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ON THE THERMAL STRESSES IN VISCOELASTIC CYLINDERS

BY

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Summary. The paper is concerned with the problem of thermal stresses in viscoelastic beams within the quasi–static theory. The body is subjected to a prescibed thermal field is a general polynomial in the axial coordinate. The problem is reduced to plane strain problem.

1. Introduction. Most of papers concerned with thermal stresses in cylinders are devoted to elastic materials (see e.g.[1–4]). In this paper, by using some results established in [2,4,5], we solve the problem of thermal stresses in the context of quasi–static thermal theory of linear thermoviscoelasticity. The material is assumed to be isotropic with constitutive coefficients that are independent of the axial coordinate.

We study the defomation of a cylinder when the temperature distribution is a general polynomial in the axial coordinate. In the first part of the paper we solve the problem assuming that the temperature field is independent of the axial coordinate. Then, we use the method of induction to obtain the solution when the temperature field is a general polynomial in the axial coordinate.

The problem under consideration is solved using some results concerning Saint–Venant's problem for non–homogeneous elastic cylinders.

2. Statement of the problem. Throughout this paper B denote the interior of a right cylinder of length h, with generic cross section Σ and the lateral boundary S. We call ∂B the boundary of B. We use a rectangular Cartesian coordinate system $Ox_k(k = 1, 2, 3)$. The Catesian coordinate frame is chosen such that x_3 axis is parallel to the generators of B, and the x_1Ox_2 plane contains one of the terminal cross sections. We denote by Γ the boundary of the generic cross section Σ.

In what follows we shall employ the usual summation and differential conventions: Greek subscripts are understood to range over the integers (1,2) and Latin subscripts over (1,2,3); summation over repeated subscripts is implied, and subscripts preceded by a comma denote differentiation with respect to the corresponding Cartesian coordinate. Moreover, we use a superposed dot to denote partial differentiation with respect to the time t.

Let u_i denote the components of the displacement vector field. Then, the components of the infinitesimal strain field are given by

$$\varepsilon_{ij}\frac{1}{2}(u_{i,j}+u_{j,i}). \tag{2.1}$$

We study the quasi–static theory of viscoelasticity. The equilibrium equations, in the absence of body forces, are

$$t_{ij,j}=0, \tag{2.2}$$

on $B\times I$, where t_{ij} are the components of the stress tensor.

We suppose that B is composed of a nonhomogeneous isotropic viscoelastic material which is assumed to have been at rest up until time $t=0$. We denote by L, M and β the relaxation functions and assume that

$$L=L(x_1,x_2,t),\ M=M(x_1,x_2,t),\ \beta=\beta(x_1,x_2,t)\ (x,t)\in B\times I, \tag{2.3}$$

where $I=[0,\infty)$. Moreover, we suppose that L, M and β are smooth functions on $\overline{B}\times[0,\infty)$ and that

$$3L(0)+2M(0)>0, \qquad M(0)>0. \tag{2.4}$$

Here we have suppressed the dependence upon the spatial variables. The constitutive relations are [6–8]

$$\begin{aligned} t_{ij} &= L(0)\varepsilon_{rr}\delta_{ij}+2M(0)\varepsilon_{ij}+ \\ &+\int_0^t[\dot{L}(t-\tau)\varepsilon_{rr}(\tau)\delta_{ij}+2\dot{M}(t-\tau)\varepsilon_{ij}(\tau)-\beta(t-\tau)\delta_{ij}\dot{T}(\tau)]d\tau, \end{aligned} \tag{2.5}$$

where T is the temperature measured from the constant absolute temperature of the body in its reference state.

In what follows we assume that the functions L, M, β and T are prescribed.

We study the deformation of the cylinder caused by the temperature field T and assume that the mechanical loading are absent. On the lateral surface S we have the conditions

$$t_{\alpha i} n_\alpha = 0 \tag{2.6}$$

on $S \times I$, where $(n_1, n_2, 0)$ are the components of the outward normal to the lateral surface. We assume that the loading applied on the end located at $x_3 = 0$ is statically equivalent to zero. Then, for $x_3 = 0$ we have the conditions

$$\int_\Sigma t_{3\alpha} da = 0, \tag{2.7.1}$$

$$\int_\Sigma t_{33} da = 0, \tag{2.7.2}$$

$$\int_\Sigma x_\alpha t_{33} da = 0, \tag{2.7.3}$$

$$\int_\Sigma \varepsilon_{3\alpha\beta} x_\alpha t_{\beta 3} da = 0, \tag{2.7.4}$$

where ε_{ijk} is the alternating symbol.

The problem consists of solving Eqs. (2.1), (2.2), (2.5) on $B \times I$ with the conditions (2.6)–(2.7.4).
We assume that

$$T = \sum_{k=0}^{r} T_k(x_1, x_2, t) x_3^k, \tag{2.8}$$

where T_k are given. In what follows we study the problem under consideration using some results established for Saint–Venant problem for viscoelastic cylinders. The solution of Saint–Venant problem for anisotropic viscoelastic cylinders has been presented in [9].

Let us denote by $P^{(n)}$ the problem of solving Eqs. (2.1), (2.2), (2.5) with the conditions (2.6)–(2.7.4) assuming that the function T has the form

$$T = T_n(x_1, x_2, t)x_3^n, \tag{2.9}$$

where n is a positive integer or zero and the function T_n is known. If we know the solution of the problem $P^{(n)}$ for any n, then, according to the linearity of the theory, we can determine the solution when the temperature has the form (2.8). To solve the initial problem we use the method of induction. First, we solve the problem $P^{(0)}$, then we establish the solution of the problem $P^{(n+1)}$ when the solution of the problem $P^{(n)}$ is known.

If we introduce the notation [6]

$$(k \circ f)(t) = k(0)f(t) + \int_0^t \dot{k}(t-\tau)f(\tau)d\tau, \tag{2.10}$$

then the constitutive equations become

$$t_{ij} = L \circ \varepsilon_{rr}\delta_{ij} + 2M \circ \varepsilon_{ij} - \Lambda\delta_{ij}, \tag{2.11}$$

where

$$\Lambda(x,t) = \int_0^t \beta(x, t-\tau)\dot{T}(x,\tau)d\tau. \tag{2.12}$$

3. Auxiliary plane–strain problems. The isothermal state of plane strain of the cylinder B, parallel to the plane x_1Ox_2, is characterized by

$$u_\alpha = u_\alpha(x_1, x_2, t),\ u_3 = 0,\ T = 0,\ (x_1, x_2) \in \Sigma,\ t \in I. \tag{3.1}$$

This restriction implies $t_{ij} = t_{ij}(x_1, x_2, t)$, and that

$$t_{\alpha\beta} = L \circ \varepsilon_{\rho\rho}\delta_{\alpha\beta} + 2M \circ \varepsilon_{\alpha\beta}, \tag{3.2}$$

where

$$\varepsilon_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha}). \tag{3.3}$$

Given the body forces $F_\alpha = F_\alpha(x_1, x_2, t),\ F_3 = 0,\ (x_1, x_2) \in \Sigma,\ t \in I$, the equations of equilibrium reduce to

$$t_{\alpha i,\alpha} + F_\alpha = 0, \tag{3.4}$$

on $\Sigma \times I$. We consider the boundary conditions

$$t_{\alpha\beta} n_\beta = \widetilde{t}_\alpha, \tag{3.5}$$

on Γ, where $\widetilde{t}_\alpha$ are prescribed functions independent of x_3. We assume that the domain Σ, the relaxation functions and the given forces satisfy the smoothness hypoteses requested by the existence theorem (cf. Fichera [10]). The necessary and sufficient conditions for the existence of the solutions for the quasi–static plane strain problem (3.2)–(3.5) are [10].

$$\begin{aligned} &\int_\Sigma F_\alpha da + \int_\Gamma \widetilde{t}_\alpha ds = 0, \\ &\int_\Sigma \varepsilon_{3\alpha\beta} x_\alpha F_\beta da + \int_\Gamma \varepsilon_{3\alpha\beta} x_\alpha \widetilde{t}_\beta ds = 0. \end{aligned} \tag{3.6}$$

In what follows we will have the occasion to use the solutions of three particular planestrain problems denoted by $D^{(k)}$ $(k = 1, 2, 3)$. We denote by $u_\alpha^{(s)}, \varepsilon_{\alpha\beta}^{(s)}$ and $t_{\alpha\beta}^{(s)}$ the components of the displacement vector field, the components of the stress tensor field, respectively, from the problem $D^{(s)}$. The problems $D^{(k)}$ are characterized by the equations

$$\begin{aligned} &t_{\alpha\beta}^{(k)} = L \circ \varepsilon_{\rho\rho}^{(k)} \delta_{\alpha\beta} + 2M \circ \varepsilon_{\alpha\beta}^{(k)}, \quad 2\varepsilon_{\alpha\beta}^{(k)} = u_{\alpha,\beta}^{(k)} + u_{\beta,\alpha}^{(k)}, \\ &t_{\alpha\beta,\beta}^{(\rho)} + (Lx_\rho)_{,\alpha} = 0, \quad t_{\alpha\beta,\beta}^{(3)} + L_{,\alpha} = 0,\ (\rho = 1, 2), \end{aligned} \tag{3.7}$$

on Σ, and the following boundary conditions on $\partial\Sigma$

$$t_{\alpha\beta}^{(\rho)} n_\beta = -L x_\rho n_\alpha, \quad t_{\alpha\beta}^{(3)} n_\beta = -L n_\alpha, \tag{3.8}$$

for each $t \in [0, \infty)$. It is easy to see that the necessary and sufficient conditions (3.6) for the existence of the solution are satisfied for each boundary value problem $D^{(k)},\ (k = 1, 2, 3)$. In what follows we shall assume that the functions $u_\alpha^{(k)}$ $(k = 1, 2, 3)$ are known.

4. Plane temperature field. We first consider the problem of a cylinder subjected to the temperature distribution

$$T = f(x_1, x_2, t), \quad (x_1, x_2) \in \Sigma,\ t \in I, \tag{4.1}$$

where f is a prescribed function of class C^2.

We seek the solution of this problem in the form

$$u_\alpha = -\tfrac{1}{2}a_\alpha(t)x_3^2 + \sum_{k=1}^{3} a_k \circ u_\alpha^{(k)} + \nu_\alpha(x_1, x_2, t), \qquad u_3 = [a_1(t)x_1 + a_2(t)x_2 + a_3(t)]x_3, \tag{4.2}$$

where $u_a^{(k)}$ are the components of the displacement field from the problem $D^{(k)}$, ν_α are unknown functions of the variables x_1, x_2 and t, and a_i are unknown continuous functions of the variable t.

It follows from (2.1) and (4.2) that

$$\varepsilon_{\alpha\beta} = \gamma_{\alpha\beta} + \sum_{s=1}^{3} a_s \circ \varepsilon_{\alpha\beta}^{(s)}, \quad \varepsilon_{\alpha 3} = 0, \quad \varepsilon_{33} = a_1x_1 + a_2x_2 + a_3, \tag{4.3}$$

where

$$\gamma_{\alpha\beta} = \frac{1}{2}(\nu_{\alpha,\beta} + \nu_{\beta,\alpha}). \tag{4.4}$$

By (2.5) and (4.3) we obtain

$$\begin{aligned} t_{\alpha\beta} &= L \circ (a_1x_1 + a_2x_2 + a_3)\delta_{\alpha\beta} + \sum_{s=1}^{3} a_s \circ t_{\alpha\beta}^{(s)} + \sigma_{\alpha\beta}, \ t_{\alpha 3} = 0, \\ t_{33} &= (L + 2M) \circ (a_1x_1 + a_2x_2 + a_3) + \sum_{s=1}^{3} a_s \circ t_{33}^{(s)} + L \circ \gamma_{\alpha\alpha} - H, \end{aligned} \tag{4.5}$$

where

$$\sigma_{\alpha\beta} = L \circ \gamma_{\rho\rho}\delta_{\alpha\beta} + 2M \circ \gamma_{\alpha\beta} - H\delta_{\alpha\beta}, \tag{4.6}$$

and

$$\begin{aligned} H(x_1, x_2, t) &= \int_0^t \beta(x_1, x_2, t - \tau)\dot{f}(x_1, x_2, \tau)d\tau, \\ t_{33}^{(s)} &= L \circ \varepsilon_{\alpha\alpha}^{(s)}, \ (s = 1, 2, 3). \end{aligned} \tag{4.7}$$

It follows from (3.7) and (4.5), that the equations of equilibrium (2.2) reduce to

$$\sigma_{\alpha\beta,\beta} = 0, \tag{4.8}$$

on $\Sigma \times I$. The boundary conditions (2.6) become

$$\sigma_{\alpha\beta} n_\beta = 0, \tag{4.9}$$

on Γ. Thus, from (4.4), (4.6), (4.8) and (4.9) we conclude that the functions ν_α are the components of the displacement vector field in a thermoviscoelastic plane–strain problem corresponding to the temperature distribution f. Clearly, the necessary and sufficient conditions for the existence of the functions ν_α are satisfied. Therefore, we can assume that the functions ν_α are known.

The conditions (2.7) and (2.7.4) are satisfied on the basis of the relations (4.5). From (2.7.2) and (2.7.3) we obtain for the functions a_i the following Volterra integral equations

$$D_{ij} \circ a_j = d_i, \tag{4.10}$$

where

$$\begin{aligned} &D_{\alpha\beta} = \int\limits_\Sigma x_\alpha [(L+2M)x_\beta + t_{33}^{(\beta)}]da, \quad && D_{\alpha 3} = \int\limits_\Sigma x_\alpha [L+2M+t_{33}^{(3)}]da, \\ &D_{3\alpha} = \int\limits_\Sigma [(L+2M)x_\alpha + t_{33}^{(\alpha)}]da, && D_{33} = \int\limits_\Sigma [L+2M+t_{33}^{(3)}]da, \\ &d_\alpha = \int\limits_\Sigma x_\alpha [H - L \circ \gamma_{\rho\rho}]da, && d_3 = \int\limits_\Sigma [H - L \circ \gamma_{\rho\rho}]da. \end{aligned} \tag{4.11}$$

It is known (see [4]) that the matrix $(D_{ij}(0))$ is symmetric and inversable. Since d_i are continuous on $(0, t_1)$, we conclude that the system (4.10) has one and only one solution continuous on $[0, t_1)$.

Thus the problem $P^{(0)}$ is solved.

5. Recurrence process. We denote by $u_i^*, \varepsilon_{ij}^*$ and t_{ij}^* the components of the displacement vector, the components of strain tensor and the components of the stress tensor from the problem $P^{(n)}$, respectively, and by $u_i, \varepsilon_{ij}, t_{ij}$ the analogous functions from the problem $P^{(n+1)}$. Let us establish the solution of the problem $P^{(n+1)}$ when the solution of the problem $P^{(n)}$ is known. We shall use the method given in [2]. As the solution of the problem $P^{(n)}$ is supposed to be known for any T_n, we known the solution of the probelm $P^{(n)}$ when $T = T_{n+1}(x_1, x_2, t)x_3^n$. Thus, the problem can be presented as follows: find the functions $u_i, \varepsilon_{ij}, t_{ij}$ which satisfy the equations (2.1),(2.2), (2.5) on $B \times I$ and the conditions (2.6)–(2.7.4) when the temperature field has the form

$$T = g(x_1, x_2, t)x_3^{n+1}, \tag{5.1}$$

assuming that the functions $u_i^*, \varepsilon_{ij}^*, t_{ij}^*$ which satisfy the equations (2.1), (2.2), (2.5) on $B \times I$ and the conditions (2.6)–(2.7.4) for the temperature distribution

$$T = g(x_1, x_2, t)x_3^n, \tag{5.2}$$

are known. We assume that g is prescribed function of class C^2.

We seek the solutions of the problem in the form

$$\begin{aligned} u_\alpha = \; & (n+1)\left[\int_0^{x_3} u_\alpha^* dx_3 - \tfrac{1}{2}b_\alpha(t)x_3^2 + \sum_{s=1}^{3} b_s \circ u_\alpha^{(s)} + \right. \\ & \left. + \varepsilon_{3\beta\alpha} b_4(t) x_\beta x_3 + w_\alpha(x_1, x_2, t)\right], \\ u_3 = \; & (n+1)\left[\int_0^{x_3} u_3^* dx_3 + (b_1(t)x_1 + b_2(t)x_2 + b_3(t))x_3 + \right. \\ & \left. + b_4 \circ \phi + \Psi(x_1, x_2, t)\right], \end{aligned} \tag{5.3}$$

where $u_\alpha^{(s)}$ are the components of the displacement field from the problem $D^{(s)}, w_\alpha, \phi$ and Ψ are unknown functions of the variables x_1, x_2 and t of class C^2, and $b_n (n = 1, 2, 3, 4)$ are unknown continuous functions of the variable t.

From (2.1) and (5.3) we obtain

$$\begin{aligned} \varepsilon_{\alpha\beta} = \; & (n+1)\left[\int_0^{x_3} \varepsilon_{\alpha\beta}^* dx_3 + \sum_{s=1}^{3} b_s \circ u_\alpha^{(s)} + e_{\alpha\beta}\right], \\ \varepsilon_{\alpha 3} = \; & (n+1)\left\{\int_0^{x_3} \varepsilon_{\alpha 3}^* dx_3 + \tfrac{1}{2}[b_4 \circ \phi_{,\alpha} + \Psi_{,\alpha} + \varepsilon_{3\beta\alpha} b_4 x_\beta + \right. \\ & \left. + u_\alpha^*(x_1, x_2, 0, t)]\right\}, \\ \varepsilon_{33} = \; & (n+1)\left[\int_0^{x_3} \varepsilon_{33}^* dx_3 + b_1 x_1 + b_2 x_2 + b_3 + u_3^*(x_1, x_2, 0, t)\right], \end{aligned} \tag{5.4}$$

where

$$e_{\alpha\beta} = \frac{1}{2}(w_{\alpha,\beta} + w_{\beta,\alpha}). \tag{5.5}$$

In view of (2.11), (5.1)–(5.4) we obtain

$$
\begin{aligned}
t_{\alpha\beta} = \quad & (n+1)\left\{\int_0^{x_3} t^*_{\alpha\beta}dx_3 + L\circ[b_1x_1+b_2x_2+b_3+\right.\\
& \left.+u^*_3(x_1,x_2,0,t)]\delta_{\alpha\beta} + \sum_{s=1}^{3} b_s\circ t^{(s)}_{\alpha\beta} + \pi_{\alpha\beta},\right\}\\
t_{33} = \quad & (n+1)\left\{\int_0^{x_3} t^*_{33}dx_3 + (L+2M)\circ[b_1x_1+b_2x_2+b_3+\right.\\
& \left.+u^*_3(x_1,x_2,0,t)] + \sum_{s=1}^{3} b_s\circ t^{(s)}_{33} + L\circ e_{\rho\rho}\right\},\\
t_{\alpha 3} = \quad & (n+1)\left\{\int_0^{x_3} t^*_{\alpha 3}dx_3 + M\circ[b_4\circ\phi_{,\alpha} + \Psi_{,\alpha} - \varepsilon_{3\alpha\beta}x_\beta b_4+\right.\\
& \left.+u^*_\alpha(x_1,x_2,0,t)]\right\},
\end{aligned}
\tag{5.6}
$$

where

$$
\pi_{\alpha\beta} = L\circ e_{\rho\rho}\delta_{\alpha\beta} + 2M\circ e_{\alpha\beta}. \tag{5.7}
$$

We consider the following boundary value problem for the function ϕ

$$
(M\circ\phi_{,\alpha})_{,\alpha} = h \text{ on } \Sigma\times I, \qquad M\circ\phi_{,\alpha}n_\alpha = g \text{ on } \Gamma, \tag{5.8}
$$

where

$$
h = \varepsilon_{3\alpha\beta}(Mx_\beta)_{,\alpha}, \qquad g = \varepsilon_{3\alpha\beta}x_\beta n_\alpha M. \tag{5.9}
$$

The necessary and sufficient condition for the existence of the boundary –value problem (5.8) is (Fichera) [10])

$$
\int_\Sigma h\ da = \int_\Gamma g\ ds. \tag{5.10}
$$

If we take into account the relations (5.9) we see that the condition (5.10) is satisfied. Therefore, the function ϕ can be assumed to be known.

The equation of equilibrium (2.2) reduce to

$$
\pi_{\alpha\beta,\beta} + G_\alpha = 0, \tag{5.11}
$$

and

$$
(M\circ\Psi_{,\alpha})_{,\alpha} = P, \tag{5.12}
$$

on Σ, for each $t \in I$. We have used the notations

$$
\begin{aligned}
G_\alpha &= t^*_{\alpha 3}(x_1, x_2, 0, t) + \{[L \circ u^*_3](x_1, x_2, 0, t)\}_{,\alpha}\,, \\
P &= -t^*_{33}(x_1, x_2, 0, t) - \{[L \circ u^*_\alpha](x_1, x_2, 0, t)\}_{,\alpha}\,.
\end{aligned} \tag{5.13}
$$

The boundary conditions (2.6) reduce to the conditions

$$\sigma_{\alpha\beta} n_\beta = q_\alpha, \tag{5.14}$$

and

$$M \circ \Psi_{,\alpha} n_\alpha = S, \tag{5.15}$$

on Γ, for each $t \in I$. We have used the notations

$$q_\alpha = -n_\alpha [L \circ u^*_3](x_1, x_2, 0, t), \qquad S = -n_\alpha [M \circ u^*_\alpha](x_1, x_2, 0, t). \tag{5.16}$$

The necessary and sufficient condition for the existence of the solution of the boundary value problem (5.12), (5.15) becomes

$$\int_\Sigma t^*_{33}(x_1, x_2, 0, t) da = 0.$$

We note that this condition is satisfied since t^*_{33} satisfies the condition (2.7.2). We conclude that the functions w_α are the components of the displacement field in the isothermal plane strain problem characterized by the equations (5.5), (5.7), (5.11) on Σ and the boundary conditions (5.14). It follows from (5.13), (5.16) and the divergence theorem that

$$
\begin{aligned}
&\int_\Sigma G_\alpha da + \int_\Gamma q_\alpha ds = \int_\Sigma t^*_{\alpha 3}(x_1, x_2, 0, t) da, \\
&\int_\Sigma \varepsilon_{3\alpha\beta} x_\alpha G_\beta da + \int_\Gamma \varepsilon_{3\alpha\beta} x_\alpha q_\beta ds = \int_\Sigma \varepsilon_{3\alpha\beta} x_\alpha t^*_{\beta 3}(x_1, x_2, 0, t) da.
\end{aligned} \tag{5.17}
$$

Since the components t^*_{ij} satisfy the conditions (2.7.1)– (2.7.4), wee conclude from (5.17) that the necessary and sufficient conditions (3.6) for the existence of a solution to the boundary value problem (5.5), (5.7), (5.11), (5.14) are satisfied. We will assume that the functions w_α are known.

If we take into account (5.6) and (5.8), then from (2.7.4) we obtain the following equation for b_4

$$D \circ b_4 = -\int_\Sigma \varepsilon_{3\alpha\beta} M \circ [\Psi_{,\beta} + u^*_\beta(x_1, x_2, 0, t)] da, \tag{5.18}$$

where

$$D = \int_{\Sigma} M \circ (\varepsilon_{3\alpha\beta} x_\alpha \phi_{,\beta} + x_1^2 + x_2^2) da. \tag{5.19}$$

We note that $D(0)$ is the torsional rigidity in classical elasticity, so that $D(0) > 0$. The equation (5.18) uniquely determines the function b_4.
In view of (5.6) the conditions (2.7.2) and (2.7.3) reduce to

$$D_{ij} \circ b_j = A_i, \tag{5.20}$$

where D_{ij} are given by (4.11) and

$$\begin{aligned} A_\alpha &= -\int_{\Sigma} x_\alpha [L \circ e_{\rho\rho} + (L + 2M) \circ u_3^*(x_1, x_2, 0, t)] da, \\ A_3 &= -\int_{\Sigma} [L \circ e_{\rho\rho} + (L + 2M) \circ u_3^*(x_1, x_2, 0, t)] da. \end{aligned} \tag{5.21}$$

The system (5.20) has the same form as the system (4.10). It follows that the functions b_i are determined by (5.20).

We note that

$$\begin{aligned} \int_{\Sigma} t_{3\alpha} da &= \int_{\Sigma} (t_{3\alpha} + x_\alpha t_{i3,i}) da = \int_{\Sigma} [(x_\alpha t_{3\beta})_{,\beta} + x_\alpha t_{33,3}] da = \\ &= \int_{\Gamma} x_\alpha t_{3\beta} n_\beta ds + (n+1) \int_{\Sigma} x_\alpha t_{33}^* da. \end{aligned} \tag{5.22}$$

In view of the conditions (2.6) and (2.7.3), the relations (5.22) imply that the conditions (2.7.1) are identically satisfied. Thus, the initial problem is solved.

REFERENCES

1. B o l e y, B.A. and W e i n e r, J.H.– *Theory of Thermal Stresses*, John Wiley, New York (1960).
2. I e ş a n, D.– *Thermal stresses in composite cylinders*, J. Thermal Stresses, vol.**3**, pp.495–508 (1980).
3. N a p p a, L.– *Thermal stresses in microstretch elastic cylinders*, J. Thermal Stresses, vol.**18**, pp.537–550 (1995).
4. I e ş a n, D.– *Saint–Venant's Problem*. Lecture Notes in Mathematics, vol.**1279**, Springer, Berlin (1987).

5. I e ş a n, D.– *On Saint–Venant's problem.* Arch. Rational Mech. Anal., vol.**51**, pp.363–373 (1986).
6. L e i t m a n, M.J. and F i s h e r, G.M.C.– *The Linear Theory of Viscoelasticity.* In "Handbuch der Physik", vol.VI a/3, (edited by C.A.Truesdell), Springer, Berlin (1973).
7. I e ş a n, D.– *Sur la thèorie de la thermoviscoelasticitè coupleè*, C. R. Acad. Sc. Paris, Serie B., **267**, pp.667–670 (1968).
8. G u r t i n, M.E.– *Time–reversal and symmetry in the thermodynamic of materials with memory*, Arch. Rational Mech. Anal., vol.**44**, pp.387–399 (1972).
9. C h i r i ţ ă, S.– *Saint–Venant's problem and semi–inverse solutions in linear viscoelasticity*, Acta Mechanica, vol.**94**, pp.221–232 (1992).
10. F i c h e r a, G.–*Existence Theorem in Elasticity.* In "Handbuch der Physik", vol.VI a/2 (Edited by C.A.Truesdell), Springer, Berlin (1972).

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