ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL.I.CUZA" IAŞI
Tomul XLIII, s.I.a, Matematică, 1997, f1.

SPINOR BUNDLE OF ORDER TWO ON THE INTERNAL DEFORMED SYSTEM

BY

P.C.STAVRINOS, V.BALAN, N.PREZAS and P.MANOUSELIS

Abstract. Using the Hilbert–Palatini technique for a Utiyama–type Lagrangian density in the deformed spinor bundle $S^{(2)}M \times R$, the authors determine the explicit expressions of the field equations, generalising previous results; also, the equivalence principle is shown to represent an extension for the corresponding one from $S^{(2)}M$.

1. Introduction. In previous papers [4], [5], was presented the concept of a spinor bundle $S^{(2)}M$ and its relation to the Poincarè group. This group, consisting of the set of rotations, boosts and translations, gives an exact meaning to the terms: "momentum", "energy", "mass", and "spin" and is used to determine characteristics of the elementary particles. Also, it is a gauge, acting locally in the space-time. Hence we may perform Poincarè transformations for a physical approach. In that study we considered a base manifold $(M, g_{\mu\nu}(x, \xi, \overline{\xi}))$ where the metric tensor depends on the position coordinates and the spinor (Dirac) variables $(\xi_\alpha, \overline{\xi}^\alpha) \in C^4 \times C^4$. A spinor bundle $S^{(1)}(M)$ can be constructed from one of the principal fiber bundles with fiber $F = C^4$. Each fiber is diffeomorphic with one proper Lorentz group.

In our study we apply an analogous method as in the theory of deformed bundles developed in [6], for the case of a **spinor bundle** $S^{(2)}M = M \times C^{4\cdot 2}$ in connection with a deformed **internal fibre** R. Namely our space has the form $S^{(2)}M \times R$. The cosideration of Miron d–connections [1], which preserve the h– and v–distributions is of vital importance in our approach, as in the previous work. This standpoint enables us to use a more general group $G^{(2)}$, called the structural group of all rotations and translations, that is isomorphic to the Poincarè Lie algebra. A **spinor** is an element of

the spinor bundle $S^{(2)}(M) \times R$ where R represents the **internal fibre of deformation.** The local variables are in this case

$$(x^\mu, \xi_\alpha, \overline{\xi}^\alpha, \lambda) \in S^{(2)}(M) \times R = \tilde{S}^{(2)}(M), \lambda \in R.$$

The non–linear connection on $\tilde{S}^{(2)}(M)$ is defined analogously, as for the vector bundles of order two [2], [3]

$$T(\tilde{S}^{(2)}M) = H(\tilde{S}^2 M) \oplus \mathcal{F}^{(1)}(\tilde{S}^{(2)}M) \oplus \mathcal{F}^{(2)}(\tilde{S}^{(2)}M) \oplus R,$$

where $\mathcal{H}$,$\mathcal{F}^{(1)}$,$\mathcal{F}^{(2)}$, R represent the horizontal, vertical, normal and deformation distributions respectively.

The fundamental gauge 1–form fields which take values from the Lie algebra of the Poincarè group will have the following form in the local bases of their corresponding distributions

$$\rho_\mu(x, \xi, \overline{\xi}, \lambda) = \frac{1}{2}\omega_\mu^{*ab}\mathcal{J}_{ab} + h_\mu^a(x, \xi, \overline{\xi}, \lambda)P_a \tag{1}$$

$$\zeta_\alpha = \frac{1}{2}\theta_\alpha^{(*)ab}J_{ab} + \psi_\alpha^a P_a \tag{2}$$

$$\overline{\zeta}^\alpha = \frac{1}{2}\overline{\theta}^{(*)\alpha ab}J_{ab} + \overline{\psi}^{\alpha a}P_a \tag{3}$$

$$\rho_o = \frac{1}{2}\omega_o^{ab}J_{ab} + L_o^a P_a \tag{4}$$

where, J_{ab}, P_a are the generators of the four–dimensional Poincarè group, namely the angular momentum and linear momentum, $\omega_\mu^{(*)ab}$ represent the Lorentz – spin connection coefficients, $\overline{\Psi}^{\alpha a}$, Ψ_α^a, $\theta_\alpha^{(*)ab}$, $\overline{\theta}^{(*)\alpha ab}$ are the given spin–tetrad and spin – connection coefficients, and L_o^a deformed tetrad coefficients. We use Greek letters $\lambda, \mu, \nu, \ldots$ for space–time indices, $\alpha, \beta, \gamma, \ldots$ for the spinor, $a, b, c, \ldots$ for Lorentz ones, and the index (o) represents the deformed variable; λ, α, $a = 1, \ldots, 4$. The general transformations of coordinates on $\tilde{S}^{(2)}M$ are

$$x'^\mu = x'^\mu(x^\nu), \xi'_\alpha = \xi'_\alpha(\xi_\beta, \overline{\xi}^\beta), \overline{\xi'}^\alpha = \overline{\xi'}^\alpha(\overline{\xi}^\beta, \xi_\beta), \lambda' = \lambda \tag{5}$$

2. Connections. We define the following gauge covariant derivatives

$$(a)\mathcal{D}^{(*)}_{\mu}=\frac{\delta}{\delta x^{\mu}}+\frac{1}{2}\omega^{(*)ab}_{\mu}\mathcal{J}_{ab}\qquad (b)\mathcal{D}^{(*)\alpha}=\frac{\delta}{\delta\xi_{\alpha}}+\frac{1}{2}\overline{\Theta}^{(*)\alpha ab}\mathcal{J}_{ab}$$

$$(c)\mathcal{D}^{(*)}_{\alpha}=\frac{\partial}{\partial\overline{\xi}^{\alpha}}+\frac{1}{2}\Theta^{(*)ab}_{\alpha}\mathcal{J}_{ab}\qquad (d)\mathcal{D}^{(*)}_{o}=\frac{\partial}{\partial\lambda}+\omega^{ab}_{o}\mathcal{J}_{ab} \tag{6}$$

where,

$$\frac{\delta}{\delta x^{\mu}}=\frac{\partial}{\partial x^{\mu}}+\mathcal{N}_{\alpha\mu}\frac{\partial}{\partial\xi_{\alpha}}-\overline{\mathcal{N}}^{\alpha}_{\mu}\frac{\partial}{\partial\overline{\xi}^{\alpha}}-\mathcal{N}^{o}_{\mu}\frac{\partial}{\partial\lambda}$$

$$\frac{\delta}{\delta\xi_{\alpha}}=\frac{\partial}{\partial\xi_{\alpha}}-\overline{\mathcal{N}}^{\alpha\beta}_{o}\frac{\partial}{\partial\overline{\xi}^{\beta}},\frac{\partial}{\partial\lambda}=L^{\mu}_{o}\frac{\partial}{\partial x^{\mu}} \tag{7}$$

The non–linear connection coefficients are defined further. The space –time, Lorentz, spin frames and the deformed frame are connected by the relations

$$(a)\frac{\delta}{\delta x^{\mu}}=h^{a}_{\mu}\frac{\delta}{\delta x^{a}}\qquad (b)\frac{\partial}{\partial\xi_{\alpha}}=\overline{\Psi}^{\alpha a}\frac{\partial}{\partial x^{a}} \tag{8}$$

$$(b')\frac{\partial}{\partial\overline{\xi}^{\alpha}}=\Psi^{a}_{\alpha}\frac{\partial}{\partial x^{a}}\qquad (c)\frac{\partial}{\partial\lambda}=L^{\mu}_{o}\frac{\partial}{\partial x^{\mu}}$$

The relation $(8a)$ is a generalization of the well – known principle of equivalence in the total space of the spinor bundle $S^{(2)}M$. In addition, the relations $(8a,b,b',c)$ represent a generalized form of the equivalence principle, since the considered deformed spinor bundle contains spinors as internal variables.

The absolute differential of an arbitrary contravariant vector X^{ν} in $\tilde{S}^{(2)}M$, has the form

$$\mathcal{D}X^{\nu}=(\mathcal{D}^{(*)}_{\mu}X^{\nu})dx^{\mu}+(\mathcal{D}^{(*)\alpha}X^{\nu})d\xi_{\alpha}+ \tag{9}$$

$$+(\mathcal{D}^{(*)}_{\alpha}X^{\nu})d\overline{\xi}^{\alpha}+(\mathcal{D}^{(*)}_{o}X^{\nu})d\lambda$$

The differentials $\mathcal{D}\xi_{\alpha}$, $\mathcal{D}\overline{\xi}^{\alpha}$, $\mathcal{D}\lambda$ can be written

$$\mathcal{D}\xi_{\beta}=d\xi_{\beta}+\mathcal{N}^{o}_{\alpha\beta}d\overline{\xi}^{\alpha}+\tilde{\mathcal{N}}^{o\alpha}_{\beta}d\xi_{\alpha}+\mathcal{N}_{\beta\mu}dx^{\mu}$$

(10) $$\mathcal{D}\overline{\xi}^{\beta} = d\overline{\xi}^{\beta} + \mathcal{N}^{\beta}_{o\alpha} d\overline{\xi}^{\alpha} - \tilde{\mathcal{N}}^{\beta\alpha}_{o} d\xi_{\alpha} - \overline{\mathcal{N}}^{\beta}_{\mu} dx^{\mu}$$

$$\mathcal{D}_{\circ}\lambda = d\lambda + \mathcal{N}^{\circ}_{\kappa} dx^{\kappa} - \tilde{\mathcal{N}}^{\alpha}_{\circ} d\xi_{\alpha} - \mathcal{N}^{\circ}_{\alpha} d\overline{\xi}^{\alpha}$$

where,

$$\mathcal{N} = \{\mathcal{N}^{\circ}_{\alpha\beta}, \mathcal{N}_{\beta\mu}, \tilde{\mathcal{N}}^{o\alpha}_{\beta}, \overline{\mathcal{N}}^{\beta}_{\mu}, \tilde{\mathcal{N}}^{\beta\alpha}_{\circ}, \mathcal{N}^{\beta}_{o\alpha}, \mathcal{N}^{\circ}_{\kappa}, \tilde{\mathcal{N}}^{\beta}_{\circ}, \mathcal{N}^{\circ}_{\alpha}\}$$

represent the coefficients of the non-linear connection which are given by

$$\mathcal{N}_{\beta\mu} = \frac{1}{2}\omega^{(*)ab}_{\mu}\mathcal{J}_{ab}\xi_{\beta}, \qquad \overline{\mathcal{N}}^{\beta}_{\mu} = -\frac{1}{2}\omega^{(*)ab}_{\mu}\mathcal{J}_{ab}\overline{\xi}^{\beta}, \mathcal{N}^{\circ}_{\mu} = \frac{1}{2}\omega^{ab}_{o\mu}\mathcal{J}_{ab}$$

(11) $$\tilde{\mathcal{N}}^{o\alpha}_{\beta} = \frac{1}{2}\overline{\theta}^{(*)\alpha ab}\mathcal{J}_{ab}\xi_{\beta}, \tilde{\mathcal{N}}^{\alpha\beta}_{\circ} = -\frac{1}{2}\overline{\theta}^{(*)\alpha ab}\mathcal{J}_{ab}\overline{\xi}^{\beta},$$

$$\tilde{\mathcal{N}}^{\alpha}_{\circ} = \frac{1}{2}\omega^{ab}_{\circ}\mathcal{J}_{ab}\overline{\xi}^{\alpha},$$

$$\mathcal{N}^{\circ}_{\alpha\beta} = \frac{1}{2}\theta^{(*)ab}_{\alpha}\mathcal{J}_{ab}\xi_{\beta}, \mathcal{N}^{\beta}_{o\alpha} = -\frac{1}{2}\theta^{(*)ab}_{\alpha}\mathcal{J}_{ab}\overline{\xi}^{\beta}, \mathcal{N}^{\circ}_{\alpha} = \frac{1}{2}\omega^{ab}_{\circ}\mathcal{J}_{ab}\xi_{\alpha}.$$

The metrical structure in the deformed spinor bundle $\tilde{S}^{(2)}M$ has the form

(12) $$G = g_{\mu\nu}(x,\xi,\overline{\xi},\lambda)dx^{\mu}\otimes dx^{\nu} + g_{\alpha\beta}(x,\xi,\overline{\xi},\lambda)\mathcal{D}\overline{\xi}^{\alpha}\otimes\mathcal{D}\overline{\xi}^{*\beta} +$$

$$+g^{\alpha\beta}(x,\xi,\overline{\xi},\lambda)\mathcal{D}\xi_{\alpha}\otimes\mathcal{D}\xi^{*}_{\beta} + g_{oo}(x,\xi,\overline{\xi},\lambda)\mathcal{D}\lambda\otimes\mathcal{D}\lambda$$

where $'^{*'}$ denotes Hermitean conjugation. The local adapted frame is given by

$$\frac{\delta}{\delta\zeta^{A}} = \{\frac{\delta}{\delta x^{\lambda}}, \frac{\delta}{\delta\xi_{\alpha}}, \frac{\partial}{\partial\overline{\xi}^{\alpha}}, \frac{\partial}{\partial\lambda}\}$$

and the associated dual frame

(13) $$\delta\zeta^{A} = \{\mathcal{D}x^{\kappa} \equiv dx^{\kappa}, \mathcal{D}\xi_{\beta}, \mathcal{D}\overline{\xi}^{\beta}, \mathcal{D}_{\circ}\lambda\},$$

where the terms $\frac{\delta}{\delta x^{\lambda}}, \frac{\delta}{\delta\xi_{\alpha}}, \mathcal{D}_{\circ}\lambda, \mathcal{D}x^{\kappa}, \mathcal{D}\xi_{\beta}, \mathcal{D}\overline{\xi}^{\beta}$, are provided by the relations (7), (9), (10).

The considered connection in $\tilde{S}^{(2)}(M)$ is a *d*- -connection, having with respect to the adapted basis the coefficients

(14) $$\Gamma^{A}_{BC} = \{\Gamma^{(*)\mu}_{\nu\rho}, C^{\mu}_{\nu\alpha}, \overline{C}^{\mu\alpha}_{\nu}, \Gamma^{(*)\mu}_{\nu o}, \overline{\Gamma}^{(*)\alpha}_{\beta\lambda}, \tilde{C}^{\beta}_{\alpha\gamma}, \tilde{C}^{\beta\alpha}_{\gamma}, \overline{\Gamma}^{(*)\beta}_{\circ\gamma},$$

$$\Gamma^{(*)\beta}_{\alpha\nu}, C^{\gamma}_{\beta\alpha}, C^{\gamma\alpha}_{\beta}, C^{\alpha}_{\beta\circ}, \Gamma^{(*)\circ}_{\circ\mu}, \overline{C}^{\circ\alpha}_{\circ}, C^{\circ}_{\circ\alpha}, L^{\circ}_{\circ\circ}\}$$

defined by the generic relations

$$\mathcal{D}_{\frac{\delta}{\delta z^C}} \frac{\delta}{\delta z^B} = \Gamma^A_{BC} \frac{\delta}{\delta z^A}, \qquad \frac{\delta}{\delta^A_z} \in \left\{ \frac{\delta}{\delta x^\mu}, \frac{\delta}{\delta \xi_\alpha}, \frac{\partial}{\partial \overline{\xi}^\alpha}, \frac{\partial}{\partial \lambda} \right\}. \tag{15}$$

It preserves the distributions $\mathcal{H}, \mathcal{F}^{(1)}, \mathcal{F}^{(2)}, R$, and its coefficients are defined by

$$\mathcal{D}_{\frac{\delta}{\delta x^\rho}} \frac{\delta}{\delta x^\nu} = \Gamma^{(*)\mu}_{\nu\rho} \frac{\delta}{\delta x^\mu}, \qquad \mathcal{D}_{\frac{\partial}{\partial \overline{\xi}^\alpha}} \frac{\delta}{\delta x^\nu} = C^{\mu}_{\nu\alpha} \frac{\delta}{\delta x^\mu},$$

$$\mathcal{D}_{\frac{\delta}{\delta \xi_\alpha}} \frac{\delta}{\delta x^\nu} = \overline{C}^{\mu\alpha}_{\nu} \frac{\delta}{\delta x^\mu}, \qquad \mathcal{D}_{\frac{\partial}{\partial \lambda}} \frac{\delta}{\delta x^\nu} = \Gamma^{(*)\mu}_{\nu o} \frac{\delta}{\delta x^\mu},$$

$$\mathcal{D}_{\frac{\delta}{\delta x^\lambda}} \frac{\delta}{\delta \xi_\alpha} = \overline{\Gamma}^{(*)\alpha}_{\beta\lambda} \frac{\delta}{\delta \xi_\beta}, \qquad \mathcal{D}_{\frac{\partial}{\partial \overline{\xi}_\alpha}} \frac{\delta}{\delta \xi_\beta} = \tilde{C}^{\beta}_{\alpha\gamma} \frac{\delta}{\delta \xi_\gamma},$$

$$\mathcal{D}_{\frac{\delta}{\delta \xi_\alpha}} \frac{\delta}{\delta \xi_\beta} = \tilde{C}^{\beta\alpha}_{\gamma} \frac{\delta}{\delta \xi_\gamma}, \qquad \mathcal{D}_{\frac{\partial}{\partial \lambda}} \frac{\delta}{\delta \xi_\beta} = \overline{\Gamma}^{(*)\beta}_{o\gamma} \frac{\delta}{\delta \xi_\gamma},$$

$$\mathcal{D}_{\frac{\delta}{\delta x^\nu}} \frac{\partial}{\partial \overline{\xi}^\alpha} = \Gamma^{(*)\beta}_{\alpha\nu} \frac{\partial}{\partial \overline{\xi}^\beta}, \qquad \mathcal{D}_{\frac{\partial}{\partial \overline{\xi}^\alpha}} \frac{\partial}{\partial \overline{\xi}^\beta} = C^{\gamma}_{\beta\alpha} \frac{\partial}{\partial \overline{\xi}^\gamma},$$

$$\mathcal{D}_{\frac{\delta}{\delta \xi_\alpha}} \frac{\partial}{\partial \overline{\xi}^\beta} = C^{\gamma\alpha}_{\beta} \frac{\partial}{\partial \overline{\xi}^\gamma}, \qquad \mathcal{D}_{\frac{\partial}{\partial \lambda}} \frac{\partial}{\partial \overline{\xi}^\beta} = C^{\alpha}_{\beta o} \frac{\partial}{\partial \overline{\xi}^\alpha},$$

$$\mathcal{D}_{\frac{\delta}{\delta x^\mu}} \frac{\partial}{\partial \lambda} = \Gamma^{(*)\circ}_{\circ\mu} \frac{\partial}{\partial \lambda}, \qquad \mathcal{D}_{\frac{\delta}{\delta \xi_\alpha}} \frac{\partial}{\partial \lambda} = \overline{C}^{\circ\alpha}_{\circ} \frac{\partial}{\partial \lambda}$$

$$\mathcal{D}_{\frac{\partial}{\partial \overline{\xi}^\alpha}} \frac{\partial}{\partial \lambda} = C^{\circ}_{\circ\alpha} \frac{\partial}{\partial \lambda}, \qquad \mathcal{D}_{\frac{\partial}{\partial \lambda}} \frac{\partial}{\partial \lambda} = L^{\circ}_{\circ\circ} \frac{\partial}{\partial \lambda},$$

The covariant differentiation of tensors, spin-tensors and Lorentz – type tensors of arbitary rank is defined as follows:

$$\nabla_\kappa T^{\mu\ldots}_{\nu\ldots} = \frac{\delta T^{\mu\ldots}_{\nu\ldots}}{\delta x^\kappa} + \Gamma^{(*)\mu}_{\rho\kappa} T^{\rho\ldots}_{\nu\ldots} + \ldots - \Gamma^{(*)\rho}_{\nu\kappa} T^{\mu\ldots}_{\rho\ldots}$$

$$\nabla^\alpha T^{\mu\ldots}_{\nu\ldots} = \frac{\delta T^{\mu\ldots}_{\nu\ldots}}{\delta \xi_\alpha} + \overline{C}^{\mu\alpha}_{\kappa} T^{\kappa\ldots}_{\nu\ldots} + \ldots - \overline{C}^{\kappa\alpha}_{\nu} T^{\mu\ldots}_{\kappa\ldots}$$

$$\nabla_\alpha T^{\mu\ldots}_{\nu\ldots} = \frac{\partial T^{\mu\ldots}_{\nu\ldots}}{\partial \overline{\xi}^\alpha} + C^{\mu}_{\kappa\alpha} T^{\kappa\ldots}_{\nu\ldots} + \ldots - \overline{C}^{\kappa}_{\nu\alpha} T^{\mu\ldots}_{\kappa\ldots}$$

$$\nabla_{\circ} T^{\mu\ldots}_{\nu\ldots} = \frac{\partial T^{\mu\ldots}_{\nu\ldots}}{\partial\lambda} + \Gamma^{(*)\mu}_{\kappa o} T^{\kappa\ldots}_{\nu\ldots} - \ldots + \Gamma^{(*)\kappa}_{\nu o} T^{\mu\ldots}_{\kappa\ldots}$$

$$\nabla_{\kappa} \Phi^{\alpha\ldots}_{\beta\ldots} = \frac{\delta \Phi^{\alpha\ldots}_{\beta\ldots}}{\delta x^{\kappa}} - \overline{\Gamma}^{(*)\gamma}_{\beta\kappa} \Phi^{\alpha\ldots}_{\gamma\ldots} - \ldots + \Phi^{\gamma\ldots}_{\beta\ldots} \overline{\Gamma}^{(*)\alpha\ldots}_{\gamma\kappa} \tag{16}$$

$$\nabla^{\delta} \Phi^{\alpha\ldots}_{\beta\ldots} = \frac{\delta \Phi^{\alpha\ldots}_{\beta\ldots}}{\delta \xi_{\delta}} - \tilde{C}^{\gamma\delta}_{\beta} \Phi^{\alpha\ldots}_{\gamma\ldots} - \ldots + \Phi^{\gamma\ldots}_{\beta\ldots} \tilde{C}^{\alpha\delta}_{\gamma}$$

$$\nabla_{\delta} \Phi^{\alpha\ldots}_{\beta\ldots} = \frac{\partial \Phi^{\alpha\ldots}_{\beta\ldots}}{\partial \overline{\xi}^{\delta}} - C^{\gamma}_{\beta\delta} \Phi^{\alpha\ldots}_{\gamma\ldots} - \ldots + \Phi^{\gamma\ldots}_{\beta\ldots} C^{\alpha}_{\gamma\delta}$$

$$\nabla_{\circ} \Phi^{\alpha\ldots}_{\beta\ldots} = \frac{\partial \Phi^{\alpha\ldots}_{\beta\ldots}}{\partial\lambda - \overline{\Gamma}^{(*)\gamma}_{o\beta}} \Phi^{\alpha\ldots}_{\gamma\ldots} - \ldots + \Phi^{\gamma\ldots}_{\beta\ldots} \Gamma^{(*)\alpha}_{o\gamma}$$

$$\nabla^{(*)}_{\mu} V^{a\ldots}_{c\ldots} = \frac{\delta V^{a\ldots}_{c\ldots}}{\delta x^{\mu}} + \omega^{(*)a}_{\mu b} V^{b\ldots}_{c\ldots} + \ldots - \omega^{(*)b}_{\mu c} V^{a\ldots}_{b\ldots}$$

$$\nabla^{(*)\alpha} V^{a\ldots}_{c\ldots} = \frac{\delta V^{a\ldots}_{c\ldots}}{\delta \xi_{\alpha}} + \ldots + \overline{\theta}^{(*)a}_{\alpha b} V^{b\ldots}_{c\ldots} + \ldots - \theta^{(*)b}_{\alpha c} V^{a\ldots}_{b\ldots}$$

$$\nabla^{(*)}_{\alpha} V^{a\ldots}_{c\ldots} = \frac{\partial V^{a\ldots}_{c\ldots}}{\partial \overline{\xi}_{\alpha}} + \theta^{(*)a}_{\alpha b} V^{b\ldots}_{c\ldots} + \ldots - \theta^{(*)b}_{\alpha c} V^{a\ldots}_{b\ldots}$$

$$\nabla^{(*)}_{\circ} V^{a\ldots}_{c\ldots} = \frac{\partial V^{a\ldots}_{c\ldots}}{\partial\lambda} + \omega^{(*)a}_{ob} V^{b\ldots}_{c\ldots} + \ldots - \omega^{(*)b}_{oc} V^{a\ldots}_{b\ldots}$$

The covariant derivatives of the metric tensor $g_{\mu\nu}$ are postulated to be zero:

$$\mathcal{D}^{(*)}_{\mu} g_{\kappa\lambda} = 0, \mathcal{D}^{(*)\alpha} g_{\kappa\lambda} = 0, \mathcal{D}^{(*)}_{\alpha} g_{\kappa\lambda} = 0, \mathcal{D}^{(*)}_{(o)} g_{\kappa\lambda} = 0.$$

3.Curvatures and Torsions.

From the relations (6) we obtain the curvatures and torsions of the space $\tilde{S}^{(2)}M$

$$[\mathcal{D}^{(*)}_{\mu}, \mathcal{D}^{(*)}_{\nu}] = \mathcal{D}^{(*)}_{\mu} \mathcal{D}^{(*)}_{\nu} - \mathcal{D}^{(*)}_{\nu} \mathcal{D}^{(*)}_{\mu} = R^{a}_{\mu\nu} P_a + \frac{1}{2} R^{a}_{\mu\nu} J_{ab} \tag{18}$$

with their explicit expressions given by

$$\begin{cases} R^{a}_{\mu\nu} = \frac{\delta h^{a}_{\mu}}{\delta x^{\nu}} - \frac{\delta h^{a}_{\nu}}{\delta x^{\mu}} + \omega^{(*)a}_{\mu b} h^{b}_{\nu} - \omega^{(*)a}_{\nu b} h^{b}_{\mu} \\ R^{ab}_{\mu\nu} = \frac{\delta \omega^{(*)ab}_{\mu}}{\delta x^{\nu}} - \frac{\delta \omega^{(*)ab}_{\nu}}{\delta x^{\mu}} + \omega^{(*)a\rho}_{\mu} \omega^{(*)b}_{\nu\rho} - \omega^{(*)\rho a}_{\nu} \omega^{(*)b}_{\mu\rho} \end{cases}$$

$$[\mathcal{D}^{(*)}_{\mu}, \mathcal{D}^{(*)}_{\alpha}] = P^{a}_{\mu\alpha} P_{a} + \frac{1}{2} P^{ab}_{\mu\alpha} J_{ab} \tag{19}$$

$$\begin{cases} P^{ab}_{\mu\alpha} = \frac{\delta \theta^{(*)ab}_{\alpha}}{\delta x^{\mu}} - \frac{\partial \omega^{(*)ab}_{\mu}}{\partial \bar{\xi}^{\alpha}} + \theta^{(*)b}_{\alpha c} \omega^{(*)ac}_{\mu} - \theta^{(*)a}_{\alpha c} \omega^{(*)cb}_{\mu} \\ P^{a}_{\mu\alpha} = \frac{\delta \psi^{a}_{\alpha}}{\delta x^{\mu}} - \frac{\partial h^{a}_{\mu}}{\partial \bar{\xi}^{\alpha}} + \omega^{(*)a}_{\mu c} \psi^{c}_{\alpha} - \theta^{(*)a}_{\alpha c} h^{c}_{\mu} \end{cases}$$

Similarly to [5], the other four curvatures and torsions result from the commutation relations

$$[\mathcal{D}^{(*)}_{\mu}, \mathcal{D}^{\alpha}] = \overline{P}^{a\alpha}_{\mu} P_{a} + \frac{1}{2} \overline{P}^{ab\alpha}_{\mu} J_{ab} \tag{20}$$

$$[\mathcal{D}^{(*)}_{\alpha}, \mathcal{D}^{(*)\beta}] = S^{\beta a}_{\alpha} P_{a} + \frac{1}{2} S^{ab\beta}_{\alpha} J_{ab} \tag{21}$$

$$[\mathcal{D}^{(*)}_{\alpha}, \mathcal{D}^{(*)}_{\beta}] = Q^{a}_{\alpha\beta} P_{a} + \frac{1}{2} Q^{ab}_{\alpha\beta} J_{ab} \tag{22}$$

$$[\mathcal{D}^{(*)\alpha}, \mathcal{D}^{(*)\beta}] = \tilde{Q}^{\alpha\beta a} P_{a} + \frac{1}{2} \tilde{Q}^{ab\alpha\beta} J_{ab} \tag{23}$$

The contribution of the λ – covariant derivative $\mathcal{D}^{(*)}_{\circ}$ provides us the following curvatures and torsions

$$[\mathcal{D}^{(*)}_{\circ}, \mathcal{D}^{(*)}_{\mu}] = R^{a}_{o\mu} P_{a} + \frac{1}{2} R^{ab}_{o\mu} J_{ab} \tag{24}$$

$$\begin{cases} R^{a}_{o\mu} = \frac{\delta L^{a}_{\mu}}{\delta \lambda} - \frac{\delta h^{a}_{\circ}}{\delta x^{\mu}} + \omega^{(*)a}_{\mu b} L^{b}_{\circ} - \omega^{(*)a}_{bo} h^{b}_{\mu} \\ R^{ab}_{o\mu} = \frac{\delta \omega^{(*)ab}_{\mu}}{\delta \lambda} - \frac{\delta \omega^{(*)ab}_{\circ}}{\delta x^{\mu}} + \omega^{(*)a\rho}_{\mu} \omega^{(*)b}_{o\rho} - \omega^{(*)\rho a}_{\circ} \omega^{(*)b}_{\mu\rho} \end{cases}$$

$$[\mathcal{D}_\circ^{(*)}, \mathcal{D}_\circ^{(*)}] = 0, \qquad R_{\circ\circ}^{ab} = 0, \qquad R_{oo}^{a} = 0. \tag{25}$$

$$[\mathcal{D}_\circ^{(*)}, \mathcal{D}^{(*)\alpha}] = \overline{P}_\circ^{a\alpha} P_a + \frac{1}{2}\overline{P}_\circ^{ab\alpha} J_{ab} \tag{26}$$

$$\begin{cases} \overline{P}_\circ^{a\alpha} = \frac{\partial \overline{\psi}^{a\alpha}}{\partial\lambda} - \frac{\delta L_\circ^a}{\delta \xi_\alpha} + \omega_{oc}^{(*)a}\overline{\psi}^{\alpha a} - \overline{\theta}_c^{(*)a}\alpha L_\circ^c \\ \overline{P}_\circ^{ab\alpha} = \frac{\partial \overline{\theta}^{(*)\alpha ab}}{\partial\lambda} - \frac{\delta \omega_\circ^{(*)ab}}{\delta\xi_\alpha} + \overline{\theta}_c^{(*)ab}\omega_\mu^{(*)ac} - \overline{\theta}_c^{(*)a\alpha}\omega_\circ^{(*)ab} \end{cases}$$

$$[\mathcal{D}_\circ^{(*)}, \mathcal{D}_\alpha^{(*)}] = P_{\circ\alpha}^{a} P_a + \frac{1}{2} P_{o\alpha}^{ab} J_{ab} \tag{27}$$

$$\begin{cases} P_{o\alpha}^{a} = \frac{\partial \psi_\alpha^a}{\partial\lambda} - \frac{\partial L_\circ^a}{\partial \overline{\xi}^\alpha} + \omega_{oc}^{(*)a}\psi_\alpha^c - \theta_{\alpha c}^{(*)a} L_\circ^c \\ P_{o\alpha}^{ab} = \frac{\partial \theta_\alpha^{(*)ab}}{\partial\lambda} - \frac{\delta \omega_\circ^{(*)ab}}{\delta\overline{\xi}^\alpha} + \theta_{ac}^{(*)b}\omega_\circ^{(*)ac} - \theta_{\alpha c}^{(*)a}\omega_\circ^{(*)cb} \end{cases}$$

4. Field Equations. In the following, we derive by means of the Palatini method the field equations, using a Lagrangian of the form

$$\mathcal{L} = h(R + P + \overline{P} + Q + \tilde{Q} + R_\circ + \overline{P}_\circ + Po) \tag{28}$$

which depends on the tetrads and on the connection coefficients,

$$\mathcal{L}(\kappa^A, \delta_M \kappa^A) = \mathcal{L}(h, \omega^{(*)}, \psi, \overline{\psi}, \theta^{(*)}, \overline{\theta}^{(*)}, \omega_\circ^{(*)})$$

where,

$$\kappa^A \in \{(h_\mu^a(z), \omega_\mu^{(*)ab}(z), \psi_\alpha^a(z), \overline{\psi}^{\alpha a}(z), \theta_\alpha^{(*)ab}(z), \overline{\theta}^{(*)\alpha ab}(z), \omega_\circ^{(*)ab}(z))\},$$

$$\delta_M = \frac{\delta}{\delta z^M} \in \left\{ \frac{\delta}{\delta x^\mu}, \frac{\delta}{\delta \xi_\alpha}, \frac{\partial}{\partial \overline{\xi}^\alpha}, \frac{\partial}{\partial\lambda} \right\}, \qquad z = (z^M) = (x^\mu, \xi_\alpha, \overline{\xi}^\alpha, \lambda) \tag{29}$$

and

$$\begin{cases} R = h^{\mu}_{a} h^{\kappa}_{c} R^{c}_{\mu\kappa}, & P = h^{\mu}\psi^{\alpha}_{c} P^{c}_{\mu\alpha}, & \overline{P} = h^{\mu}\overline{\psi}_{\alpha c}\overline{P}^{c\alpha}_{\mu}, \\ Q = Q^{ab}_{\alpha\beta}\overline{\psi}^{\alpha}_{a}\overline{\psi}^{\beta}_{b}, & \tilde{Q} = \tilde{Q}^{ab\alpha\beta} = \psi_{\alpha a}\psi_{\beta b}, & S = \overline{\psi}^{\alpha}_{a}\psi_{\beta b}S^{ab\beta}_{\alpha}, \\ R_{\circ} = L^{\circ}_{\kappa} h^{\mu}_{c} h^{\kappa}_{a} R^{ac}_{\circ\mu}, & \overline{P}_{\circ} = L^{\circ}_{\kappa} h^{\kappa}_{a}\overline{\psi}_{\alpha c}\overline{P}^{ac\alpha}_{\circ}, & P_{\circ} = L^{\circ}_{\kappa} h^{\kappa}_{a}\overline{\psi}^{\alpha}_{c} P^{ac}_{\circ\alpha}. \end{cases}$$

The Euler–Lagrange equations are generally given by

$$\frac{\delta\mathcal{L}}{\delta\kappa^{(A)}} = \partial_{M}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{M}\kappa^{(A)})}\right) - \frac{\partial\mathcal{L}}{\partial\kappa^{(A)}} = 0, \tag{30}$$

with $\partial_{M} = \frac{\partial}{\partial z^{M}} \in \left\{\frac{\partial}{\partial x^{\mu}}, \frac{\partial}{\delta\partial\xi_{\alpha}}, \frac{\partial}{\partial\overline{\xi}^{\alpha}}, \frac{\partial}{\partial\lambda}\right\}$.

From the relation (30), the variation of $\mathcal{L}$ with respect to the tetrads h^{ν}_{b} gives us the first field equation

$$\partial_{\mu}\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}h^{\nu}_{b})} + \partial^{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial^{\alpha}h^{\nu}_{b})} + \partial_{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha}h^{\nu}_{b})} + \partial_{\circ}\frac{\partial\mathcal{L}}{\partial(\partial_{\circ}h^{\nu}_{b})} - \frac{\partial\mathcal{L}}{\partial h^{\nu}_{b}} = 0, \tag{31}$$

where we denoted $\partial_{\circ} = \frac{\partial}{\partial\lambda}$. Finally, after some calculations we get,

$$\tilde{H}^{b}_{\nu} - \frac{1}{2}h^{b}_{\nu}\tilde{H} = 0, \tag{32}$$

where we put,

$$\tilde{H} = R + P + \overline{P} + Q + \tilde{Q} + S + R_{\circ} + \overline{P}_{\circ} + P_{\circ},$$

$$\tilde{H}^{b}_{\nu} = 2R^{b}_{\nu} + P^{b}_{\nu} + \overline{P}^{b}_{\nu} + R^{b}_{o\nu} + \overline{P}^{b}_{o\nu} + P^{b}_{(o)\nu},$$

$$\begin{cases} R^{b}_{\nu} = h^{\kappa}_{c} R^{bc}_{\nu\kappa}, & P^{b}_{\nu} = \overline{\psi}^{\alpha}_{c} P^{bc}_{\nu\alpha}, & R^{b}_{\circ\nu} = L^{\circ}_{\kappa} h^{\kappa} R^{b}_{o\nu} + L^{\circ}_{\nu} h^{\mu}_{c} R^{bc}_{\circ\mu}, \\ \overline{P}^{b}_{\nu} = \psi_{\alpha c}\overline{P}^{bc\alpha}_{\nu}, & \overline{P}^{b}_{o\nu} = L^{\circ}_{\nu}\overline{\psi}_{\alpha c}\overline{P}^{bc\alpha}_{\circ}, & P^{b}_{\circ\nu} = L^{\circ}_{\nu}\overline{\psi}^{\alpha}_{c} P^{bc}_{\circ\alpha} \end{cases}$$

The equation (32) is the **Einstein equation for empty space**, in the framework of our consideration. Also, the variation of $\mathcal{L}$ with respect to $\omega^{(*)ab}_{\mu}$ gives,

$$\partial_{\nu}\frac{\partial\mathcal{L}}{\partial(\partial_{\nu}\omega^{(*)ab}_{\mu})} + \partial^{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial^{\alpha}\omega^{(*)ab}_{\mu})} + \partial_{\alpha}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha}\omega^{(*)ab}_{\mu})} - \frac{\partial\mathcal{L}}{\partial\omega^{(*)ab}_{\mu}} = 0. \tag{33}$$

From this relation we get the second field equation in the following form,

$$\partial_\nu[h(h^\mu_a h^\nu_b - h^\mu_b h^\nu_a)] - \partial^\alpha[hh^\mu_a\psi_{\alpha b}] - \partial_\alpha[hh^\mu_a\overline{\psi}^\alpha_b] - \partial_\circ[hL^\circ_a h^\mu_b] - \tag{34}$$
$$-h[\omega^{(*)d}_{\kappa(a}h^{(\kappa}_{b)}h^{\mu)}_d + h^\mu_a(\overline{\psi}^\alpha_c\theta^{(*)c}_{\alpha b} + \psi_{\alpha c}\overline{\theta}^{(*)\alpha c}_b) +$$
$$+h^\mu_d(\overline{\psi}^\alpha_b\theta^{(*)d}_{\alpha a} + \psi_{\alpha b}\theta^{(*)\alpha d}_a) - L^\circ_\kappa\omega^{(*)c}_{ob}h^\mu_{[c}h^\kappa_{a]} = 0,$$

where the parantheses () and [] are used to denote symmetrization and antisymmetrization respectively.

The variation of $\mathcal{L}$ with respect to ψ^a_α provides the field equation

$$\partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu\psi^a_\alpha)} + \partial^\beta\frac{\partial\mathcal{L}}{\partial(\partial^\beta\psi^a_\alpha)} + \partial_\beta\frac{\partial\mathcal{L}}{\partial(\partial_\beta\psi^a_\alpha)} + \partial_\circ\frac{\partial\mathcal{L}}{\partial(\partial_\circ\psi^a_\alpha)} - \frac{\partial\mathcal{L}}{\partial\psi^a_\alpha} = 0, \tag{35}$$

having the explicit form

$$\frac{1}{2}\overline{\psi}^\beta_c S^{c\alpha}_{\beta a} + \frac{1}{2}P^{ba}_{\mu\alpha}h^\mu_b + \tilde{Q}^{d\gamma\alpha}_a\psi_{\gamma d} = 0. \tag{36}$$

From the variation with respect to $\overline{\psi}^{\alpha a}$

$$\partial_\nu\frac{\partial\mathcal{L}}{\partial(\partial_\nu\overline{\psi}^{\alpha a})} + \partial^\beta\frac{\partial\mathcal{L}}{\partial(\partial^\beta\overline{\psi}^{\alpha a})} + \partial_\beta\frac{\partial\mathcal{L}}{\partial(\partial_\beta\overline{\psi}^{\alpha a})} + \partial_\circ\frac{\partial\mathcal{L}}{\partial(\partial_\circ\overline{\psi}^{\alpha a})} - \frac{\partial\mathcal{L}}{\partial\overline{\psi}^{\alpha a}} = 0, \tag{37}$$

we get the fourth field equation

$$\frac{1}{2}\overline{P}^{ba\alpha}_\mu h^\mu_b + \frac{1}{2}\psi_{\beta b}S^{b\beta}_{a\alpha} + \overline{\psi}^\gamma_d Q^d_{a\gamma\alpha} = 0. \tag{38}$$

Finally, we write down the other three field equations which are derived from the variation of $\mathcal{L}$ with respect to the connection coefficients $\theta^{(*)ab}_\alpha$, $\overline{\theta}^{(*)\alpha ab}$ and $\omega^{(*)ab}_{(\circ)}$

$$\partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\Omega^{(\star)})}) + \partial^\beta(\frac{\partial\mathcal{L}}{\partial(\partial^\beta\Omega^{(\star)})}) + \partial_\beta(\frac{\partial\mathcal{L}}{\partial(\partial_\beta\Omega^{(\star)})}) + \partial_\circ(\frac{\partial\mathcal{L}}{\partial(\partial_\circ\Omega^{(\star)})}) = 0 \tag{39}$$

with $\Omega^{(*)} \in \{\theta^{(*)ab}_\alpha, \overline{\theta}^{(*)\alpha ab}, \omega^{(*)ab}_{(\circ)}\}$, which have the corresponding detailed forms

$$\begin{cases} \partial_\nu(hh^\nu_a\psi^\alpha_b) + \partial_\beta(h\psi^{(\beta}_a\overline{\psi}^{\alpha)}_b) - \partial^\beta(h\overline{\psi}^\alpha_a\psi_{\beta b}) + \\ \partial_\circ(hL^\circ_\kappa h^\kappa_a\overline{\psi}^\alpha_b) - -h[\omega^{(*)d}_{\mu b}h^\mu_{(d}\psi^\alpha_{a)} + \overline{\psi}^{[\gamma}_d\overline{\psi}^{\alpha]}_{(a}\theta^{(*)d}_{b)\gamma} + \\ \theta^{(*)\beta d}_b\overline{\psi}^\alpha_{(a}\psi_{d)\beta} + L^\circ_\kappa\omega^{(*)}_{ob}h^\kappa_{(d}\overline{\psi}^\alpha_{a)}] = 0 \end{cases} \tag{40}$$

$$(41)\qquad \begin{cases} \partial_\nu(hh^\nu_a\overline{\psi}_{\alpha b})+ & \partial^\beta(h\psi_{a[\beta}\psi_{\alpha]b})+ & \partial_\beta(h\overline{\psi}^\beta_a\psi_{\alpha b})+ \\ \partial_\circ(hL^\circ_\kappa h^\kappa_a\psi_{\alpha b})- & h[\omega^{(*)d}_{b\mu}h^\mu_{[d}\overline{\psi}_{a]\alpha}+ & \theta^{(*)d}_{\beta a}\overline{\psi}^\beta_{[d}\psi_{b]\alpha}+ \\ 2\theta^{(*)\gamma d}_a\psi_{\gamma[d}\psi_{b]\alpha}+ & L^\circ_\kappa(h^\kappa_d\overline{\psi}_{\alpha a}\omega^{(*)d}_{ob}- & \overline{\psi}_{\alpha d}h^\kappa_b\omega^{(*)d}_{oa})]=0 \end{cases}$$

$$(42)\qquad \begin{cases} hL^\circ_\kappa[h^\mu_a h^\kappa_d\omega^{(*)d}_{\mu b}- & h^\mu_d h^\kappa_b\omega^{(*)d}_{\mu a}+ & h^\kappa_a\overline{\psi}^\alpha_c\theta^{(*)c}_{\alpha b}- \\ \theta^{(*)d}_{\alpha a}\overline{\psi}^\alpha_b h^\kappa_d+ & h^\kappa_a\overline{\psi}_{\alpha c}\overline{\theta}^{(*)\alpha c}_b- & \overline{\theta}^{(*)\alpha d}_a\overline{\psi}_{\alpha b}h^\kappa_d]+ \\ \partial_\nu[hL^\circ_\kappa h^\nu_b h^\kappa_\alpha]+ & \partial_\beta[hL^\circ_\kappa h^\kappa_a\overline{\psi}^\beta_b]+ & \partial^\beta[hL^\circ_\kappa h^\kappa_a\overline{\psi}_{\beta b}]=0 \end{cases}$$

Conclusions. In this paper we studied the spinor bundle of order two $\tilde{S}^{(2)}(M)$, which is a foliation of the structure of the spinor bundle presented in [4], [5]. In the present approach the generalized tetrads and the spin–tetrads define, by means of the relations (8), a generalized principle of equivalence in the spinor bundle $\tilde{S}^{(2)}(M)$. Also, employing the Miron – type connections, we cover all the possibilities for the S - bundle connections, which represent the gauge potential in physical interpretation. These have, in the framework of our considerations, the remarkable property of isotopic spin conservation. The introduction of the internal deformed system (as a fibre) in $\tilde{S}^{(2)}M)$, is expected to produce as a natural consequence , for a definite value κ, ϕ^α (where κ is a constant and ϕ^α a scalar field), the Higgs field. This will be derived within the developed theory, in a forecoming paper.

REFERENCES

1. M i r o n, R. and A n a s t a s i e i, M. – *The Geometry of Lagrange Spaces: Theory and Applications*, Kluwer Academic Publishers, 1994.
2. M i r o n, R. and A t a n a s i u, Gh. – *Lagrange Geometry of Second Order*, Math. Comput. Model - Vol 20, No 4/5, Pergamon Press, 1994, pp. 41-56.
3. M u n t e a n u, G. and A t a n a s i u, Gh. – *On Miron Connections in Lagrange Spaces of Second Order*, Tensor N.S., Vol. 50 (1994), pp.241-247.
4. S t a v r i n o s, P.C. – *On the Differential Geometry of Nonlocalized Field Theory*, Poincarè Gravity. Proceedings of 8-th Nat. Sem. on Finsler, Lagrange and Hamilton Spaces 1994, to appear.
5. S t a v r i n o s, P.C. and M a n o u s e l i s, P. – *Nonlocalized Field Theory over Spinor Bundles: Poincarè Gravity and Yang - Mills Fields*, Rep. Math. Phys., to appear.

6. S t a v r i n o s, P.C. and I k e d a, S. – *A Geometrical Structure in the λ - parameter Family of Generalized Metric Spaces*, Tensor N.S., to appear.

Received: 10.XII.1995

P.C.Stavrinos, N.Prezas, P.Manouselis,
Department of Mathematics,
University of Athens, 157 84 Panepistimiopolis,
Athens, Greece.
V.Balan,
Department of Mathematics I,
Politehnica University of Bucharest,
Splaiul Independenţei 313, RO-77206,
Bucharest,
ROMANIA.