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FIBRE STRUCTURES ON $OSC^{(k)}(M)$ BUNDLE

BY

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Dedicated to Professor Gh.Gheorghiev on the occasion of his 90-th birthday.

The geometry of $Osc^{(k)}(M)$ bundle meets some difficulties for $k > 1$ because its fibre structure is not a vectorial one.

There are many ways to approach this bundle but a systematic study of it was done in the last years by R.Miron and Gh.Atanasiu ([6], [7]) by using some adapted distributions in tangent spaces to $Osc^{(k)}(M)$ manifold.

In the present paper we shall analyse two other points of view in study of this geometry. The first consists in introducing, by means of a nonlinear connection on base manifold M, of some adapted local coordinates on $Osc^{(k)}(M)$ that detrmine a vector bundle structure.

The second one is to view the $Osc^{(k)}(M)$ bundle as an associated bundle to the k–frames bundle. This point of view is useful in gauge theories.

1. Vector bundle structure on $Osc^{(k)}(M)$. Let M be a differentiable manifold, $\dim M = m$. For begining, we shall prove that $Osc^{(k)}(M) = E_k$ is a subbundle of the tangent bundle over E_{k-1}.

Let be $E_1 = Osc^{(1)}(M)$ the tangent bundle $\Pi_\circ : TM \longrightarrow M$ and $\Pi^T : TTM \longrightarrow TM$ its tangent map. We have also the bundle $\Pi_1 : TTM \longrightarrow TM$. Let us consider the subbundle ([1]):

$$E_2 = \{z \in T(E_1)/\Pi_1(z) = \Pi^T_\circ(z)\}.$$

Locally, if $z = (x, y, X, Y)$ is a point in $TTM = TE_1$ then $z \in E_2$ iff $z = (x, y, X, Y)$. It follows that a point in E_2 is locally characterized by the

coordinates $z=(x,y,Y)$ with the change rules:

$$\begin{aligned}&\widetilde{x}^i=\widetilde{x}^i(x),\\ &\widetilde{y}^i=\frac{\partial\widetilde{x}^i}{\partial x^j}\cdot y^j,\\ &\widetilde{Y}^i=\frac{\partial\widetilde{y}^i}{\partial x^j}\cdot y^j+\frac{\partial\widetilde{y}^i}{\partial y^j}\cdot Y^j.\end{aligned}\tag{1.1}$$

Then, notations $x^i,y^i=y^{(1)i},Y^i=2y^{(2)i}$ one deduces that (1.1) gives the local change of coordinates on $Osc^{(2)}(M)$ ([7]). Therefore, E_2 is a canonical choice of $Osc^{(2)}(M)$ bundle.

Further, inductively let be on $E_\alpha=Osc^{(\alpha)}(M)$ the next change rules known fron [6] for $u=(x,y^1,\ldots,y^{(\alpha)})$:

$$\begin{aligned}&\widetilde{x}^i=\widetilde{x}^i(x)\\ &\widetilde{y}^{(1)i}=\frac{\partial\widetilde{x}^i}{\partial x^j}\cdot y^j\\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ &\alpha\widetilde{y}^{(\alpha)}=\frac{\partial\widetilde{y}^{(\alpha-1)i}}{\partial x^j}\cdot y^{(1)j}+2\frac{\partial\widetilde{y}^{(\alpha-1)i}}{\partial y^{(1)j}}y^{(2)j}+\ldots+\alpha\frac{\partial\widetilde{y}^{(\alpha-1)i}}{\partial y^{(\alpha-1)j}}y^{(\alpha)j}\end{aligned}\tag{1.2}$$

For the bundles $\Pi_\alpha:T(E_\alpha)\longrightarrow E_\alpha$ and $\Pi^\alpha_{\alpha-1}:E_\alpha\longrightarrow E_{\alpha-1}$ we define the subbundle of $T(E_\alpha)$:

$$E_{\alpha+1}=\{z\in T(E_\alpha)/(\Pi^\alpha_{\alpha-1})^T(z)=\Pi^\alpha_\circ(z)\},$$

which implies the comutativity of diagram:

$$\begin{array}{ccc}\mathrm{T(E_\alpha)} & \xrightarrow{\quad\Pi_\alpha\quad} & \mathrm{E_\alpha}\\ (\Pi^\alpha_{\alpha-1})^{\mathrm{T}}\big\downarrow & & \big\uparrow\Pi^\alpha_0\\ \mathrm{E_\alpha} & \xrightarrow{\quad\Pi^\alpha_0\quad} & \mathrm{M}\end{array}$$

Under the inductive assuption that $z\in E_{\alpha+1}$ we conclude iff $z=(x,y^{(1)},\ldots,y^{(\alpha)})$ and after the notation $Y^{(\alpha+1)}=(\alpha+1)y^{(\alpha)}$ the local coordinates satisfy (1.2) including the same rule for $\widetilde{y}^{(\alpha+1)}$.

Let us consider $E_2\subset TTM$ as subbundle and $X_u=y^{(1)i}\cdot\frac{\partial}{\partial x^i}+$ $+2y^{(2)i}\frac{\partial}{\partial y^{(1)i}}$ a local field. We shall replace the natural base $\{\frac{\partial}{\partial x^i}\}$ with $\{\frac{\delta}{\delta x^i}=\frac{\partial}{\partial x^i}-\underset{(1)}{N}{}^j_i\frac{\partial}{\widetilde{y}^{(1)j}}\}$ that admits the simple change: $\frac{\partial}{\partial x^i}=\frac{\partial\widetilde{x}^j}{\partial x^i}\frac{\delta}{\delta\widetilde{x}^j}$ if $\underset{(1)}{N}{}^j_i(x,y)$ are the coefficients of a nonlinear connection on M ([5]). Then the field X_u takes the form: $X_u=y^{(1)i}\cdot\frac{\delta}{\delta x^i}+(2y^{(2)i}+\underset{(1)}{N}{}^i_j y^{(1)j})\frac{\partial}{\partial y^{(1)i}}$.

This suggests to take on E_2 the local adapted coordinates $z=(x,u^{(1)},u^{(1)})$, where:

$$x^i;\ u^{(1)i}=y^{(1)i};\ u^{(2)i}=2y^{(2)i}+\underset{(1)}{N}{}^i_j y^{(1)j} \tag{1.3}$$

If $\underset{(1)}{N}{}^i_j$ are the coefficients of a nonlinear connection then the adapted local coordinates are changing according to

$$\widetilde{x}^i=\widetilde{x}^i(x),\ \widetilde{u}^{(\alpha)}=\frac{\partial\widetilde{x}^i}{\partial x^j}u^{(\alpha)j},\ \alpha=1,2. \tag{1.3$'$}$$

In consequence, with respect to the adapted coordinates (1.3), E_2 has a vector bundle structure isomorphic to $TM\oplus TM$.

By induction, let us replace the local coordinates $z=(x,y^{(1)},\ldots,y^{(k)})$ on a local chart in E_k by the adapted coordinates $z=(x,u^{(1)},\ldots,u^{(k)})$ defined by:

$$\begin{aligned}
&u^{(1)i}=y^{(1)i}\\
&u^{(2)i}=2y^{(2)i}+\underset{(1)}{N}{}^i_j y^{(1)j}\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&u^{(k)i}=k!y^{(k)i}+(k-1)!\underset{(1)}{N}{}^i_j y^{(k-1)j}+\\
&\quad+\ldots+2!\underset{(k-2)}{N}{}^i_j y^{(2)j}+\underset{(k-1)}{N}{}^i_j y^{(1)j}
\end{aligned} \tag{1.4}$$

If $\underset{(1)}{N},\ldots,\underset{(k-1)}{N}$ are the coefficients of an adapted base on E_{k-1}, $\{\frac{\delta}{\delta x^1}=\frac{\partial}{\partial x^i}-\underset{(1)}{N}{}^j_i\cdot\frac{\partial}{\partial u^{(1)j}}-\ldots-\underset{(k-1)}{N}{}_i^j\frac{\partial}{\partial u^{(k-1)j}},\frac{\partial}{\partial u^{(1)i}},\ldots,\frac{\partial}{\partial u^{(k-1)i}}\}$ that satisfy the conditions:

$$\underset{(\alpha)}{\widetilde{N}}{}^h_k\frac{\partial}{\partial x^i}=\frac{\partial\widetilde{u}^{(\alpha)h}}{\partial u^{(\alpha)k}}\underset{(\alpha)}{N}{}^k_i-\frac{\partial u^{(\alpha)h}}{\partial x^i},\alpha=\overline{1,k-1}, \tag{1.5}$$

then the local adapted coordinates (1.4) are changing as follows:

$$\widetilde{x}^i=\widetilde{x}^i(x),\widetilde{u}^{(\alpha)i}=\frac{\partial\widetilde{x}^i}{\partial\widetilde{x}^j}\cdot u^{(\alpha)j},\alpha=\overline{1,k} \tag{1.6}$$

Therefore, we have a vector bundle structure on E_k with respect to the adapted coordinates (1.4) if the functions $\underset{(\alpha)}{N}{}^i_j(x,u^{(1)}\ldots,u^{(\alpha)})$ satisfy (1.5).

Theorem 1.1. *If on the manifold M is it given a nonlinear connection $N_i^j(x,y)$ then the k–osculator bundle E_k admits a vector bundle structure with fibre R^{mk} and base manifold M, isomorphic to $\oplus_k TM$.*

Proof. The functions $\underset{(1)}{N}{}_j^i = N_j^i$ and $\underset{(a)}{N}{}_j^i = \frac{\partial N_j^i}{\partial y^{(1)k}} u^{(a)k}$, $a = \overline{2, k-1}$ satisfy (1.5). It follows that on E_k we have a vector bundle structure with respect to the adapted local coordinates (1.4).

Proposition 1.2. *Let N be a nonlinear connection on M. Then E_k admits a vector bundle structure over $E_h, o \leq h < k$, with fibre $R^{(k-h)m}$.*

Now, the geometry of E_k can be approached in adapted coordinates according to the known techniques from the theory of vector bundle (see, for instance [5]) taking into account that $T(E_k) = \underset{k}{\oplus} T(TM)$.

A nonlinear connection on E_k is a splitting of an exact sequence, that determines the decomposition $T(E_k) = HE_k \oplus VE_k$, where $VE_k = Ker(\Pi_k)^T$ is the vertical distribution locally spanned by $\{\frac{\partial}{\partial u^{(1)}}, \ldots, -\frac{\partial}{\partial u^{(k)}}\}$. A derivation law on E_k is expressed as a sum of derivation laws on tangent bundle TM.

2. The associated bundle structure to k–frames bundle. Let be $\mathcal{P}^{(k)}(M)$ the k–frames bundle of the manifold M, namely the bundle of k–jets of local diffeomorphism of R^m to M.

$\Pi_0^k : \mathcal{P}^{(k)}(M) \longrightarrow M$ is a principal bundle $\mathcal{P}^{(k)}(M), M, G_m^k)$ ([4]), where G_m^k is the k–prolongation of linear group $Gl(m,R)$.

Let us consider $F = R^{mk}$ and the action of G_m^k on F, given by:

$$(2.1)\quad a \cdot p = (a^i_{j_1} p^{(1)j_1}; a^i_{j_1 j_2} p^{(1)j_1} p^{(1)j_2} + a^i_{j_1} p^{(2)j_1}; a^i_{j_1 j_2 j_3} p^{(1)j_1} p^{(1)j_2} p^{(1)j_3} +$$

$$+ a^i_{j_1 j_2} p^{(1)j_1} p^{(2)j_2} + a^i_{j_1 j_3} p^{(1)j_1} p^{(2)j_3} + a^i_{j_2 j_3} p^{(1)j_2} p^{(2)j_3} + a^i_{j_1} p^{(2)j_1}; \ldots)$$

where $a = (a^i_{j_1}, a^i_{j_1 j_2}, \ldots, a^i_{j_1 j_2} \ldots j_k) \in G_m^k$, $p = (p^{(1)i}, \ldots, p^{(k)i}) \in F$.

The associated bundle $E_k = \mathcal{P}^{(k)}(M) \times F/G_m^k$ has as elements the classes $[z,p] = \{(za, z^{-1}p),\ a \in G_m^k\}$, and the trivizing of $\widetilde{\Pi}_0^k : E_k \longrightarrow M$ is given by $[z,p] \longrightarrow (\Pi_0^k(z), \varphi'(z) \cdot p)$.

By (2.1) the element $u = (\Pi_0^k(z), \varphi'(z) \cdot p)$ can be written in the form

$$(2.2)\qquad u = (x^i, y^{(1)i}, 2y^{(2)i}, \ldots, ky^{(k)i})$$

(the coefficients were considered for convenience).

A change of local coordinates in manifold E_k is done with the elemets of $G_m^k, u \longrightarrow \widetilde{u}(\Pi_0^k(z), a \cdot \varphi^{-1}(z) \cdot p) = (\widetilde{x}^i, \widetilde{y}^{(1)}, \widetilde{y}^{(2)}, \ldots, \widetilde{y}^{(k)})$.

If $(U,\varphi),(V,\psi)$ are local charts in $x\in M$ then $a=j_0^k(\psi\circ\varphi^{-1})$, and because $\psi\circ\varphi^{-1}$ is expressed by $\widetilde{x}^i=\widetilde{x}^i(x)$ it follows that

$$a=\left(\frac{\partial\widetilde{x}^i}{\partial x^{j_1}},\frac{\partial^2\widetilde{x}^i}{\partial x^{j_1}\partial x^{j_2}},\ldots,\frac{\partial^k\widetilde{x}^i}{\partial x^{j_1}\ldots\partial x^{j^k}}\right)$$

From (2.1) it is deduced that in associated bundle E_k of $\mathcal{P}^{(k)}(M)$ we have the same change rules as (1.2) ($\alpha=k$), hence $Osc^{(k)}(M)$ bundle is just the associated bundle to k– frames bundle.

Let be $q:\mathcal{P}^{(k)}(M)\times F\longrightarrow E_k$ the canonical projection and $V=Ker(\Pi_0^k)^T$, $\widetilde{V}=Ker(\Pi_0^k)^T$ the vertical subbundles in $\mathcal{P}^{(k)}(M)$ and in the associated bundle E_k, respectively.

The restriction $(q^T)^V:V\times TF\longrightarrow\widetilde{V}$ is well defined and from the morphisms in exact sequences ([3]):

$$\begin{array}{ccccccccc}(2.3)\ 0&\longrightarrow&V\times TF&\longrightarrow&T\mathcal{P}^{(k)}(M)\times TF&\longrightarrow&\Pi_0^{*k}(TM)\times TF&\longrightarrow&0\\ &&\downarrow(q^T)^\gamma&&&&&&\\ 0&\longrightarrow&\widetilde{V}&\longrightarrow&TE&\longrightarrow&\widetilde{\Pi}_0^{*k}TM&\longrightarrow&0\end{array}$$

it results that a connection $\mathcal{C}$ in $\mathcal{P}^{(k)}(M), T_z\mathcal{P}^{(k)}(M)=H_z\oplus V_z$, $R_a\cdot(H_z)=H_{za}, a\in G_m^k$, induces a nonlinear connection $\widetilde{\mathcal{C}}$ in associated bundle $E_k=Osc^{(k)}(M)$,

$$\widetilde{\mathcal{C}}\circ q^T=(q^T)^V\circ(\mathcal{C}\times\ \mathrm{id}_{\ TF}) \tag{2.4}$$

Let us consider h and v the projectors on $T_z\mathcal{P}^{(k)}(M)$ and let ω be the connection form, $\omega:T_z(\mathcal{P}^{(k)}(M)\longrightarrow g_m^k, \omega(hZ)=0, \omega(vZ)=\omega(Z), g_m^k$ the Lie algebra of G_m^k. Let be $\omega^1,\omega^2,\ldots,\omega^k$ the components of connection form ω.

Now, let us recall from [4] the notion of connection of order k. Denote by $\theta:T(\mathcal{P}^{(k)}(M))\longrightarrow R^m+S_1(R^m,R^m)+\ldots+S_{k-1}(R^m,R^m)$ the fundamental form on $\mathcal{P}^{(k)}(M)$ defined by: $\theta(Z)=(z^T)^{-1}\rho_{k-1}^k\beta_z^{-1}(Z)$, where β_z is the isomorphism between $J_z^k(TM)$ and $T_z(\mathcal{P}^{(k)}(M))$, $\rho_{k-1}^k:J^k(TM)\longrightarrow J^{k-1}(TM)$ the projection, and $z\in\mathcal{P}^{(k)}(M)$.

Let $(\theta^0,\theta^1,\ldots,\theta^{k-1})$ be the components of θ. Then, there are satisfied the structure equations on $\mathcal{P}^{(k)}(M)$:

$$d\theta^h=-\frac{1}{2}\{[\theta^0\wedge\theta^{h+1}]+\ldots+[\theta^{h+1}\wedge\theta^0]\}\qquad h=\overline{0,k-2} \tag{2.5}$$

The fundamental form Θ is vanishing, $\theta(Z)=0$ iff Z is a vertical field of the bundle $\Pi_{k-1}^k:\mathcal{P}^{(k)}(M)\longrightarrow\mathcal{P}^{(k)}(M), Z\in V_k=Ker(\Pi_{k-1}^k)^T$. On the vertical distribution $V=Ker(\Pi_0^k)^T$ only θ^0 does vanish.

Definition ([4]). *A connection of order k on $\mathcal{P}^{(k)}(M)$ is a connection $\mathcal{C}$ for which all components $\theta^1, \theta^2, \theta^3, \ldots, \theta^{k-1}$ are vanishing.*

For a connection of order k we can consider $\omega^1 = \theta^1, \omega^2 = \theta^2, \ldots,$ $\omega^{k-1} = \theta^{k-1}$ and consequently, it is essential is to know the 1–forms $\omega^k : T(\mathcal{P}^{(k)}(M)) \longrightarrow S_k(R^m, R^m)$.

Let us consider the curvature and torsion forms of a connection of order k:

$$\Omega_z(X,Y) = d\omega_z^k(hX, hY), \Theta_z(X,Y) = d\theta_z^{k-1}(hX, hY) \tag{2.6}$$

In addition to (2.5) one results the following structure equations ([4]):

$$d\theta^{k-1} = -[\theta^0 \wedge \omega^k] - \frac{1}{2}\{[\omega^1 \wedge \omega^{k-1}] + \ldots + [\omega^{k-1} \wedge \omega^1]\} + \Theta \tag{2.7}$$

$$d\omega^k = -[\omega^1 \wedge \omega^k] - \frac{1}{2}\{[\omega^2 \wedge \omega^{k-1}] + \ldots + [\omega^{k-1} \wedge \omega^2]\} + \Omega.$$

Now let us consider the associated bundle $E_k = Osc^{(k)}(M)$. Let $\widetilde{V}_1 = \widetilde{V} = Ker(\widetilde{\Pi}_0^k)^T$ be the vertical distribution and $\widetilde{H}$ the supplementary distribution to $\widetilde{V}$, determined by the nonlinear connection $\widetilde{\mathcal{C}} = \widetilde{\mathcal{C}}l_1$ induced by $\mathcal{C}$ on $E_k, T_u(E_k) = \underset{1,u}{\widetilde{V}} \oplus \widetilde{H}_u$. The vertical distribution $\widetilde{V}_{1,u}$ is spanned by $\{\frac{\partial}{\partial y^{(1)}}, \ldots, \frac{\partial}{\partial y^{(k)}}\}$ and in $\widetilde{H}_u$ a local adapted base is $\{\frac{\delta}{\delta x^1} = \frac{\partial}{\partial x^i} - \underset{(1)}{N}{}_i^j \frac{\partial}{\partial y^{(1)j}} - \ldots - \underset{(k)}{N}{}_i^j \cdot \frac{\partial}{\partial y^{(k)j}}\}$ where $\underset{(\alpha)}{N}{}_i^j$ are the coeficients of a nonlinear connection in $Osc^{(k)}(M)$ bundle ([6]). Denoting by $\widetilde{\omega}^1 = \widetilde{\mathcal{C}}_1$ the 1–form, $\widetilde{\omega}^1 : T(E_k) \longrightarrow V_1$ and by $\widetilde{\theta}^0 = \widetilde{h}$, the natural almost k–tangent structure on E_k is J. Consider $\widetilde{\theta}^\alpha = J^{k-\alpha}.\alpha = 1, k-1$. It follows that $\widetilde{\theta}'(Z) = 0$ iff $\widetilde{\omega}'(Z) = 0$.

As in [6] we introduce the sequence of m–dimensional distributions $\widetilde{N}_0 = \widetilde{H}, \widetilde{N}_1 = J(\widetilde{N}_0), \ldots, \widetilde{N}_{k-1} = J^{k-1}(\widetilde{N}_0)$, which satisfy the conditions: $\widetilde{N}_\alpha \subset \widetilde{V}_\alpha = Ker(\widetilde{\Pi}_{\alpha-1}^k)^T$. Then we have the decomposition $TE_{k,u} =$ $= \widetilde{N}_{0,u} \oplus \widetilde{N}_{l,u} \oplus \ldots \oplus \widetilde{N}_{k-1,u} \oplus \widetilde{V}_{k,u}$. We have also the inclusions $\widetilde{V}_1 \supset \widetilde{V}_2, \ldots, \supset \widetilde{V}_k$. Let us note that $\widetilde{N}_1$ is a complementary distribution to $\widetilde{V}_2$ in $\widetilde{V}_1$, in consequence it determines a nonlinear connection $\widetilde{\mathcal{C}}_2$ in the bundle $\widetilde{\Pi}_1^k \longrightarrow E_1$. Let $\widetilde{\omega}^{(2)} = \widetilde{\mathcal{C}}_2 \circ (\Pi_0^k)^T$ be the connection form. It is easy to see that it is vanishing iff $\widetilde{\theta}^2$ is vanishing. Inductively it is defined $\omega^{(k)} = \widetilde{\mathcal{C}}_k \circ (\Pi_{k-2}^k)^T$ the connection form in $\widetilde{V}_k = Ker(\widetilde{\Pi}_{k-1}^k)^T$.

Let us consider the Berwald connection in $E_k = Osc^{(k)}(M)$ ([6]):

$$\widehat{D}_{\frac{\delta}{\delta x^i}} \frac{\delta}{\delta y^{(\alpha)i}} = \frac{\delta N_i^m}{\delta y^{(1)j}} \cdot \frac{\delta}{\delta y^{(\alpha)m}}, \widehat{D}_{\frac{\delta}{\delta y^{(\alpha)j}}} \frac{\delta}{\delta y^{(\beta)i}} = 0 \quad \begin{matrix} \alpha = \overline{1,k} \\ \beta = \overline{0,k} \end{matrix} \tag{2.8}$$

$\widehat{D}$ is a derivative law valued in vertical bundle V_γ for any $\alpha, \beta \leq \gamma$.

Now, let us consider the projections on $T_u(E_k)$, $v^{(\alpha)} = \widetilde{v}_\alpha + \widetilde{v}_{\alpha+1} + \ldots + \widetilde{v}_k$ and $h^{(\alpha)} = I - v^{(\alpha)}$.

By α–absolute derivative of a q–form θ with values in vertical bundle V_α we mean the $(q+1)$ form:

$$\nabla\Phi(X_0, X_1, \ldots, X_q) = \widehat{d}\Phi(h^{(\alpha)}X_0, h^{(\alpha)}X_1, \ldots, h^{(\alpha)}X_q), \ X_q \in \lambda(E_k),$$

where $v^{(\alpha_0)} = h$ and:

$$\widehat{d}\Phi(X_0, X_1, \ldots, X_q) = \sum_{i=0}^{q} (-1)^i \widehat{D}_{x_i}(\Phi(X_0, \ldots, X_i, \ldots, X_q)) +$$

$$+ \sum_{0=i<j} (-1)^{i+j} \Phi([X_i, X_j], \ldots, \overline{X_i}, \ldots, \overline{X_j}, \ldots, X_q).$$

The nonlinear connection $\mathcal{C}_1$ determines the folowing 2–forms of connection:

$$\Omega^{(\alpha)} = -\nabla\omega^{(\alpha)}, \quad \alpha = \overline{1,k} \tag{2.9}$$

Let us consider the Nijenjuis tensor $N_{V^{(\alpha)}}$ associated to the projector $v^{(\alpha)}$ on the distribution V_α.

Theorem 2.1. *The following structure equations hold:*

$$\Omega^{(\alpha)} = N_{V^{(\alpha)}}, \quad \alpha = \overline{1,k}$$

The proof is similar to that from [5] for $\alpha = 1, V_\alpha$ being an integrable distribution.

From the proof of the theorem it results that $\Omega^{(1)}$ for example has only $\Omega^{(1)}(h^{(1)}X, h^{(1)}Y) = v^{(1)}[h^{(1)}X, h^{(1)}Y]$ as nonvanishing component. It results that $[h^{(1)}X, h^{(1)}Y] = h^{(1)}[h^{(1)}X, h^{(1)}Y] + v^{(1)}[h^{(1)}X, h^{(1)}Y] =$
$= h^{(1)}[h^{(1)}X, h^{(1)}Y] - \Omega^{(1)}(X, Y)$.

Analoguosly it will be proved that:

Proposition 2.2. *For any* $X, Y \in \Xi(E_k)$ *we have:*

$$[h^{(\alpha)}X, h^{(\alpha)}Y] = h^{(\alpha)}[h^{(\alpha)}X, h^{(\alpha)}Y] - \Omega^{(\alpha)}(X, Y), \alpha = \overline{1, k}$$

The 2–forms of torsion for $\mathcal{C}_1$ is defined by $\widetilde{\theta}^{(\alpha)} = -\nabla\widetilde{\theta}^{(\alpha)}$, $\alpha = \overline{1, k-1}$.

Taking into account the above construction of connections forms $\widetilde{\omega}^{(\alpha)}$ and fundamental form $\widetilde{\theta}^{(\alpha)}$ in associated bundle $E_k = Osc^{(k)}(M)$ it results that these satisfy equations of structure analogous to (2.5) and (2.7).

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