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AFFINE STRUCTURE ON COMPLEX FOLIATED MANIFOLDS

BY

GHEORGHE MUNTEANU and CRISTIAN IDA

Abstract. In this paper the affine complex manifold notion is defined and some local and global aspects on such a manifold: complex Liouville vector, semispray, complex Lagrange structure, etc., are studied.

These notions are introduced here by analogy with the real case ([V1, V2, V3]), but there exist specific particularities.

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1. Affine complex manifold. Let $\mathcal{M}$ be a complex foliated manifold ([V1]) with the same dimension and codimension n and $u = (z^i, \eta^i)_{i=\overline{1,n}}$ complex coordinates in a local map $(U_\alpha, \varphi_\alpha)$.

Definition 1.1. *$\mathcal{M}$ is said to be an affine complex manifold if the local changes $(U_\alpha, \varphi_\alpha) \to (U_\beta, \varphi_\beta)$ are given by*

$$z'^i = z'^i(z) \quad ; \quad \eta'^i = \frac{\partial z'^i}{\partial z^j}\eta^j + B'^i(z) \tag{1.1}$$

where z'^i and B'^i are holomorphic functions on z^j variables and $\det(\frac{\partial z'^i}{\partial z^j}) \neq 0$.

The leafs of this manifold, denoted by $\mathcal{V}$, are characterized by $z^i = const$.

Let J be the natural complex structure of the manifold and $T'\mathcal{M}$ and $T''\mathcal{M} = \overline{T'\mathcal{M}}$ be its holomorphic and antiholomorphic subbundles, respectively. By $T_C\mathcal{M} = T'\mathcal{M} \oplus T''\mathcal{M}$ we denote the complexified tangent bundle of $\mathcal{M}$. From (1.1) it result the following changes for the natural local frames on $T'_u\mathcal{M}$:

$$\begin{aligned} \frac{\partial}{\partial z^k} &= \frac{\partial z'^i}{\partial z^k}\frac{\partial}{\partial z'^i} + \Big(\frac{\partial^2 z'^i}{\partial z^j \partial z^k}\eta^j + \frac{\partial B'^i}{\partial z^k}\Big)\frac{\partial}{\partial \eta'^i} \\ \frac{\partial}{\partial \eta^k} &= \frac{\partial z'^i}{\partial z^k}\frac{\partial}{\partial \eta'^i}. \end{aligned} \tag{1.2}$$

By conjugation over all in (1.2) we obtain the change rules of the local frames on $T''_u\mathcal{M}$, and then the behaviour of the J complex structure

$$J\Big(\frac{\partial}{\partial z^k}\Big) = i\frac{\partial}{\partial z^k}\ ;\ J\Big(\frac{\partial}{\partial \eta^k}\Big) = i\frac{\partial}{\partial \eta^k}\ ;\ J\Big(\frac{\partial}{\partial \bar{z}^k}\Big) = -i\frac{\partial}{\partial \bar{z}^k}\ ;\ J\Big(\frac{\partial}{\partial \bar{\eta}^k}\Big) = -i\frac{\partial}{\partial \bar{\eta}^k}. \tag{1.3}$$

Note that inside of (1.1) we can take into account the more general affine changes $\eta'^i = A'^i_j(z)\eta^j + B'^i(z)$, where A'^i_j are also holomorphic functions.

As well as in (1.1) we can consider the particular case $B'^i = 0$, when $\mathcal{M}$ is identified with the holomorphic tangent bundle of a complex manifold, $\mathcal{M} = T'M$. In this last situation we will say that $T_C\mathcal{M}$ is of *holomorphic vector type.*

However, the general discussion $A'^i_j \neq \frac{\partial z'^i}{\partial z^j}$ limits the opportunity of other significant structures on $\mathcal{M}$ be considered. Such important structure for the geometry of $T_C\mathcal{M}$, excluded by the general case, is

$$S\Big(\frac{\partial}{\partial z^k}\Big) = \frac{\partial}{\partial \eta^k}\ ;\ S\Big(\frac{\partial}{\partial \eta^k}\Big) = 0\ ;\ S\Big(\frac{\partial}{\partial \bar{z}^k}\Big) = \frac{\partial}{\partial \bar{\eta}^k}\ ;\ S\Big(\frac{\partial}{\partial \bar{\eta}^k}\Big) = 0. \tag{1.4}$$

Proposition 1.1. *S is a global tangent structure acting on $T_C\mathcal{M}$. Moreover, S is integrable.*

Proof. We easily check that $S^2 = 0$ and the (1.4) definition is preserved at (1.2) changes. Since $N_s(X,Y) := [SX, SY] - S[SX, Y] - S[X, SY] + S^2[X,Y] = 0$, for any $X, Y \in \Gamma T_C\mathcal{M}$, it follows that S is integrable. □

Proposition 1.2. $(S, J, J\circ S = S\circ J)$ *determines a commutative and integrable semiquaternionic structure ([Mu1]) on* $T_C\mathcal{M}$.

Consider $T'\mathcal{V} = span\{\frac{\partial}{\partial \eta^i}\} \subset T'\mathcal{M}$, the *vertical distribution*, which in view of (1.2) is an integrable one.

Let $\Gamma_\alpha := \eta^i \frac{\partial}{\partial \eta^i}$ be the Liouville complex field, ([Mu2]), locally considered in the map $(U_\alpha, \varphi_\alpha)$. Note that if $T_C\mathcal{M}$ is of holomorphic vectorial type, then Γ_α glue up to a global defined vector field. Let us pay more attention how other requests ensure the global existence of the Liouville complex vector field. This is one important question in the following.

By $\mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V})$ we denote the module of affine leafwise holomorpfic vertical functions, namely $\{a_i(z)\eta^i + b(z)\}$, where $a_i, b \in \Omega^0_{pr}(\mathcal{M})$ are holomorphic functions on $\mathcal{M}$. For the sheaf of corresponding germs we have the following exact sequence

$$0 \to \Omega^0_{pr}(\mathcal{M}) \xrightarrow{i} \mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V}) \xrightarrow{p} \Omega^0_{pr}(\mathcal{M}) \otimes T'^*\mathcal{V} \to 0 \tag{1.5}$$

explicitly given by $b \xrightarrow{i} a_i\eta^i + b \xrightarrow{p} a_i dz^i$.

We note that $\Gamma_{\alpha\beta} := \Gamma_\beta - \Gamma_\alpha = (\frac{\partial z'^i}{\partial z^j}\eta^j + B'^i - \eta^i)\frac{\partial}{\partial \eta'^i}$ is a complex vector field on $U_\alpha \cap U_\beta$ with its coefficients in $\mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V})$, and therefore $\Gamma_{\alpha\beta}$ yields a cocicle, $(\delta\Gamma)_{\alpha\beta\gamma} = \Gamma_{\alpha\beta} - \Gamma_{\alpha\gamma} + \Gamma_{\beta\gamma} = 0$; hence its class in Céch cohomology, $[\Gamma_\alpha]$, will be with coefficients in $\mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V})$. Thus $[\Gamma_\alpha] \in H^1(\mathcal{M},\mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V}))$, the first group of cohomology, and will be called the *complex linear obstruction.* Clearly, if $[\Gamma_\alpha] = 0$, then the Liouville complex field is globally defined on $\mathcal{M}$ and it will be simply denoted by Γ. Moreover, it results that $\Gamma_\alpha = \Gamma\mid_{U_\alpha} + E_\alpha$, where $E_\alpha = E^i(z)\frac{\partial}{\partial \eta^i}$ is a projectable vector field on U_α, called the Euler complex field.

In [P-M] we proved a Grothendieck-Dolbeault lemma on a complex manifold endowed with a complex Finsler metric. We can consider here $\bar{\Gamma}_\alpha = \bar{\eta}^i\frac{\partial}{\partial \bar{\eta}^i}$, the conjugate Liouville complex vector and the differential operator $d = d' + d''$ on the affine complex manifold $\mathcal{M}$. All the same, a Grothendieck-Dolbeault lemma on an affine complex manifold (endowed possible with a complex Finsler metric) seems to be a difficult problem. Hence d'', usually, don't determine a fine resolution and consequently to prove the classical isomorphism between $H^1(\mathcal{M},\mathcal{A}^0_{pr}(\mathcal{M},\mathcal{V}))$ and the $1-$dimensional de Rham group is not anyway possible. Remains for a further work to furnish some examples for which we can do it.

Now with respect to Γ_α, as in the classical way (a bit generalized), [M-A, Mu2, V3], we will introduce the affine complex semispray.

Definition 1.2. *An affine complex semispray in* $(U_\alpha, \varphi_\alpha)$ *is a complex vector field* G_α *on* $T'\mathcal{M}$, *for which* $S \circ G_\alpha - \Gamma_\alpha$ *is projectable, i.e.* $S \circ G_\alpha = \Gamma_\alpha + A^i(z)\frac{\partial}{\partial \eta^i}$.

By this definition it is easily deduced that G_α have the following writing:

$$G_\alpha = (\eta^i + A^i)\frac{\partial}{\partial z^i} - 2G^i\frac{\partial}{\partial \eta^i}, \tag{1.6}$$

where $G^i(z,\eta)$ are complex valued functions on $\mathcal{M}$, called the coefficients of the semispray.

If we consider (U_β, φ_β) other local chart and G_β an affine complex semispray with locall coefficients given by $G'^i(z', \eta')$, using (1.2) and (1.6), directly is obtained

$$\begin{aligned} A'^i &= A^k\frac{\partial z'^i}{\partial z^k} - B'^i; \\ G'^i &= G^k\frac{\partial z'^i}{\partial z^k} - \frac{1}{2}(\eta^k + A^k)(\frac{\partial^2 z'^i}{\partial z^j \partial z^k}\eta^j + \frac{\partial B'^i}{\partial z^k}). \end{aligned} \tag{1.7}$$

Now, let us make a useful remark. The affine character of the complex semispray is related to the chosen local chart. If in $(U_\alpha, \varphi_\alpha)$, we operate in (1.1) the following translation for the coordinates: $\tilde{z}^i = z^i$ and $\tilde{\eta}^i = \eta^i - A^i$, then G_α looses his affine character and, in a such atlas, $G_{\alpha\beta} := G_\beta - G_\alpha$ has coefficients in the form $a_{ij}(z)\eta^i\eta^j + a_i(z)\eta^i + b(z)$, which belongs to the shaves of 2-jets vertical affine leafwise germs, denoted by $J^2\mathcal{A}^0_{pr}(\mathcal{M}, \mathcal{V})$.

It results that

$$0 \to \Omega^0_{pr}(\mathcal{M}) \xrightarrow{i} \mathcal{A}^0_{pr}(\mathcal{M}, \mathcal{V}) \xrightarrow{i^2} J^2\mathcal{A}^0_{pr}(\mathcal{M}, \mathcal{V}) \xrightarrow{\pi} \Omega^0_{pr}(\mathcal{M}) \otimes (T'^*\mathcal{M})^2 \to 0$$

is an exact sequence.

Obviously, in the mentioned atlas, $G_{\alpha\beta}$ defines a cocicle in the Céch cohomology, its coefficients being in $J^2\mathcal{A}^0_{pr}(\mathcal{M}, \mathcal{V})$. Let $[G_\alpha]$ be the cohomology class defined by G_α; $[G_\alpha] \in H^1(M, J^2\mathcal{A}^0_{pr}(\mathcal{M}, \mathcal{V}))$ and its vanishing is a condition for the global existence of the complex semispray G_α, called *the second order obstruction.*

The interest for the existence of a global complex semispray is major. It is well known, [Mu2], that in our purpose of "linearizing" for the geometry of $\mathcal{M}$, a special meaning plays the complex nonlinear connection (in brief (*c.n.c.*)), which is intrinsically related to the notion of complex semispray. Therefore, subsequently, let us analyze in which consist the obstructions for globalization of a (*c.n.c.*).

Let $T'\mathcal{H}$ be a supplementary (transversal) distribution of $T'\mathcal{V}$ in $T'\mathcal{M}$, locally spanned in $(U_\alpha, \varphi_\alpha)$ by $\{\frac{\delta}{\delta z^k} = \frac{\partial}{\partial z^k} - N^j_k\frac{\partial}{\partial \eta^j}\}$. From (1.2) and (1.4) it results $S(\frac{\delta}{\delta z^k}) = \frac{\partial}{\partial \eta^k}$, and then the global existence of the tangent structure S implies

$$\frac{\delta}{\delta z^k} = \frac{\partial z'^i}{\partial z^k}\frac{\delta}{\delta z'^i}. \tag{1.8}$$

Hence, at the local changes of the charts, we have the following rules of change for the coefficients $N^j_k(z,\eta)$ of the $(c.n.c.)$

$$\frac{\partial z'^l}{\partial z^k}N'^i_l = N^l_k\frac{\partial z'^i}{\partial z^l} - (\frac{\partial^2 z'^i}{\partial z^j \partial z^k}\eta^j + \frac{\partial B'^i}{\partial z^k}). \tag{1.9}$$

Proposition 1.3. *In the translated atlas* $\{(\tilde{z}^i = z^i \ ; \ \tilde{\eta}^i = \eta^i - A^i)\}$, *in which* $G_{\alpha\beta}$ *has coefficients in the shaves of 2-jets vertical affine leafwise germs,* $N^j_k = \partial G^j/\partial\tilde{\eta}^k$ *determine the coefficients of a* $(c.n.c.)$.

Proof. By differentiate (1.7), in the mentioned atlas, yields

$$\frac{\partial G'^i}{\partial \tilde{\eta}^m} = N^l_m\frac{\partial \tilde{z}'^i}{\partial \tilde{z}^l} - \frac{1}{2}(\frac{\partial^2 \tilde{z}'^i}{\partial \tilde{z}^j\partial \tilde{z}^m}\tilde{\eta}^j + \frac{\partial B'^i}{\partial \tilde{z}^m}) - \frac{1}{2}\frac{\partial^2 \tilde{z}'^i}{\partial \tilde{z}^k\partial \tilde{z}^m}\tilde{\eta}^k.$$

Since $\frac{\partial G'^i}{\partial \tilde{\eta}^m} = \frac{\partial \tilde{z}'^l}{\partial \tilde{z}^m}N'^i_l$, precisely the (1.9) rule of change results for the coefficients of the $(c.n.c.)$. □

It deduces that the global existence of G_α, that is $[G_\alpha] = 0$, is a sufficient restriction for the global existence of a $(c.n.c.)$.

We recall, [Mu2], that a $(c.n.c.)$ can exist independently of a complex semispray (for instance, the Chern-Finsler $(c.n.c.)$ does not come anyway from a semispray). From the definition of the adapted frames $\frac{\delta}{\delta z^k}$ and (1.9) we may conclude that the obstructions for the existence of a $(c.n.c.)$ are linear but their coefficients are not always affine projectable functions.

We can continue our talk with the study of the global existence for other geometric objects, related to a $(c.n.c.)$, such as: $d-$complex linear connection, their torsions and curvatures, etc. But these may be the topic for a forthcoming paper.

2. Complex Lagrange structures. Let $L_\alpha : \mathcal{M} \to \mathbb{R}$ be a Lagrangian function, defined on $U_\alpha \subset \mathcal{M}$, domain of local chart.

Definition 2.1. *We say that a family* $\{\mathcal{M}, L_\alpha\}$ *is a local complex Lagrange structure on* $\mathcal{M}$*, if there exists an atlas such that* $g_{i\bar{j}} = \partial^2 L_\alpha/\partial\eta^i\partial\bar{\eta}^j$ *glue up to a global Hermitian metric on* $\mathcal{M}$.

If $\{\mathcal{M}, L_\alpha\}$ defines a global Lagrange structure on $\mathcal{M}$, by integration of $g_{i\bar{j}}$, we obtain a Lagrangian $L : \mathcal{M} \to \mathbb{R}$ such $L_\alpha = L\mid_{U_\alpha} + l_\alpha$, where l_α is an affine real valued vertical form on M, i.e. there exist $\overset{\alpha}{A}_i(z)$ and $\overset{\alpha}{B}(z) \in \mathbb{R}^+$ such that

$$L_\alpha = L\mid_{U_\alpha} + \overset{\alpha}{A}_i(\eta^i + \bar{\eta}^i) + \overset{\alpha}{B}. \tag{2.1}$$

On the intersection $U_\alpha \cap U_\beta$ we can define a cocycle, L being closed with respect to differential $(\delta L)_{\alpha\beta\gamma} := L_{\alpha\beta} - L_{\alpha\gamma} + L_{\beta\gamma} = 0$.

Proposition 2.1. *Denote by* $[L_\alpha]$ *the cohomology class defined by the cocycle* $L_{\alpha\beta}$.

Then $[L_\alpha] \in H^1(\mathcal{M}, \mathcal{A}^0_R(\mathcal{M}, \mathcal{V} + \overline{\mathcal{V}}))$ *and* $[L_\alpha] = 0$ *yields* L *is globally defined.*

Let us see when $[L_\alpha] = 0$. Certainly, generally the class $[L_\alpha]$ is distinct to $[G_\alpha]$.

By analogy with (1.5) we can construct an exact sequence over affine real valued functions, without requesting their holomorphy,

$$0 \to \Phi^0_R(\mathcal{M}) \xrightarrow{i} \mathcal{A}^0_R(\mathcal{M}, \mathcal{V} \oplus \overline{\mathcal{V}}) \xrightarrow{\pi} \Phi^{(1,0)}_R(\mathcal{M}) \to 0,$$

explicitly given by the correspondence $\overset{\alpha}{B} \to \overset{\alpha}{A}_i(\eta^i + \bar{\eta}^i) + \overset{\alpha}{B} \to \overset{\alpha}{A}_i\, d\eta^i$.

This induces the following exact sequence of cohomology groups

$$\begin{array}{rl} 0 \ \to & H^1(\mathcal{M}, \Phi^0_R(\mathcal{M})) \xrightarrow{i^*} H^1(\mathcal{M}, \mathcal{A}^0_R(\mathcal{M}, \mathcal{V} \oplus \overline{\mathcal{V}})) \xrightarrow{\pi^*} \\ & H^1(\mathcal{M}, \Phi^{(1,0)}_R(\mathcal{M})) \xrightarrow{\bar{\partial}} H^2(\mathcal{M}, \Phi^{(2,0)}_R(\mathcal{M})) \xrightarrow{\bar{\partial}} \ldots. \end{array}$$

Let be $[L_\alpha]_1 = \pi^*[L_\alpha] \in H^1(\mathcal{M}, \Phi^{(1,0)}_R(\mathcal{M}))$. If $[L_\alpha]_1 = 0$ then $[L_\alpha]_1 \in ker\ \pi^* = Im\ i^*$ and therefore there exists $[L_\alpha]_2 \in H^1(\mathcal{M}, \Phi^0_R(\mathcal{M}))$ such that $i^*[L_\alpha]_2 = [L_\alpha]$. Hence we can state

Proposition 2.2. $[L_\alpha] = 0$ *if and only if* $[L_\alpha]_1 = [L_\alpha]_2 = 0$.

These are the main obstructions to a global complex Lagrange structure on $\mathcal{M}$.

In a complex Lagrange space the Euler $(1,0)-$form plays a special rolé ([Mu2]). With respect to a local Lagrange structure L_α, the Euler form is $\theta_\alpha = \frac{\partial L_\alpha}{\partial \eta^i} dz^i$. The difference $C_{\alpha\beta} := \theta_\beta - \theta_\alpha = \{\overset{\beta}{A}_j \frac{\partial z'^j}{\partial z^i} - \overset{\alpha}{A}_i\} dz^i$ defines a cocycle and its cohomology class is in $H^1(\mathcal{M},\Phi_R^{(1,0)}(\mathcal{M}))$. Then $[L_\alpha]_1=0$ implies θ_α is global defined and conversely. So, θ_α glue up to a global complex Euler form and L_α differ from $L\mid_{U_\alpha}$ only by the term $\overset{\alpha}{B}$.

On the other hand, for a fixed globally defined ($c.n.c.$) which spans the distribution $T'\mathcal{H}$, we can consider the dual adapted base $\delta\eta^i=d\eta^i+N_j^i dz^j$ and accordingly the following form $\omega_\alpha:=\frac{\partial L_\alpha}{\partial\eta^i}\delta\eta^i=\frac{\partial(L|_{U_\alpha})}{\partial\eta^i}\delta\eta^i+\overset{\alpha}{A_i}\ \delta\eta^i$. Since $\delta\eta^i$ is globally defined, it results that θ_α is globally defined if and only if ω_α is globally defined, too.

We note that

$d''^v\omega_\alpha=\frac{\partial^2 L_\alpha}{\partial\eta^i\partial\bar{\eta}^j}dz^i\wedge\delta\bar{\eta}^j=g_{i\bar{j}}\ dz^i\wedge\delta\bar{\eta}^j:=\Theta.$

Since $g_{i\bar{j}}$ the metric tensor of a local complex Lagrange structure is globally defined, then Θ is global too. Thus, in conclusion we obtain

Proposition 2.3. $[L_\alpha]_1=0$ *if and only if* Θ *is a* d''^v-*exact form.*

Let us obtain further a characterization condition for $[L_\alpha]_2=0$. Let us assume that $[L_\alpha]_1=0$, namely $\Theta=d''^v\omega$ is an exact form. Then $\theta_\alpha=\omega\mid_{U_\alpha}+\xi_\alpha$, where $\xi_\alpha=\overset{\alpha}{\xi_i}(z)dz^i$ is a foliated $(1,0)-$form on $\mathcal{M}$. From (2.1) it results $\frac{\partial(L_\beta-L_\alpha)}{\partial\eta^i}=\overset{\beta}{\xi_i}-\overset{\alpha}{\xi_i}$; that means $L_\beta-L_\alpha=(\overset{\beta}{\xi_i}-\overset{\alpha}{\xi_i})(\eta^i+\bar{\eta}^i)+\overset{\alpha\beta}{B}$, where $(\overset{\beta}{\xi_i}-\overset{\alpha}{\xi_i})(z)$ and $\overset{\alpha\beta}{B}(z)$ are real valued on $U_\alpha\cap U_\beta$.

After the change of the local complex structure, $\tilde{L}_\alpha:=L_\alpha-\overset{\alpha}{\xi_i}(\eta^i+\bar{\eta}^i)$, is obtained a cocycle $\tilde{L}_{\alpha\beta}=\tilde{L}_\beta-\tilde{L}_\alpha$ which coincides with $L_{\alpha\beta}$, but $[L_\alpha]_1=0$ implies $\tilde{L}_\alpha=L\mid_{U_\alpha}+\overset{\alpha}{\tilde{B}}$.

Obviously we have $(d'^v+d''^v)\overset{\alpha}{\tilde{B}}=0$, because $\overset{\alpha}{\tilde{B}}$ depends only on z.

Theorem 2.1. *The family* $\{L_\alpha\}$ *defines a global complex Lagrange structure on* $\mathcal{M}$ *if and only if, for a fixed global horizontal distribution* $T'\mathcal{H}$, *the* $2-$*form* Θ *is exact and there exist a family* $\{\tilde{L}_\alpha\}$ *of local complex Lagrangians such that* $\{(d'^v+d''^v)\tilde{L}_\alpha\}$ *define a* $1-$*global form on* $\mathcal{M}$.

Finally we remark that the above discussion can be easily applied for the case of complex Lagrange geometry, when $\mathcal{M}=T'M$ is endowed with a local complex Lagrangian, intensively studied in [Mu2].

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Transilvania Univ. of Braşov,
Faculty of Mathematics and Informatics,
gh.munteanu@inf o.unitbv.ro,