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FIELD OF CONES ON A POLYSYMPLECTIC MANIFOLD

BY

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Abstract. The aim of the paper is to generalize the results obtained by Pătrăşcoiu in ([8]), i.e. to define fields of cones on a polysymplectic manifold and to associate to a field of cones on the k-tangent bundle, a natural field of cones on the k-cotangent bundle in order to define a natural, positive and monotone operator between the k-tangent and the k-cotangent bundle ([3]), ([5]). The motivation comes from the classical mechanics, because the cotangent bundle of a manifold is the phase space of mechanical systems. Using the canonical symplectic structure of the cotangent bundle given by the Liouville 1-form, we define a natural field of cones.

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1. Introduction. Recall that a polysymplectic manifold (M, ω) is a smooth manifold M together with a closed, nondegenerate $\mathbb{R}^k$-valued 2-form $\omega = \sum_{A=1}^{k} \omega_A \otimes e_A$, where $\{e_1, ..., e_k\}$ is the canonical basis of $\mathbb{R}^k$. If (M, ω) is a polysymplectic manifold, then each tangent space $(T_xM, \omega_{Ax}), x \in M, 1 \leq A \leq k$, is a symplectic vector space.

We will endow the k-tangent and k-cotangent bundles of a polysymplectic manifold with natural fields of cones.

Consider V a real vector space.

Definition 1.1. *A cone of V is a subset $K \subset V$ such that for any $v \in K$ and $\lambda \in \mathbb{R}_+ \Rightarrow \lambda v \in K$.*
A convex cone of V is a cone K such that for any $v_1, v_2 \in K \Rightarrow v_1 + v_2 \in K$.
A pointed cone of V is a cone K such that if $v \in K$ and $-v \in K \Rightarrow v = 0$.

Let (E, p, M) be a regular vector bundle (a vector bundle for which the base is a paracompact, connected and without boundary manifold).

Definition 1.2. ([4]) *A field of cones on (E, p, M) is a map $K : M \to E, x \in M \mapsto K(x) \subset E_x = p^{-1}(x)$, that satisfies:*
(1) for any $x \in M, K(x)$ is a convex, pointed, closed cone with interior points of E_x;
(2) $\bigcup_{x\in M} IntK(x)$ and $\bigcup_{x\in M}(E_x - K(x))$ are open in E.

Denote by $[(E, p, M); K]$ ([6]) a regular vector bundle endowed with a field of cones K. Then the structures $[(E, p, M); K]$ are the objects of a category whose morphisms are the morphisms of vector bundles $f : [(E, p, M); K] \to [(E', p', M'); K']$, such that $f(K(x)) \subset K'(f(x))$, for any $x \in M$.

2. Field of cones on the k-tangent and k-cotangent bundles. We shall describe the way we associate to a field of cones on the Whitney sum $T^1_k M = TM \oplus ... \oplus TM$ of k-copies of the tangent bundle TM of a polysymplectic manifold, a natural field of cones on the the Whitney sum $(T^1_k)^* M = T^*M \oplus ... \oplus T^*M$ of k-copies of the cotangent bundle T^*M.

Proposition 2.1. *Let (M, ω) be a polysymplectic manifold (paracompact, connected and without boundary). If there exists K a field of cones on the vector bundle $(T^1_k M, \tau, M)$, then there exists a natural field of cones K^* on the vector bundle $((T^1_k)^* M, \tau^*, M)$ such that the structures $[(T^1_k M, \tau, M);$ $K]$ and $[((T^1_k)^* M, \tau^*, M); K^*]$ are isomorphic.*

Proof. Because $(T_x M, \omega_{Ax})$ is a symplectic vector space, for any $x \in M, 1 \leq A \leq k$ ([1]), we can define the isomorphisms:

$$h_{Ax} : T_x M \to T^*_x M, h_{Ax}(X_x) = i_{X_x}\omega_{Ax}$$

and

$$h_x : (T^1_k)_x M \to (T^1_k)^*_x M, h_x(X_{1x}, ..., X_{kx}) = (i_{X_{1x}}\omega_{1x}, ..., i_{X_{kx}}\omega_{kx}).$$

Therefore we obtain the isomorphism:

$$h : T^1_k M \to (T^1_k)^* M, h \mid (T^1_k)_x M = h_x.$$

Then $K^* = k \circ K$ is a field of cones on $((T^1_k)^* M, \tau^*, M)$ and h is an isomorphism between the structures $[(T^1_k M, \tau, M); K]$ and $[((T^1_k)^* M, \tau^*, M); K^*]$.□

Remark 2.1. We associated to the regular vector bundle $((T_k^1)^*M, \tau^*, M)$ a natural field of cones K^*, starting with a field of cones K on the regular vector bundle $(T_k^1 M, \tau, M)$. One can also associate different other fields of cones, for example $\overline{K}^*$, where $\overline{K}^*(x) = \{\overline{\alpha}_x = (\overline{\alpha}_{1x}, ..., \overline{\alpha}_{kx}) \in (T_k^1)^*_x M : \overline{\alpha}_{Ax}(X_{Ax}) \geq 0$, for any $X_x = (X_{1x}, ..., X_{kx}) \in K(x)\}$.

Let $\chi^k(M) = \chi(M) \times ... \times \chi(M)$, where $\chi(M)$ is the set of the sections of the tangent bundle and $\pi : \chi^k(M) \to M, \pi(X_{1x}, ..., X_{kx}) = x$. On the above set, we can define an ordering relation $\leq$ such that $(\chi^k(M), \leq)$ becomes an ordered vector space directed on both sides, as follows:

$$X, Y \in \chi^k(M), X \leq Y \Leftrightarrow Y_x - X_x \in K(x), \forall x \in M.$$

Using this ordering relation on $\chi^k(M)$, we can construct a convex, pointed cone K_+ of $(\chi^k(M), \pi, M)$, as follows:

$$K_+ = \{X = (X_1, ..., X_k) \in \chi^k(M) : X \geq 0\}.$$

The dual problem can be treated similarly. On $\Omega_k^1(M) = \Omega^1(M) \times ... \times \Omega^1(M)$, where $\Omega^1(M)$ is the set of the sections of the cotangent bundle and $\pi^* : \Omega_k^1(M) \to M, \pi^*(\alpha_{1x}, ..., \alpha_{kx}) = x$, define the ordering relation $\leq$

$$\alpha, \beta \in \Omega_k^1(M), \alpha \leq \beta \Leftrightarrow \beta_x - \alpha_x \in K^*(x), \forall x \in M$$

and construct in the same manner like before the convex, pointed cone K_+^* of $(\Omega_k^1(M), \pi^*, M)$

$$K_+^* = \{\alpha = (\alpha_1, ..., \alpha_k) \in \Omega_k^1(M) : \alpha \geq 0\}.$$

In the general case discussed by Papuc ([7]), for $[(E, p, M); K]$ a regular vector bundle endowed with a field of cones, if we denote by Σ the set of all global continuous sections of (E, p, M) and by K_Σ the set of all global continuous and positive sections (i.e. $K_\Sigma = \{\sigma \in \Sigma : \forall x \in M, \sigma(x) \in K(x)\}$) and take an element $Z \in IntK_\Sigma$, Σ is not necessary Z-measurable. It was proved that for a fixed $Z \in IntK_\Sigma$, the set of all Z-measurable elements is a closed, non-empty, linear subspace of Σ and the two sets coincides if and only if M is compact ([7]).

Consider now that M is a compact manifold.

For $Z \in IntK_+$, $\chi^k(M)$ is Z-measurable, i.e. for any $X \in K_+$, there exists a $\lambda \in \mathbb{R}_+$ such that $-\lambda Z \leq X \leq \lambda Z$. Fixing $Z \in IntK_+$, we can define on $\chi^k(M)$ a norm $|\cdot|_Z$,

$$| X |_Z = min\{\lambda \in \mathbb{R}_+ : -\lambda Z \leq X \leq \lambda Z\}.$$

It is a monotone norm, i.e. $0 \leq X \leq Y \Rightarrow 0 \leq \mid X \mid_Z \leq \mid Y \mid_Z$ ([4]).

Similarly, for $\gamma \in IntK^*_+$, $\Omega^1_k(M)$ is γ-measurable and if we fix $\gamma \in IntK^*_+$, we define on $\Omega^1_k(M)$ the monotone norm $\mid \cdot \mid_\gamma$,

$$\mid \alpha \mid_\gamma = min\{\lambda \in \mathbb{R}_+ : -\lambda\gamma \leq \alpha \leq \lambda\gamma\}.$$

Proposition 2.2. *If (M, ω) is a polysymplectic manifold such that there exists a field of cones K on the k-tangent fiber bundle $(T^1_k M, \tau, M)$, then there exists a positive, monotone operator between the isomorphic structures $[(T^1_k M, \tau, M); K]$ and $[((T^1_k)^* M, \tau^*, M); K^*]$, where K^* is the field of cones defined in the Proposition* 2.1.

Proof. The isomorphism h defined in the Proposition 2.1 is the positive, monotone operator between $[(T^1_k M, \tau, M); K]$ and $[((T^1_k)^* M, \tau^*, M); K^*]$. It is positive, as $h \mid (T^1_k)_x M(K(x)) = h_x(K(x)) = K^*(x)$, for any $x \in M$ and it is monotone, as for $X = (X_1, ..., X_k), Y = (Y_1, ..., Y_k) \in \chi^k(M), X \leq Y \Rightarrow Y_x - X_x \in K(x)$, for any $x \in M \Rightarrow h_x(Y_{1x} - X_{1x}, ..., Y_{kx} - X_{kx}) \in K^*(x)$, for any $x \in M \Leftrightarrow (i_{X_1}\omega_1, ..., i_{X_k}\omega_k) \leq (i_{Y_1}\omega_1, ..., i_{Y_k}\omega_k) \Leftrightarrow h(X) \leq h(Y)$. $\square$

Moreover, h is a regular operator because every smoothly parameterized family of k-vector fields is transformed into a smooth family of k-covector fields.

Consider J_A k almost complex structures on M, compatible with ω_A (i.e. k sections of $End(TM)$ such that ${J_A}^2 = -id_{TM}, \omega_A(X, J_A X) > 0$, for any $X \in \chi(M) - \{0\}$) and $1 \leq A \leq k$. Using these structures, define the Riemannian metrics g_A on M:

$$g_A(X, Y) = \omega_A(X, J_A Y) - \omega_A(J_A X, Y),$$

which are J_A-invariant. Indeed,

$$\begin{aligned} g_A(J_A X, J_A Y) &= \omega_A(J_A X, J_A J_A Y) - \omega_A(J_A J_A X, J_A Y) \\ &= -\omega_A(J_A X, Y) + \omega_A(X, J_A Y) \\ &= g_A(X, Y), \forall X, Y \in \chi(M). \end{aligned}$$

Using a Riemannian metric we will give sufficiently conditions for the tangent and cotangent bundles of $T^1_k M$ to posses a field of cones. The metric g we shall use is the Witney sum of the Riemannian metrics g_A.

Proposition 2.3. *Let (M, ω) be a polysymplectic manifold such that there exists $X \in \chi(T^1_k M)$ with $X_x \neq 0$, for any $x = (Y_1, ..., Y_k) \in T^1_k M$. Then there exists a field of cones K on $(T(T^1_k M), \tau_k, M)$, where $\tau_k : T(T^1_k M) \to M, \tau_k(X_{(Y_{1x}, ..., Y_{kx})}) = x$.*

Proof. Using the Riemannian metric g, we construct the field of cones K on $(T(T_k^1M), \tau_k, M)$, where

$$\begin{aligned} K(x) &= \{Y_x \in T_x(T_k^1M) : \frac{1}{2}[g_x(X_x, X_x)g_x(Y_x, Y_x)]^{\frac{1}{2}} \leq g_x(X_x, Y_x) \\ &\leq [g_x(X_x, X_x)g_x(Y_x, Y_x)]^{\frac{1}{2}}\}. \end{aligned}$$

Indeed, one can easily check that for any $x \in M$, $K(x)$ is a convex, pointed, closed cone with interior points of $T_x(T_k^1M)$ and the fact that the sets $\bigcup_{x\in T_k^1M} IntK(x)$ and $\bigcup_{x\in T_k^1M}(T_x(T_k^1M) - K(x))$ are open in $T(T_k^1M)$ is a consequence of the Theorem 1.3.1 in ([7]). □

Corollary 2.4. *Let (M, ω) be a polysymplectic manifold such that there exists $X \in \chi(T_k^1M)$ with $X_x \neq 0$, for any $x = (Y_1, ..., Y_k) \in T_k^1M$. Then there exists a field of cones on $(T^*(T_k^1M), \tau_k^*, M)$, where $\tau_k^* : T^*(T_k^1M) \to M, \tau_k^*(\alpha_{(X_{1x},...,X_{kx})}) = x$.*

Proof. Using the Propositions 2.1 and 2.3. □

As in the symplectic case, one can define a polysymplectic structure on the Whitney sum of the cotangent bundle of a differential manifold.

Proposition 2.5. *There exists a polysymplectic structure $\widetilde{\omega}$ on the manifold $(T_k^1)^*M$.*

Proof. Denote by $\tau^*_{\alpha_x} : T_{\alpha_x}((T_k^1)^*M) \to T_{\tau^*(\alpha_x)}M = T_xM$ the differential of the projection $\tau^* : (T_k^1)^*M \to M, \tau^*(\alpha_{1x}, ..., \alpha_{kx}) = x$ in $\alpha_x = (\alpha_{1x}, ..., \alpha_{kx}) \in (T_k^1)^*_xM$ and define the k 1-forms $\widetilde{\theta}_{A\alpha_x} \in T^*_{\alpha_x}((T_k^1)^*M)$, $\widetilde{\theta}_{A\alpha_x}(X_{\alpha_x}) = \alpha_{Ax}(\tau^*_{*\alpha_x}(X_{\alpha_x}))$. Set $\widetilde{\omega}_A = -d\widetilde{\theta}_A$ and $\widetilde{\omega} = \sum_{A=1}^k \widetilde{\omega}_A \otimes e_A$ will be a polysymplectic structure on $(T_k^1)^*M$. □

Define $\widetilde{\theta} = \sum_{A=1}^k \widetilde{\theta}_A \otimes e_A$ and notice that $\widetilde{\omega} = \sum_{A=1}^k \widetilde{\omega}_A \otimes e_A = \sum_{A=1}^k -d\widetilde{\theta}_A \otimes e_A = -d(\sum_{A=1}^k \widetilde{\theta}_A \otimes e_A) = -d\widetilde{\theta}$, i.e. $\widetilde{\theta}$ is the Liouville $\mathbb{R}^k$-valued 1-form.

Using the isomorphism

$$\widetilde{h} : T((T_k^1)^*M) \to T^*((T_k^1)^*M), \widetilde{h}(X_\alpha) = i_{X_{\alpha_x}}\widetilde{\omega}_\alpha$$

and the Liouville $\mathbb{R}^k$-valued 1-form $\widetilde{\theta}$ on $(T_k^1)^*M$, we can define a vector field on $(T_k^1)^*M$, $X^{\widetilde{\theta}} = \widetilde{h}^{-1} \circ \widetilde{\theta}$.

Having defined the vector field $X^{\widetilde{\theta}}$, we can construct the field of cones $\widetilde{K}^{\widetilde{\theta}}$ on $(T((T_k^1)^*M), \widetilde{\tau}, (T_k^1)^*M)$, where $\widetilde{\tau} : T((T_k^1)^*M) \to (T_k^1)^*M, \widetilde{\tau}(X_\alpha) = \alpha$,

$\widetilde{K}^{\widetilde{\theta}}(\alpha) = \{Y_\alpha \in T_\alpha((T_k^1)^*M) : \frac{1}{2}[\widetilde{g}_\alpha(X_\alpha^{\widetilde{\theta}}, X_\alpha^{\widetilde{\theta}})\widetilde{g}_\alpha(Y_\alpha, Y_\alpha)]^{\frac{1}{2}} \leq \widetilde{g}_\alpha(X_\alpha^{\widetilde{\theta}}, Y_\alpha) \leq [\widetilde{g}_\alpha(X_\alpha^{\widetilde{\theta}}, X_\alpha^{\widetilde{\theta}})\widetilde{g}_\alpha(Y_\alpha, Y_\alpha)]^{\frac{1}{2}}\}$, where $\widetilde{g}$ is the Riemannian metric on $(T_k^1)^*M$, $\widetilde{g}(X,Y) = \widetilde{\omega}(X, \widetilde{J}Y) - \widetilde{\omega}(\widetilde{J}X, Y)$ for $\widetilde{J}$ an almost complex structure on $(T_k^1)^*M$, compatible with $\widetilde{\omega}$. According to the Proposition 2.1, there exists a field of cones $\widetilde{K}^{*\widetilde{\theta}}$ on $(T^*((T_k^1)^*M), \widetilde{\tau}^*, (T_k^1)^*M)$, where $\widetilde{\tau}^* : T^*((T_k^1)^*M) \to (T_k^1)^*M, \widetilde{\tau}^*(\alpha_\beta) = \beta$.

The lift of a diffeomorphism to the cotangent bundles preserves the canonical symplectic forms, hence it is a symplectomorphism. Moreover, it preserves even the Liouville 1-forms corresponding to the canonical symplectic structures. We can state the following generalization for the case of the k-cotangent bundles.

Proposition 2.6. *Any diffeomorphism $\phi : M_1 \to M_2$ can be lifted to a symplectomorphism $\phi^{T_*^k} : (T_k^1)^*M_1 \to (T_k^1)^*M_2$. Moreover, $(\phi^{T_*^k})^*\widetilde{\theta}_{M_2} = \widetilde{\theta}_{M_1}$.*

Proof. Define $\phi^{T_*^k}(\alpha_{1x}, ..., \alpha_{kx}) = (\alpha_{1x} \circ \phi_{*x}^{-1}, ..., \alpha_{kx} \circ \phi_{*x}^{-1})$. Indeed, $\phi^{T_*^k}$ is a diffeomorphism and it preserves the Liouville $\mathbb{R}^k$-valued 1-forms (hence the symplectic forms).

First notice the following equalities: $(\tau_2^* \circ \phi^{T_*^k})(\alpha_{1x}, ..., \alpha_{kx}) = \phi(x)$ and $\tau_2^* \circ \phi^{T_*^k} = \phi \circ \tau_1^*$, where $\tau_i^* : (T_k^1)^*M_i \to M_i, \tau_i^*(\alpha_{1x}, ..., \alpha_{kx}) = x, i = 1, 2$. Indeed, let $\alpha_x = (\alpha_{1x}, ..., \alpha_{kx}) \in (T_k^1)_x^*M_1, x \in M_1$. Then $(\tau_2^* \circ \phi^{T_*^k})(\alpha_x) = \tau_2^*(\alpha_{1x} \circ \phi_{*x}^{-1}, ..., \alpha_{kx} \circ \phi_{*x}^{-1}) = \phi(x) = \phi(\tau_1^*(\alpha_x)) = (\phi \circ \tau_1^*)(\alpha_x)$. We have

$$\begin{aligned}
((\phi^{T_*^k})^*\widetilde{\theta}_{M_2})_{\alpha_x} X_{\alpha_x} &= \widetilde{\theta}_{M_2 \phi^{T_*^k}(\alpha_x)}((\phi^{T_*^k})_{*\alpha_x} X_{\alpha_x}) \\
&= \sum_{A=1}^{k} (\widetilde{\theta}_{M_2 A \phi^{T_*^k}(\alpha_x)}((\phi^{T_*^k})_{*\alpha_x} X_{\alpha_x})) \otimes e_A \\
&= \sum_{A=1}^{k} ((\alpha_{Ax} \circ \phi_{*x}^{-1})(\tau^*_{2*\phi^{T_*^k}(\alpha_x)}((\phi^{T_*^k})_{*\alpha_x} X_{\alpha_x}))) \otimes e_A \\
&= \sum_{A=1}^{k} (\alpha_{Ax}(((\phi^{-1})_{*\phi(x)} \circ \tau^*_{2*\phi^{T_*^k}(\alpha_x)} \circ (\phi^{T_*^k})_{*\alpha_x}) X_{\alpha_x})) \otimes e_A \\
&= \sum_{A=1}^{k} (\alpha_{Ax}(\tau^*_{1*\alpha_x} X_{\alpha_x})) \otimes e_A
\end{aligned}$$

$$= \sum_{A=1}^{k} (\widetilde{\theta}_{M_1 A\alpha_x} X_{\alpha_x}) \otimes e_A$$
$$= \widetilde{\theta}_{M_1 \alpha_x} X_{\alpha_x}.$$

It follows that $(\phi^{T^k_*})^* \widetilde{\omega}_{M_2} = (\phi^{T^k_*})^*(-d\widetilde{\theta}_{M_2}) = -d((\phi^{T^k_*})^* \widetilde{\theta}_{M_2}) = -d\widetilde{\theta}_{M_1} = \widetilde{\omega}_{M_1}$.

The following diagram commutes ([2]):

$$\begin{array}{ccc} T((T^1_k)^* M_1) & \xrightarrow{(\phi^{T^k_*})_*} & T((T^1_k)^* M_2) \\ {\scriptstyle \widetilde{\tau}_1}\downarrow & & \downarrow{\scriptstyle \widetilde{\tau}_2} \\ (T^1_k)^* M_1 & \xleftarrow[\phi^{T^k_*}]{} & (T^1_k)^* M_2 \end{array}$$

and so we deduce that $(\phi^{T^k_*})_*$ is an isomorphism between the vector bundles $(T((T^1_k)^*M_1), \widetilde{\tau}_1, (T^1_k)^*M_1)$ and $(T((T^1_k)^*M_2), \widetilde{\tau}_2, (T^1_k)^*M_2)$.

The lift $\phi^{T^k_*}$ of the diffeomorphism ϕ preserves the vector fields corresponding to the Liouville forms. Indeed, $\widetilde{h}(X^{\widetilde{\theta}_{M_i}}) = \widetilde{\theta}_{M_i}$, for $i = 1, 2 \Leftrightarrow i_{X^{\widetilde{\theta}_{M_1}}} (\phi^{T^k_*})^* \widetilde{\omega}_{M_2} = (\phi^{T^k_*})^* \widetilde{\theta}_{M_2}, i_{X^{\widetilde{\theta}_{M_2}}} \widetilde{\omega}_{M_2} = \widetilde{\theta}_{M_2} \Leftrightarrow i_{(\phi^{T^k_*})_* X^{\widetilde{\theta}_{M_1}}} \widetilde{\omega}_{M_2} = \widetilde{\theta}_{M_2}$, $i_{X^{\widetilde{\theta}_{M_2}}} \widetilde{\omega}_{M_2} = \widetilde{\theta}_{M_2} \Rightarrow X^{\widetilde{\theta}_{M_2}} = (\phi^{T^k_*})_* X^{\widetilde{\theta}_{M_1}}$. $\square$

Let $\phi : M_1 \to M_2$ be a diffeomorphism, $\phi^{T^k_*} : (T^1_k)^* M_1 \to (T^1_k)^* M_2$ its lift defined above and J_{M_1} an almost complex structure on $(T^1_k)^* M_1$. Then $J_{M_2} = (\phi^{T^k_*})_* \circ J_{M_1} \circ ((\phi^{T^k_*})_*)^{-1}$ is an almost complex structure on $(T^1_k)^* M_2$. Indeed,

$$\begin{aligned} {J_{M_2}}^2 &= (\phi^{T^k_*})_* \circ J_{M_1} \circ ((\phi^{T^k_*})_*)^{-1} \circ (\phi^{T^k_*})_* \circ J_{M_1} \circ ((\phi^{T^k_*})_*)^{-1} \\ &= (\phi^{T^k_*})_* \circ {J_{M_1}}^2 \circ ((\phi^{T^k_*})_*)^{-1} \\ &= -(\phi^{T^k_*})_* \circ ((\phi^{T^k_*})_*)^{-1} \\ &= -id_{(T^1_k)^* M_2}. \end{aligned}$$

For $\widetilde{K}^{\widetilde{\theta}_{M_1}}$ and $\widetilde{K}^{\widetilde{\theta}_{M_2}}$ the fields of cones defined above, we have $(\phi^{T^k_*})_*(\widetilde{K}^{\widetilde{\theta}_{M_1}}) = \widetilde{K}^{\widetilde{\theta}_{M_2}}$.

So we conclude that the structures $[(T((T^1_k)^*M_1), \widetilde{\tau}_1, (T^1_k)^*M_1); \widetilde{K}^{\widetilde{\theta}_{M_1}}]$ and

$[(T((T_k^1)^*M_2), \widetilde{\tau}_2, (T_k^1)^*M_2); \widetilde{K}^{\widetilde{\theta}_{M_2}}]$ are isomorphic, an isomorphism between them being $(\phi^{T_*^k})_*$.

To every manifold $M \in Ob(Man)$ and an almost complex structure J_M we associated the structure $[(T((T_k^1)^*M), \widetilde{\tau}, (T_k^1)^*M); \widetilde{K}^{\widetilde{\theta}_M}]$ and to every diffeomorphism $\phi : M_1 \to M_2$ an isomorphism $(\phi^{T_*^k})_*$ between the structures $[(T((T_k^1)^*M_1), \widetilde{\tau}_1, (T_k^1)^*M_1); \widetilde{K}^{\widetilde{\theta}_{M_1}}]$ and $[(T((T_k^1)^*M_2), \widetilde{\tau}_2, (T_k^1)^*M_2); \widetilde{K}^{\widetilde{\theta}_{M_2}}]$.

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