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# SOME EXAMPLES OF PLURIHARMONIC AND $\varphi$–PLURIHARMONIC MAPS

BY

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## Dedicated to Professor Gh. Gheorghiev on the occasion of his 90–th birthday

**Abstract.** In this paper we study some examples of pluriharmonic and $\varphi$–pluriharmonic maps. We first give some necessary and sufficient conditions under which the canonical projection $\pi: TM \to M$ is pluriharmonic, considering several almost Hermitian structures on the tangent bundle $TM$ of the Riemannian, respective almost Hermitian, manifold $M$. We also study the relationship between pluriharmonicity and $\varphi$–pluriharmonicity for several maps defined on products of almost Hermitian or metric almost contact manifolds.

**Introduction.** In this paper we study some examples of pluriharmonic and $\varphi$ –pluriharmonic maps. We first give some necesary and sufficient conditions, [3], under which the canonical projection $\pi: TM \to M$ is pluriharmonic, considering several almost Hermitian structures on the tangent bundle $TM$ of the Riemannian, respective almost Hermitian, manifold $M$.

We study some canonical almost Hermitian or metric almost contact structures on products of two almost Hermitian or metric almost contact manifolds $M$ and $M'$. In fact, the metric almost contact structures on odd dimensional manifolds are analogues to the almost Hermitian structure for even dimensional manifolds. We also study the relationship between pluriharmonicity for several maps defined on products of such manifolds. Suppose $M, M'$ and $N$ be Riemannian manifolds, $f: M \to N$ a smooth map, and $\widetilde{f}: M \times M' \to N$, given by $\widetilde{f}(x, x') = f(x)$, for any $x \in M, x' \in M'$. The point is easier to understand from the following table:

**Table 1**

| $M$ | $M'$ | $M \times M'$ | $f: M \to N$ | Our result | $\widetilde{f}: M \times M' \to N$ |
|---|---|---|---|---|---|
| almost Hermitian | almost Hermitian | almost Hermitian | plurihar-monicity | $\Leftrightarrow$ | plurihar-monicity |
| almost Hermitian | metric almost contact | metric almost contact | plurihar-monicity | $\Leftrightarrow$ | $\widetilde{\varphi}$ –plurihar-monicity |
| metric almost contact | almost Hermitian | metric almost contact | $\varphi$ –plurihar-monicity | $\Leftrightarrow$ | $\widetilde{\varphi}$ – pluryhar-monicity |
| metric almost contact | metric almost contact | almost Hermitian | $\varphi$ –plurihar-monicity | $\Rightarrow$ | plurihar-monicity |

The manifolds, maps, vector fields etc. we consider in this paper are supposed to be smooth, i.e. differentiable of class $C^\infty$. We will denote the set of all vector fields on $M$ by $\mathcal{X}(M)$.

**1. Preliminaries.** Let $(M, g), (N, h)$ be Riemannian manifolds and $f: M \to N$ be a smooth map. We denote by $\nabla^M, \nabla^N$ the Levi–Civita connections on $M$, respectively $N$ and by $\nabla^{f^{-1}TN}$ the connection induced by $f$ on the pull–back bundle $f^{-1}TN$. Then the second fundamental form of $f$ is

$$\alpha_f(X, Y) = (\nabla df)(X, Y) = \nabla^{f^{-1}TN}_X df Y - df(\nabla^M_X Y) \tag{1}$$

for any $X, Y \in \mathcal{X}(M)$.

The tension field of $f$ is $\tau(f) =$ trace $\alpha_f$. A map $f$ is said to be harmonic if its tension field is zero.

Let $(M, g)$ be a Riemannian $2n$ – dimensional manifold. Then $M$ is almost Hermitian if there is given an endomorphism $J$ on $TM$ such that $J^2 = -I$ ($I$ being the identify on $TM$) and $g(JX, JY) = g(X, Y)$, for any $X, Y \in \mathcal{X}(M)$.

Let $(M, g, J)$ be an almost Hermitian manifold and $(N, h)$ a Riemannian one. Then a smooth map $f: M \to N$ is called pluriharmonic if its

second fundamental form verifies:

$$\alpha_f(X,Y) + \alpha_f(JX,JY) = 0 \tag{2}$$

for any $X, Y$ vector fields on $M$.

**Remark.** Any pluriharmonic map $f\colon (M,g,J) \to (N,h)$ is harmonic, [14].

Let $M$ be a real manifold of dimension $2n+1$. Suppose $M$ has an almost contact structure given by the tensor field $\varphi$ of type (1,1), a vector field $\xi$ and a 1–form $\eta$ which verify:

$$\varphi^2 = -I + \eta \otimes \xi,\ \eta(\xi) = 1. \tag{3}$$

where $I$ is the identy on $TM$. On an almost contact manifold $(M,\varphi,\eta,\xi)$, we also have:

$$\varphi \circ \xi = 0,\ \eta \circ \varphi = 0. \tag{4}$$

It is well known that there exists a metric $g$ on $M$ compatible with the almost contact manifold, i.e.

$$g(X,Y) = g(\varphi X, \varphi Y) + \eta(X)\eta(Y) \tag{5}$$

for any $X, Y$ vector fields on $M$. An almost contact manifold endowed with such a metric is called a metric almost contact manifold.

It is easy to see that if $(M,g,\varphi,\eta,\xi)$ is a metric almost contact manifold, then:

$$g(\xi,\xi) = 1,\ g(\xi,X) = \eta(X), \tag{6}$$

for any $X \in \mathcal{X}(M)$.

A map $f\colon M \to N$ between the metric almost contact manifold $(M,g,\varphi,\eta,\xi)$ and the Riemannian manifold $(N,h)$ is called $\varphi$ – pluriharmonic if

$$\alpha_f(X,Y) + \alpha_f(\varphi X, \varphi Y) = 0, \tag{7}$$

for any $X, Y \in \mathcal{X}(M)$.

**Remark.** Any $\varphi$–pluriharmonic map is harmonic, [10].

We have, [10], that if $f: M \to N$ is $\varphi-$ pluriharmonic, then

$$\alpha_f(\xi,\xi)=0,\ \alpha_f(X,\xi)=0, \tag{8}$$

for any $X \in D$, where $D$ is the distribution orthogonal to $\xi$.

We also remind here the Leibnitz formula, [11]. Let $f$ be a mapping of the product manifold $M \times M'$ into another manifold $N$. The differential $df$ at $(p,q) \in M \times M'$ can be expressed as follows. If $Z \in T_{(p,q)}(M \times M')$ coresponds to $(X,Y) \in T_pM + T_qM'$, then

$$df(Z) = df_1(X) + df_2(Y) \tag{9}$$

where $f_1: M \to N, f_2: M' \to N$ are defined by $f_1(p') = f(p',q)$ for $p' \in M$ and $f_2(q') = f(p,q')$ for $q' \in M'$.

**2. Canonical structures on products of almost hermitian and metric almost contact manifolds.** Let $(M,g,J)$ and $(M',g',J')$ be two almost Hermitian manifolds. Let $\widetilde{M} = M \times M'$ be the product manifold. Consider on $\widetilde{M}$ the metric $\widetilde{g}$ given by:

$$\widetilde{g}((X,X'),(Y,Y')) = g(X,Y) + g(X',Y') \tag{10}$$

for any $X,Y \in \mathcal{X}(M), X',Y' \in \mathcal{X}(M')$.

Consider also the endomorphism of $T\widetilde{M}$:

$$\widetilde{J}((X,X')) = (JX, J'X') \tag{11}$$

for any $X \in \mathcal{X}(M), X' \in \mathcal{X}(M')$.

It is easy to see that

$$\widetilde{J}^2 = -I$$

and

$$\widetilde{g}((X,X'),(Y,Y')) = \widetilde{g}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y')),$$

$\forall X,Y \in \mathcal{X}(M), X',Y' \in \mathcal{X}(M')$, hence $(\widetilde{M},\widetilde{g},\widetilde{J})$ is an almost Hermitian manifold.

Let $(M,g,J)$ be an almost Hermitian manifold and $(M',g',\varphi',\eta',\xi')$ a metric almost contact manifold. Consider on $\widetilde{M} = M \times M'$ the metric $\widetilde{g}$ given by (10), the tensor field $\widetilde{\varphi}$ of type (1,1), the 1–form $\widetilde{\eta}$ and the vector field $\widetilde{\xi}$ given by:

$$\widetilde{\varphi}(X,X') = (JX, \varphi'X'),\ \widetilde{\eta}(X,X') = \eta'(X'),\ \widetilde{\xi} = (0,\xi') \tag{12}$$

for any $X \in \mathcal{X}(M), X' \in \mathcal{X}(M')$.

We have then for any $X, Y \in \mathcal{X}(M), X', Y' \in \mathcal{X}(M')$.

$$\widetilde{\eta}(\widetilde{\xi}) = \eta'(\xi') = 1;$$
$$\widetilde{\varphi}^2(X, X') = \widetilde{\varphi}(JX, \varphi' X') = (J^2 X, \varphi'^2 X') = (-X, -X' + \eta'(X')\xi') =$$
$$= -(X, X') + (0, \eta'(X')\xi') = -(X, X') + \widetilde{\eta}(X, X')\widetilde{\xi}$$

and

$$\widetilde{g}(\widetilde{\varphi}(X, X'), \widetilde{\varphi}(Y, Y')) = \widetilde{g}((JX, \varphi' X'), (JY, \varphi' Y')) =$$
$$= g(JX, JY) + g(\varphi' X', \varphi' Y') = g(X, Y) + g(X', Y') - \eta'(X')\eta'(Y') =$$
$$= \widetilde{g}((X, X'), (Y, Y')) - \widetilde{\eta}(X, X')\widetilde{\eta}(Y, Y').$$

Then $(\widetilde{g}, \widetilde{\varphi}, \widetilde{\xi}, \widetilde{\eta})$ is a canonical metric almost contact structure on $\widetilde{M}$.

In a similar way, one may consider a canonical metric almost contact structure on $\widetilde{M} = M \times M'$, where $(M, g, \varphi, \eta, \xi)$ is a metric almost contact manifold and $(M', g' J')$ is an almost Hermitian manifold. Indeed, consider the metric $\widetilde{g}$ from (10) and:

$$\widetilde{\varphi}(X, X') = (\varphi X, J' X'),\ \widetilde{\eta}(X, X') = \eta(X),\ \widetilde{\xi} = (\xi, 0) \tag{13}$$

for $X \in \mathcal{X}(M), X' \in \mathcal{X}(M')$.

Finally, let $(M, g, \varphi, \eta, \xi)$ and $(M', g', \varphi', \eta', \xi')$ be metric almost contact manifolds. Consider on the product $\widetilde{M} = M \times M'$ the metric $\widetilde{g}$ given by (10). Let $\widetilde{J}$ be the endomorphism of $TM'$:

$$\widetilde{J}(X, X') = (\varphi X - \eta'(X')\xi, \varphi' X' + \eta(X)\xi') \tag{14}$$

for any $X \in \mathcal{X}(M), X' \in \mathcal{X}(M')$.

Then

$$\widetilde{J}(X, X') = \widetilde{J}(\varphi X - \eta'(X')\xi, \varphi' X' + \eta(X)\xi') =$$
$$= (\varphi^2 X - \varphi(\eta'(X')\xi) - \eta'(\varphi' X' + \eta(X)\xi')\xi,$$
$$\varphi'^2 X' + \varphi'(\eta(X)\xi') + \eta(\varphi X - \eta'(X')\xi)\xi') =$$
$$= (\varphi^2 X - \eta'(X')\varphi(\xi) - \eta'(\varphi' X') - \eta(X)\eta'(\xi')\xi,$$
$$\varphi'^2 X' - \eta(X)\varphi'(\xi') + \eta(\varphi X) + \eta'(X')\eta(\xi)\xi')$$

Next, using (3) and (4) we get:

$$\widetilde{J}^2(X, X') \equiv -(X, X')$$

We also have, for $X, Y \in \mathcal{X}(M), X', Y' \in \mathcal{X}(M')$:

$$\widetilde{g}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y')) = \widetilde{g}((\varphi X - \eta'(X')\xi, \varphi' X' - \eta(X)\xi'),$$
$$(\varphi Y - \eta'(Y')\xi, \varphi' Y' + \eta(Y)\xi')) = g(\varphi X - \eta'(X')\xi, \varphi Y - \eta(Y')\xi) +$$
$$+g'(\varphi' X' + \eta(X)\xi', \varphi' Y' + \eta(Y)\xi').$$

But $g(\varphi X - \eta'(X')\xi, \varphi Y - \eta'(Y')\xi) = g(\varphi X, \varphi Y) - \eta'(X')g(\xi, \varphi Y) -$ $-\eta'(Y')g(\varphi X, \xi) + \eta'(X')\eta'(Y')g(\xi,\xi)$ and from (4), (5) and (6) we get

$$\widetilde{g}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y')) = \widetilde{g}((X,X'),(Y,Y')).$$

Therefore the product $\widetilde{M} = M \times M'$ has a canonical almost Hermitian structure.

**3. Some examples of pluriharmonic and $\varphi-$ pluriharmonic maps.** If $(M,g)$ is a Riemannian manifold then on the tangent bundle, one can consider, in a canonical way, the almost complex structure $J$, given by:

$$J(X^h) = X^v, \ J(X^v) = -X^h, \tag{15}$$

where $X^h$ and $X^v$ are the orizontal and the vertical lifts to $TM$ of the vector field $X$ of $M$, with respect to the Levi – Civita connection on $M$. Moreover, the tangent bundle is almost Kähler if we consider the Sasaki metric $G$ on it, [5], [19], and is Kähler iff $g$ has vanishing curvature.

Let us consider the Riemannian manifold $(M,g)$ and the tangent bundle $TM$ with the almost Hermitian structure given by the Sasaki metric and $J$ from (15). Then we have, [3]:

**Theorem 1.** *The projection $\pi: TM \to M$ is pluriharmonic if and only if the curvature tensor $R$ of $g$ is zero.*

Now, let $(M,g,J)$ be an almost Hermitian manifold. One can consider on the tangent bundle the almost Hermitian structure given by the Sasaki metric and the horizontal lift $J^h$ of $J$ $TTM$ given by:

$$J^h(X^h) = (JX)^h, \ J^h(X^v) = (JX)^v. \tag{16}$$

It this case we have, [3]:

**Theorem 2.** *The projection $\pi: TM \to M$ is pluriharmonic if and only if*

$$R(X, JY)JZ + R(X,Y)Z = 0 \tag{17}$$

*for any* $X, Y$ *and* $Z$ *vector fields on* $M$ .

Consider $M, M'$ differentiable manifolds, $(N, h)$ a Riemannian manifold, $F\colon M \to N$ a smooth map, and $\widetilde{f}\colon M \times M' \to N$ , given by

$$\widetilde{f}(x, x') = f(x), \tag{18}$$

for any $x \in M, x' \in M'$ .

Consider first $(M, g, J)$ and $(M', g', J')$ almost Hermitian manifolds and $(\widetilde{M}, \widetilde{g}, \widetilde{J})$ the canonical product almost Hermitian manifold given by (10) and (11). Then we have:

**Theorem 3.** $\widetilde{f}$ *is pluriharmonic iff* $f$ *is pluriharmonic.*

**Proof.** We can see that the Levi–Civita connection on $\widetilde{M}$ associated to $\widetilde{g}$ is:

$$\nabla^{\widetilde{M}}_{(X,X')}(Y, Y') = (\nabla^M_X Y, \nabla^{M'}_{X'} Y) \tag{19}$$

for any $X, Y \in \mathcal{X}(M), X', Y' \in \mathcal{X}(M')$ , where $\nabla^M$ and $\nabla^{M'}$ are the Levi–Civita connections of $M$ and $M'$ respectively.

Note that $\widetilde{f} = f \circ \pi_1$ , where $\pi_1$ is the projection $\pi_1\colon M \times M' \to M, \pi_1(x, x') = x$ , for $x \in M, x' \in M'$ . From the Leibniz formula (9) we obtain $d\widetilde{f}(X, Y) = df(X, Y)$ and $d\pi_1(X, Y) = X$ , for any $X \in \mathcal{X}(M), X' \in \mathcal{X}(M')$ .

Then, [7], the second fundamental form of $\widetilde{f}$ is:

$$\alpha_{\widetilde{f}} = (\nabla d\widetilde{f}) = \nabla d(f \circ \pi_1) = df \circ \nabla d\pi_1 + \nabla df(d\pi_1, d\pi_1) \tag{20}$$

Consider now $X, Y \in \mathcal{X}(M), X', Y' \in \mathcal{X}(M')$ . Then:

$$\alpha_{\widetilde{f}}((X, X'), (Y, Y')) = \alpha_f(X, Y) \tag{21}$$

Indeed:

$$\begin{aligned}
&\alpha_{\widetilde{f}}((X, X'), (Y, Y')) = (\nabla d\widetilde{f})((X, X'), (Y, Y')) = \\
&= df((\nabla d\pi_1)((X, X'), (Y, Y')) + \nabla df(d\pi_1(X, X'), d\pi_1(Y, Y')) = \\
&= df(\nabla^{\pi_1^{-1}TM}_{(X,X')} d\pi_1(Y, Y') - d\pi_1(\nabla^{\widetilde{M}}_{(X,X')}(Y, Y')) + \nabla df(X, Y) = \\
&= df(\nabla^M_X Y - \nabla^M_X Y) + \nabla df(X, Y) = \alpha_f(X, Y)
\end{aligned}$$

Therefore:

$$\alpha_{\widetilde{f}}((X,X'),(Y,Y'))+\alpha_{\widetilde{f}}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y'))=$$
$$=\alpha_f(X,Y)+\alpha_{\widetilde{f}}((JX,J'X'),(\widetilde{J}(JY,J'Y'))=\alpha_f(X,Y)+\alpha_f(JX,JY),$$

and hence $\widetilde{f}$ is pluriharmonic if and only if $f$ is so. Q.E.D.

Now, let $(M,g,J)$ be an almost Hermitian manifold, $(M',g',\varphi',\eta',\xi')$ a metric almost contact manifold. Consider on $\widetilde{M}=M\times M'$ the canonical metric almost contact structure $(\widetilde{g},\widetilde{\varphi},\widetilde{\xi},\widetilde{\eta})$ given by (10) and (11).

**Theorem 4.** *$\widetilde{f}$ is $\widetilde{\varphi}$ – pluriharmonic iff $f$ is pluriharmonic.*

**Proof.** For $X,Y\in\mathcal{X}(M),X',Y'\in\mathcal{X}(M')$ we have, by (21):

$$\alpha_{\widetilde{f}}((X,X'),(Y,Y'))+\alpha_{\widetilde{f}}(\widetilde{\varphi}(X,X'),\widetilde{\varphi}(Y,Y'))=$$
$$=\alpha_{\widetilde{f}}((X,X'),(Y,Y'))+\alpha_{\widetilde{f}}((JX,\varphi'X'),(JY,\varphi'Y'))=$$
$$=\alpha_f(X,Y)+\alpha_f(JX,JY).$$

Q.E.D.

Next, consider $(M,g,\varphi,\eta,\xi)$ a metric almost contact manifold, $(M',g',J')$ an almost Hermitian manifold. Let $\widetilde{M}=M\times M'$ be endowed with the metric almost contact structure given by (10) and (13).

**Theorem 5.** *$\widetilde{f}$ is $\widetilde{\varphi}$ – pluriharmonic iff $f$ is $\varphi$ –pluriharmonic.*

**Proof.** Let $X,Y$ be vector fields on $M$ and $X',Y'$ vector fields on $M'$. By (21), we get:

$$\alpha_{\widetilde{f}}((X,X'),(Y,Y'))+\alpha_{\widetilde{f}}(\widetilde{\varphi}(X,X'),\widetilde{\varphi}(Y,Y'))=\alpha_f(X,Y)+$$
$$+\alpha_{\widetilde{f}}((\varphi X,J'X'),(\varphi Y,J'Y'))=\alpha_f(X,Y)+\alpha_f(\varphi X,\varphi Y).$$

Q.E.D.

Finally, suppose $(M,g,\varphi,\eta,\xi)$ and $M',g',\varphi',\eta',\xi')$ metric almost contact manifolds. Consider in $\widetilde{M}=M\times M'$ the almost Hermitian structure $(\widetilde{g},\widetilde{J})$ from (10) and (14).

**Theorem 6.** *If $f$ is $\varphi$ – pluriharmonic, then $\widetilde{f}$ is $\varphi$ – pluriharmonic.*

**Proof.** Let $X,Y$ be vector fields on $M$ and $X',Y'$ vector fields on $M'$. We have:

$$\begin{aligned}
&\alpha_{\widetilde{f}}((X,X'),(Y,Y')) + \alpha_{\widetilde{f}}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y')) = \\
&= \alpha_f(X,Y) + \alpha_f(\varphi X - \eta'(X')\xi, \varphi Y - \eta'(Y')\xi) = \\
&= \alpha_f(X,Y) + \alpha_f(\varphi X, \varphi Y) - \\
&-\eta'(X')\alpha_f(\xi\varphi Y) - \eta'(Y')\alpha_f(\varphi X,\xi) + \eta'(X')\eta'(Y')\alpha_f(\xi,\xi).
\end{aligned}$$

Since $f$ is $\varphi$–pluriharmonic, $\alpha_f(X,Y) + \alpha_f(\varphi X, \varphi Y)$ and from (8):

$$\alpha_f(\xi,\xi) = 0,\ \alpha_f(\varphi X,\xi) = 0,\ \alpha_f(\xi,\varphi Y) = 0.$$

for any $X, Y$ vector fields on $M$.

It follows that:

$$\alpha_{\widetilde{f}}((X,X'),(Y,Y')) + \alpha_{\widetilde{f}}(\widetilde{J}(X,X'),\widetilde{J}(Y,Y')) = 0.$$

Q.E.D.

**Remark.** It is easy to see that this time, converse is not true.

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