\Omega)$).

Since $\| -r\ \|z_\varepsilon(t)\|_{L^1(\Omega)}^{\varepsilon-1} z_\varepsilon(t)\|_{L^1(\Omega)} = r\ \|z_\varepsilon(t)\|_{L^1(\Omega)}^{\varepsilon} \le$ $\le r \cdot \max\{\|z_0\|_{L^1(\Omega)}, 1\}$, $\forall t \in \boldsymbol{R}_+$, $\forall 0 < \varepsilon < 1$, we may conclude via Baras Theorem (see also [4]) that $z_\varepsilon \to z$ in $C([0,T]; L^1(\Omega))$, $\forall T \in (0,\infty)$ on a subsequence (also denoted by z_ε).

Since z_ε is a weak solution of (2.3) we get

$$\int_\Omega z_\varepsilon(T,x)\zeta(T,x)dx - \int_\Omega z_0(x)\zeta(0,x)dx - \int_0^T\int_\Omega z_\varepsilon(t,x)(\zeta_t + \Delta_x\zeta)(t,x)dxdt$$

$$(2.5)\quad = -r\int_0^T\int_\Omega \|z_\varepsilon(t,.)\|_{L^1(\Omega)}^{\varepsilon-1} z_\varepsilon(t,x)\zeta(t,x)dxdt,\quad \forall T\in(0,\infty),\ \forall\zeta\in W$$

The boundedness of $f_\varepsilon = \|z_\varepsilon\|_{L^1(\Omega)}^{\varepsilon-1} z_\varepsilon$ in $L^1(0,T;L^1(\Omega))$, $\forall T\in(0,\infty)$ implies the existence of a Radon measure $\mu\in M((0,T)\times\Omega)$ such that

$$f_\varepsilon \to^{w^*} \mu \quad in\ M((0,T)\times\Omega)$$

(for $\varepsilon\to 0$, on a subsequence also denoted by f_ε).

Passing to the limit in (2.5) we obtain

$$\int_\Omega z(T,x)\zeta(T,x)dx - \int_\Omega z_0(x)\zeta(0,x)dx - \int_0^T\int_\Omega z(t,x)(\zeta_t+\Delta_x\zeta)(t,x)dxdt$$

$$(2.6)\qquad = -r\langle\zeta,\mu\rangle,\quad \forall T\in(0,\infty),\ \forall\zeta\in W,$$

where $\langle .,.\rangle$ is the pairing between $L^\infty((0,T)\times\Omega)$ and $M((0,T)\times\Omega)$.

Consider $\tilde T = ess\ inf\ \{t\in[0,\infty);\ z(t,x)=0,\ \ a.e.\ x\in\Omega\}\in[0,\infty)$. For $t\le\tilde T$ we have $\mu(t,\cdot) = sgn|_{L^1(\Omega)} z(t,\cdot)$.

We shall prove that $\tilde T \le \dfrac{\|z_0\|_{L^1(\Omega)}}{r}$.

Indeed, we consider $g_n\in C^{1,2}([0,\tilde T]\times\overline\Omega)$, $g_n\to g\in -r\ sgn_{L^1(\Omega)} z$ in $L^\infty(0,\tilde T;L^1(\Omega))$, $z_{0n}\in D(A)$, $z_{0n}\to z_0$ in $L^1(\Omega)$ and denote by $\tilde z_n$ the strong solution of

$$(2.7)\qquad \begin{cases} \tilde z_{n,t}(t,x) - \Delta_x\tilde z_n(t,x) = g_n(t,x), & \text{on } (0,\tilde T)\times\Omega\\ \tilde z_n(t,x) = 0 & \text{on } (0,\tilde T)\times\partial\Omega\\ \tilde z_n(0,x) = z_{0n}, & \text{on } \Omega\end{cases}$$

The compactness result of Baras assures that $\tilde z_n\to z$ in $C([0,\tilde T];L^1(\Omega))$.

Multiplying $(2.7)_1$ by

$$sgn\ \tilde z_n(t,x) = \begin{cases} 1, & \text{if } \tilde z_n(t,x)>0\\ 0, & \text{if } \tilde z_n(t,x)=0\\ -1, & \text{if } \tilde z_n(t,x)<0,\end{cases}$$

and integrating on $(0,t)\times\Omega$ we obtain

$$(2.8)\quad \|\tilde z_n(t,.)\|_{L^1(\Omega)} \le \|z_{0n}\|_{L^1(\Omega)} - \int_0^t\int_\Omega g_n(s,x)\ sgn\ \tilde z_n(s,x)dxds,\quad \forall t\in[0,\tilde T]$$

Passing to the limit in (2.8) we conclude that

$$(2.9)\qquad \|z(t,.)\|_{L^1(\Omega)} \le \|z_0\|_{L^1(\Omega)} - rt,\quad \forall t\in[0,\tilde T]$$

Since $\|z(\tilde T,.)\|_{L^1(\Omega)} = 0$ and taking into account (2.9) we may conclude that $\tilde T\le\dfrac{\|z_0\|_{L^1(\Omega)}}{r}$.

The uniqueness of the solution follows imediately from the dissipativity of $A + r\ sgn|_{L^1(\Omega)}$ and using appropriate test functions ζ.

Remark. For related results we refer to [8] (there is a very different approach).

3. Hypotheses and the main result. We shall use the following hypotheses:

$\mu\in C([0,A])$, $\mu(a)\ge 0$ for all $a\in[0,A]$;
$b\in L^\infty(0,A)$, $b(a)\ge 0$ a.e. in $(0,A)$ and $b(a)=0$ a.e. in $(0,\eta)$ (where $\eta\in(0,A)$);
$y_0\in C([0,A];L^1(\Omega))$, $y_0(a,x)\ge 0$ a.e. in $(0,A)\times\Omega$ and

$$y_0(0,x) = \int_0^A b(a)y_0(a,x)da,\quad a.e.\ x\in\Omega.$$

These hypotheses are natural; in fact, the biologists have observed that the graphs of b and μ are looking as in Fig. 1 (see [2])

Definition. *For $u\in L^1_{loc}(0,\infty;L^1((0,A)\times\Omega))$ we say that $y\in C([0,A]\times[0,\infty);L^1(\Omega))$ is a weak solution of (1.1) iff*

$$\int_0^T\int_\Omega y(A,x,t)\zeta(A,x,t)dxdt + \int_0^A\int_\Omega y(a,x,T)\zeta(a,x,T)dxda$$

$$-\int_0^T\int_\Omega\int_0^A b(a)y(a,x,t)\zeta(0,x,t)dadxdt - \int_0^A\int_\Omega y_0(a,x)\zeta(a,x,0)dxda$$

$$-\int_0^A\int_0^T\int_\Omega y(a,x,t)[\zeta_t+\zeta_a-\mu(a)\zeta+\Delta_x\zeta](a,x,t)dxdadt$$

$$= \int_0^A \int_0^T \int_\Omega u(a,x,t)\zeta(a,x,t)dxdadt, \quad \forall T \in (0,\infty),$$

$$\forall \zeta \in \mathcal{W} = \{h \in C^{1,2,1}([0,A] \times \overline{\Omega} \times [0,\infty)); \;\; h(a,x,t) = 0, \;\; \forall x \in \partial\Omega\}.$$

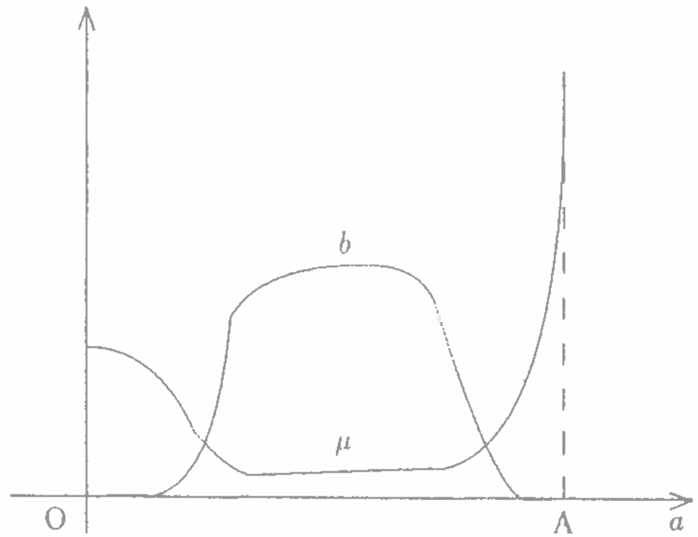

Fig. 1

Consider the following problem:

(3.1)
$$\begin{cases} y_t + y_a + \mu y - \Delta_x y \in -r\ sgn|_{L^1(\Omega)} y(a,x,t) & \text{on } (0,A) \times \Omega \times (0,\infty) \\ y(a,x,t) = 0 & \text{on } (0,A) \times \partial\Omega \times (0,\infty) \\ y(0,x,t) = y_1(x,t) & \text{on } \Omega \times (0,\infty) \\ y(a,x,0) = y_0(a,x) & \text{on } (0,A) \times \Omega, \end{cases}$$

where $y_1 \in C([0,\infty); L^1(\Omega))$. Using the results in the previous section we may conclude that system (3.1) has an unique weak solution on every segment of equation $a - t = const.$ (see Fig. 2) and since $y_0 \in C([0,A]; L^1(\Omega))$ and $y_1 \in C([0,\infty); L^1(\Omega))$ we conclude the existence of a weak solution of (3.1), i.e., $y \in C([0,A] \times [0,\infty); L^1(\Omega))$ and

$$\int_0^T \int_\Omega y(A,x,t)\zeta(A,x,t)dxdt + \int_0^A \int_\Omega y(a,x,T)\zeta(a,x,T)dxda$$

$$- \int_0^T \int_\Omega y_1(x,t)\zeta(0,x,t)dadxdt - \int_0^A \int_\Omega y_0(a,x)\zeta(a,x,0)dxda$$

$$- \int_0^A \int_0^T \int_\Omega y(a,x,t)[\zeta_t + \zeta_a - \mu(a)\zeta + \Delta_x\zeta](a,x,t)dxdadt$$

$$= -r \int_0^A \int_0^T \int_\Omega u(a,x,t)\zeta(a,x,t)dxdadt, \quad \forall T \in (0,\infty), \; \forall \zeta \in \mathcal{W}$$

(where $u(a,.,t) \in sgn|_{L^1(\Omega)} y(a,.,t)$; see also [2]).

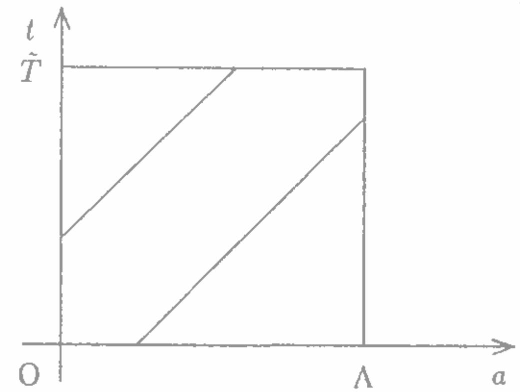

Fig. 2

Using now Banach's fixed point theorem we infer the existence of a weak solution of (1.1) with $u := -r\ sgn|_{L^1(\Omega)} y$. Choosing appropriate test functions ζ follows the uniqueness of the weak solution.

Theorem 3.1. *For $r \in (0,\infty)$ large enough, the feedback control*

$$u(a,x,t) = -r\ sgn|_{L^1(\Omega)} y(a,x,t) \tag{3.2}$$

steers the initial data $y_0 \in C([0,A]; L^1(\Omega))$ (in (1.1)) into the origin in finite time (i.e., there exists $\tilde{T} \in [0,\infty)$ such that $y(.,.,\tilde{T}) = 0$ in $L^1((0,A) \times \Omega)$).

Proof. Denote by $y_1(x,t) = \int_0^A b(a)y(a,x,t)da$. The weak solution of (1.1) with $u(a,x,t) := -r\ sgn|_{L^1(\Omega)} y(a,x,t)$ (denoted by y) is also the weak solution of (3.1). We choose now $f_n \in C^{1,2,1}([0,A]\times\overline{\Omega}\times[0,\infty))$, $f_n \to f \in -r\ sgn|_{L^1(\Omega)} y$ in $L^\infty((0,A)\times(0,\infty);L^1(\Omega))$, $y_{0n}\in C([0,A];D(A))$, $y_{0n}\to y_0$ in $C([0,A];L^1(\Omega))$ and $y_{1n}\in C([0,\infty);D(A))$, $y_{1n}\to y_1$ in $C([0,T];L^1(\Omega))$, $\forall T\in(0,\infty)$.

Denote by y_n the strong solution (which is also a weak solution) of

$$(3.3)\qquad \begin{cases} y_t + y_a + \mu(a)y - \Delta_x y = f_n & \text{on } (0,A)\times\Omega\times(0,\infty) \\ y(a,x,t) = 0 & \text{on } (0,A)\times\partial\Omega\times(0,\infty) \\ y(0,x,t) = y_{1n}(x,t) & \text{on } \Omega\times(0,\infty) \\ y(a,x,0) = y_{0n}(a,x) & \text{on } (0,A)\times\Omega, \end{cases}$$

We may conclude that $y_n \to y$ in $C([0,A]\times[0,T];L^1(\Omega))$, $\forall T\in(0,\infty)$.

Multiplying $(3.3)_1$ by

$$sgn\ y_n(a,x,t) = \begin{cases} 1, & \text{if } y_n(a,x,t) > 0 \\ 0, & \text{if } y_n(a,x,t) = 0 \\ -1, & \text{if } y_n(a,x,t) < 0, \end{cases}$$

we obtain after an easy calculation

$$\int_0^A\int_0^T\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\mu(a)\Big)\|y_n(a,.,t)\|_{L^1(\Omega)}dtda$$

$$\le \int_0^A\int_0^T\int_\Omega f_n\ sgn\ y_n dxdadt$$

$$\Rightarrow \int_0^A\|y_n(a,.,T)\|_{L^1(\Omega)}da \le \int_0^A\|y_{0n}(a,.)\|_{L^1(\Omega)}da + \int_0^T\|y_{1n}(.,t)\|_{L^1(\Omega)}dt$$

$$(3.4)\qquad + \int_0^A\int_0^T\int_\Omega f_n\ sgn\ y_n dxdadt$$

Passing to the limit we obtain

$$\|y(.,.,T)\|_{L^1((0,A)\times\Omega)} \le \|y_0\|_{L^1((0,A)\times\Omega)}$$

$$(3.5)\qquad + \int_0^T\int_0^A\int_\Omega |b(a)y(a,x,t)|dxdadt, \quad \forall T\in(0,\infty)$$

Relation (3.5) allow us to conclude that

$$(3.6)\qquad \|y(.,.,t)\|_{L^1((0,A)\times\Omega)} \le \|y_0\|_{L^1((0,A)\times\Omega)}e^{\beta t}, \quad \forall t\in[0,\infty),$$

where $\beta = \|b\|_{L^\infty(0,A)}$.

Taking now into account the "renewal equation" and (3.6) we infer:

$$\|y(0,.,t)\|_{L^1(\Omega)} \le \beta e^{\beta t}\|y_0\|_{L^1((0,A)\times\Omega)}, \quad \forall t\in \boldsymbol{R}^+$$

$$(3.7)\qquad \Rightarrow \|y_1(.,t)\|_{L^1(\Omega)} \le \beta e^{\beta(A-\eta)}\|y_0\|_{L^1((0,A)\times\Omega)}, \quad \forall t\in[0,A-\eta]$$

We choose $r = \dfrac{\beta e^{\beta(A-\eta)}\|y_0\|_{L^1((0,A)\times\Omega)}}{\eta}$. Since, y is also a weak solution on every segment of equation $a-t=const.$ and using the results from Section 2, we may conclude that

$$\|y(a,.,t)\|_{L^1(\Omega)} = 0, \quad \forall a\in[\eta,A], \quad \forall t\ge a$$

$$\Rightarrow y(0,x,t) = \int_0^A b(a)y(a,x,t)da$$

$$(3.8)\qquad = \int_\eta^A b(a)y(a,x,t)da = 0, \quad a.e.\ x\in\Omega,\ \forall t\ge A$$

Relation (3.8) implies $\|y(a,.,A+\eta)\|_{L^1(\Omega)} = 0$ for all $a\in[0,A]$ and this ends the proof.

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ON ASYMPTOTIC BEHAVIOUR OF NEUTRAL INTEGRODIFFERENTIAL EQUATIONS

BY

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Keywords and phrases: Neutral integrodifferential equation, zero-crossings, oscillations. AMS subject classification: 34 K 20, 92 A 15.

Introduction. Oscillations of delay differential equations have been extensively discussed by numerous authors. However oscillatory behaviour of integrodifferential equations especially those with infinite delay has received little attention in the literature.

In population dynamic models of integrodifferential equations, oscillation of model systems has certain ecological and evolutionary regnificance ([1–11]).

In this paper, we examine a necessary and sufficient condition for all solutions of the neutral integrodifferential equations of the form

$$(1.1)\qquad \frac{d}{dt}\left[x(t) - c\int_0^\infty H(s)x(t-s)ds\right] + a\int_0^\infty k(s)x(t-s)ds + bx(t) = 0$$

to have at least one zero on $(-\infty, +\infty)$ under the assumptions:

$$(A)\qquad \int_0^\infty k(s)ds = 1,\quad \int_0^\infty sk(s)ds < \infty,\quad K : [0,\infty) \longrightarrow [0,\infty)$$

and

$$\int_0^\infty H(s)ds < \infty,\quad \int_0^\infty sH(s)ds < \infty,\quad H : [0,\infty) \longrightarrow [0,\infty);$$