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ON SOME CARDINAL FUNCTIONS CONCERNING KATĚTOV EXTENSIONS OF INFINITE DISCRETE SPACES*

BY

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Abstract. In the present paper, an attempt has been made to investigate the cardinal functions corresponding to certain concepts associated with the Katětov H-closed extension of an infinite discrete space. The concepts of density, spread, cellularity, weight and tightness of a topological space, as given by JUHÁSZ [4], are recalled here for their determination in respect of the said Katětov extension of an infinite discrete space.

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1. Introduction. The study of H-closed spaces as a generalization of compact spaces, was initiated in 1924 with the classical paper [2] of ALEXANDROFF and URYSOHN. According to them, a Hausdorff space X is H-closed if its homeomorphic image into any Hausdorff space is closed. In 1940, KATĚTOV [5] proved the much desired result that any Hausdorff space can be densely embedded in an H-closed space; in other words, for every Hausdorff space X, there is an H-closed extension of X, of which X is a dense subset. Such an H-closed extension (constructed by Katětov) is called the Katětov extension of X and is denoted by κX. In course of the last eighty years or so, hundreds of papers have appeared which have effectively encompassed different aspects and applications of the concepts

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of H-closedness of a space and the Katětov extension κX of any Hausdorff space X. A comprehensive description in this regard can be found in Porter and Woods [7] and Alexandroff [1].

In [6] Porter and Votaw employed certain cardinality arguments to discuss some internal structures of the set of H-closed extensions of a given Hausdorff space with particular reference to the Katětov extensions. Cardinal functions in general situations for arbitrary topological spaces were extensively discussed by Juhász [4], wherein a study of cardinal functions on some special classes of spaces has also been undertaken.

In the present article, our intention is to pursue an investigation chiefly on the cardinal functions corresponding to certain concepts associated with the Katětov extension of a Hausdorff space X, where X is an infinite discrete space. It is well known that the Stone-Čech compactification $\beta\mathbb{N}$ (where $\mathbb{N}$ is the infinite discrete space of all natural numbers) has nice properties and is extremely useful for many purposes (see [3]). Our deliberation here includes some aspects of $\kappa\mathbb{N}$, in particular. The notions of density, spread, weight and tightness of a topological space, and the terms cellularity and network associated with families of subsets of a topological space, are defined in Juhász [4]. Our endeavour would be to study these concepts vis-a-vis the Katětov H-closed extension of an infinite discrete space X. We shall also classify the points of the remainder $\kappa X \setminus X$ into two categories viz. far points and proximate points. We shall derive conditions to examine the existence of either or both of these types of points in $\kappa X \setminus X$.

In what follows, by $\wp(A)$ we shall denote the power set of a given set A, and we shall adopt the notations $|A|$, $\exp|A|$, $\exp\exp|A|$ etc. to respectively denote the cardinalities of A, $\wp(A)$, $\wp(\wp(A))$ and so on. Throughout the paper, unless otherwise is stated, X will mean a Hausdorff space and κX its Katětov extension. As usual, we shall use $\mathbb{N}$ to denote the set of natural numbers which has cardinality $\aleph_0$ (the smallest infinite cardinal number).

2. Size of κX and its topology. Our main intention in this section is to calculate the size of κX and that of its topology, where X is an infinite discrete space. We shall also measure the cellularity, density and spread (to be defined explicitly, in due course) of the Katětov extensions of infinite discrete spaces. Almost all the results that will come up in this section are basically some applications of a main theorem which we feel like beginning with.

Theorem 2.1. *Every compact, Hausdorff space is a continuous image of κX, for some discrete space X.*

Proof. Let Y be a compact, Hausdorff space and D be a dense subset of Y (if Y admits no proper dense subset, we take D to be the whole of Y). We consider any discrete space X such that $|X| \geq |D|$. Then there is a surjection $f : X \to D \subseteq Y$. Since X is discrete, $f : X \to Y$ is a continuous map. As X is C^*-embedded in κX (see Proposition (p), page 309 of [7]) and Y is compact and T_2, we have a continuous map $\tilde{f} : \kappa X \to Y$ such that $\tilde{f}|_X = f$ (by Theorem 6.4 of [3]). We claim that $\tilde{f}$ is a surjection. In fact, $\tilde{f}(\kappa X)$ is an H-closed subset of Y, since H-closedness is a continuous invariant [7] and hence $\tilde{f}(\kappa X)$ is closed in Y, as Y is Hausdorff. Also, $\tilde{f}(X) = f(X) \subseteq \tilde{f}(\kappa X)$. But $f(X) = D$, since $f : X \to D$ is surjective. Thus we have $D \subseteq \tilde{f}(\kappa X)$, i.e., $Y = \overline{D} \subseteq \overline{\tilde{f}(\kappa X)} = \tilde{f}(\kappa X)$, as D is dense in Y and $\tilde{f}(\kappa X)$ is closed in Y. This shows that $\tilde{f}(\kappa X) = Y$, i.e., $\tilde{f} : \kappa X \to Y$ is surjective and as a result, Y becomes a continuous image of κX. □

It follows from the above proof that the statement of the theorem could be replaced in the following way:

Theorem 2.2. *Let D be a dense subset of a compact, Hausdorff space Y. Then for any discrete space X with $|X| \geq |D|$, Y is a continuous image of κX.*

Corollary 2.3. *Every compact, Hausdorff, separable space is a continuous image of $\kappa\mathbb{N}$.*

Proof. In a compact, Hausdorff, separable space Y, we can always have a dense subset D with $|D| \leq \aleph_0$. So we take any surjection $f : \mathbb{N} \to D$ and proceed in the same way as in Theorem 2.1 to get the desired result. □

Corollary 2.4. *Every compactification of a Tychonoff space X is a continuous image of κX.*

In the next theorem, we shall determine the size of κX and some cardinal characters of it as corollaries, where X is an infinite discrete space. For that we require the following definitions.

Definition 2.5 ([4]). (a) The density $d(X)$ of a topological space X is defined as $d(X) = \min \{|S| : S \subseteq X \text{ and } \overline{S} = X\} + \aleph_0$.

(b) The spread $s(X)$ of a topological space X is defined as $s(X) = \sup\{|D| : D \text{ is a discrete subspace of } X\} + \aleph_0$.

Definition 2.6 ([4]). A family $\mathcal{C}$ of nonempty open subsets of a topological space X is called a cellular family in X, if the members of $\mathcal{C}$ are pairwise disjoint.

The cellularity $c(X)$ of a topological space X is defined as $c(X) = \sup\{|\mathcal{C}| : \mathcal{C} \text{ is a cellular family in } X\} + \aleph_0$.

We shall also denote by $o(X)$ the size of the topology of a space X, i.e., the cardinality of the family of all open subsets of a space X.

Theorem 2.7. *If X is an infinite discrete space, then $|\kappa X| = exp\ exp|X|$.*

Proof. By Corollary 2.4, there is a surjection $f : \kappa X \to \beta X$, where βX is the Stone-Čech compactification of X. Since X is discrete, $|\beta X| = \exp\exp|X|$ (see [3]) and hence $|\kappa X| \geq \exp\exp|X|$. It is known (see [7]) that for any extension Y of any topological space X, $|Y| \leq \exp\exp|X|$, so that $|\kappa X| \leq \exp\exp|X|$. Hence $|\kappa X| = \exp\exp|X|$. □

Corollary 2.8. *For an infinite discrete space X, we have*
(a) $s(\kappa X) = \exp\exp|X|$ and
(b) $o(\kappa X) = \exp\exp\exp|X|$.

Proof. (a) Since $|\kappa X| = \exp\exp|X|$ (by Theorem 2.8), obviously $s(\kappa X) \leq \exp\exp|X|$. Also, $|\kappa X| = \exp\exp|X|$ and $|X| < \exp\exp|X|$ imply that $|\kappa X \setminus X| = \exp\exp|X|$. But $\kappa X \setminus X$ is a discrete subset of κX (see [7]) and hence $s(\kappa X) \geq |\kappa X \setminus X| = \exp\exp|X|$. Therefore we have $s(\kappa X) = \exp\exp|X|$.

(b) Obviously, $o(\kappa X) \leq |\wp(\kappa X)| = \exp|\kappa X| = \exp\exp\exp|X|$ (by Theorem 2.8). Also, $(\kappa X \setminus X)$ is a closed discrete subset of κX. Therefore all members of $\wp(\kappa X \setminus X)$ are closed in κX. Thus, $o(\kappa X) \geq |\wp(\kappa X \setminus X) = \exp|\kappa X \setminus X| = \exp\exp\exp|X|$. Hence $o(\kappa X) = \exp\exp\exp|X|$. □

Theorem 2.9. *If X is an infinite discrete space, then $d(\kappa X) = c(\kappa X) = |X|$.*

Proof. Since $\overline{X} = \kappa X$, we have $d(\kappa X) \leq |X|$. It is thus sufficient to show that if A is any subset of κX with $|A| < |X|$, then $\overline{A} \neq \kappa X$. So let $A \subseteq \kappa X$ with $|A| < |X|$. Then there exists an $x \in X$ such that $x \notin A$. Since

X is discrete and open in κX too (see [7]), x is an isolated point in κX and hence $x \notin \overline{A}$. This shows that $\overline{A} \neq \kappa X$ and thus we have $d(\kappa X) = |X|$.

Again, since for any topological space Y, $c(Y) \leq d(Y)$ (see [4]), we have $c(\kappa X) \leq d(\kappa X) = |X|$ (by above). Now, it suffices to provide a cellular family $\mathcal{A}$ in κX such that $|\mathcal{A}| = |X|$. Since X is discrete and open in κX, it follows that $\{x\}$ is open in κX, for each $x \in X$, and hence $\mathcal{A} = \{\{x\} : x \in X\}$ is a cellular family in κX with $|\mathcal{A}| = |X|$. Therefore, $c(\kappa X) = |X|$. □

3. Weight and tightness of κX. The weight of a topological space is one of the most important and well known cardinal functions in general topology. Our principal concern in this section is to determine the weight and another notion called tightness of the Katětov extensions κX of any infinite discrete space X. We shall also define a type of points in the growth of $\kappa X \setminus X$, and shall find a necessary and sufficient condition for the existence of such points in $\kappa X \setminus X$. The discussion in this connection will facilitate the deliberation concerning tightness.

To start with we need the following definitions.

Definition 3.1 ([4]). The weight $w(X)$ of a topological space X is defined as $w(X) = \min\{|\mathcal{B}| : \mathcal{B} \text{ is an open base of } X\} + \aleph_0$.

Definition 3.2 ([4]). In a topological space X, a family $\Im \subseteq \wp(X)$ is said to be a network in X, if every open set in X is a union of some members of $\Im$.

The net weight $n\omega(X)$ of a topological space X is defined as $n\omega(X) = \min\{|\Im| : \Im \text{ is a network in } X\} + \aleph_0$.

Remark 3.3. From definition it follows that an open base is a network consisting of open sets only and hence for any topological space X, we have $\omega(X) \geq n\omega(X)$.

In the following theorem we determine the weight of κX, for any discrete space X.

Theorem 3.4. *If X is an infinite discrete space, then* $\omega(\kappa X) = \exp\exp|X|$.

Proof. We first prove the following Lemma:

Lemma 3.5. *For an infinite discrete space X, $n\omega(\kappa X) = \exp\exp|X|$.*

Proof of the Lemma. Let $n\omega(\kappa X) = \alpha$. Then there exists a network $\Im$ in κX such that $|\Im| = \alpha$ (note that each network in κX is of infinite cardinality). Let $\mathcal{A}(\Im) = \{\cup\mathcal{C} : \mathcal{C} \subseteq \Im\}$. Then the topology of κX is a subcollection of $\mathcal{A}(\Im)$. Thus $|\mathcal{A}(\Im)| \geq o(\kappa X)$. Define $f : \wp(\Im) \to \mathcal{A}(\Im)$ by $f(\mathcal{C}) = \cup\mathcal{C}$, for all $\mathcal{C} \subseteq \Im$. Then f is a surjection and hence $|\wp(\Im)| \geq |\mathcal{A}(\Im)| \geq o(\kappa X)$, i.e., $\exp|\Im| \geq \exp\exp\exp|X|$ (by Corollary 2.9(c)), i.e., $\alpha \geq \exp\exp|X|$ (since $|\Im| = \alpha$), i.e., $n\omega(\kappa X) \geq \exp\exp|X|$. So it remains to provide a network $\Im$ in κX such that $|\Im| = \exp\exp|X|$. Let $\Im = \{\{x\} : x \in \kappa X\}$. Obviously, $\Im$ is a network in κX with $|\Im| = |\kappa X| = \exp\exp|X|$ (by Theorem 2.7). Therefore, for an infinite discrete space X, $n\omega(\kappa X) = \exp\exp|X|$.

Proof of the main theorem. Since for any topological space Y, $\omega(Y) \geq n\omega(Y)$, we have $\omega(\kappa X) \geq n\omega(\kappa X)$ and hence, in view of Lemma 3.5, $\omega(\kappa X) \geq \exp\exp|X|$. Thus we only need to exhibit an open base $\mathcal{B}$ of κX such that $|\mathcal{B}| = \exp\exp|X|$. To do this, we make use of the fact that $\kappa X \setminus X$ is a closed discrete subset of κX, for any Hausdorff space X (see [7]). Since $\kappa X \setminus X$ is discrete, for each $x \in (\kappa X \setminus X)$ there exists some $A \subseteq X$ such that $A \cup \{x\}$ is open in κX. So for each $x \in \kappa X \setminus X$ if we take $\mathcal{A}_x = \{A \subseteq X : A \cup \{x\} \text{ is open in } \kappa X\}$, then $|\mathcal{A}_x| \leq \exp|X|$. Let $\mathcal{A} = \cup_{x\in\kappa X\setminus X}\mathcal{A}_x$. Since $|\kappa X \setminus X| = \exp\exp|X|$ (see the proof of Corollary 2.9(b)) and $|\mathcal{A}_x| \leq \exp|X|$ for each $x \in \kappa X \setminus X$, we have $|\mathcal{A}| = \exp\exp|X|$. Also, X is a discrete open subset of κX. Therefore, all the members of $\wp(X)$ are open in κX. Let $\mathcal{B} = \mathcal{A} \cup \wp(X)$. Thus $\mathcal{B}$ is a family of open subsets of κX with $|\mathcal{B}| = \exp\exp|X|$, as $|\mathcal{A}| = \exp\exp|X|$ and $|\wp(X)| = \exp|X|$. We now prove that $\mathcal{B}$ is an open base of κX. For this, let U be an open subset of κX and $x \in \kappa X$ be such that $x \in U$. If $x \in X$, then we have $x \in \{x\} \subseteq U$, where $\{x\} \in \wp(X)$ and thereby $\{x\} \in \mathcal{B}$. So we assume that $x \in \kappa X \setminus X$. Let $K = \{y \in \kappa X \setminus X : y \in U \text{ and } y \neq x\}$.

Then $K \subseteq \kappa X \setminus X$. Since $\kappa X \setminus X$ is a closed discrete subset of κX. K is a closed set in κX with $x \notin K$. Let $G = U \setminus K$. Then G is an open set in κX with $x \in G$. Obviously, $G \cap (\kappa X \setminus X) = \{x\}$. Let $A = G \cap X$. Then $A \subseteq X$ and $G = A \cup \{x\}$. This shows that $G \in \mathcal{A} \subseteq \mathcal{B}$ with $x \in G \subseteq U$. Thus $\mathcal{B}$ is an open base for the topology of κX and hence $\omega(\kappa X) = \exp\exp|X|$. □

Digressing a bit from the course of discussion so far, we would now like to classify the points of $\kappa X \setminus X$, where X is any Hausdorff space, into two

types depending,in some sense, on their vicinity to the set X.

Definition 3.6. Let X be a Hausdorff topological space. A point x of $\kappa X \setminus X$ is said to be a proximate point if there exists some $A \subseteq X$ with $|A| < |X|$ such that $x \in \overline{A}$. If a point $x \in \kappa X \setminus X$ is not a proximate point, then we say that x is a far point.

The following theorem ensures the existence of far points in $\kappa X \setminus X$, if X is any infinite discrete space.

Theorem 3.7. *If X is an infinite discrete space, then $\kappa X \setminus X$ contains far points.*

Proof. Suppose X is an infinite discrete space. Let $\mathcal{A} = \{A \subseteq X : |A| < |X|\}$. If possible, let every point of $\kappa X \setminus X$ be a proximate point. Then for each $x \in \kappa X \setminus X$, there exists some $A \in \mathcal{A}$ such that $x \in \overline{A}$. Since X is discrete, A is clopen in X, for each $A \in \mathcal{A}$. Therefore, $\overline{A}$ is clopen in κX (see [7]), for each $A \in \mathcal{A}$. Thus $\mathcal{C} = \{\overline{A} : A \in \mathcal{A}\}$ is an open cover of κX. By H-closedness of κX, there is a finite sub-family of $\mathcal{C}$, whose union is dense in κX (see [7]). Since every member of $\mathcal{C}$ is closed in κX, we can say that there exist finitely many members $\overline{A_1}, \overline{A_2}, \ldots, \overline{A_n}$ (say) in $\mathcal{C}$, whose union equals κX, i.e., $\kappa X = \cup_{i=1}^{n} \overline{A_i} = \overline{\cup_{i=1}^{n} A_i}$. Now, since each A_i is a member of $\mathcal{A}$, we have $|A_i| < |X|$, for $i = 1, 2, \ldots, n$. Then $|\cup_{i=1}^{n} A_i| < |X|$. So there exists some $x_0 \in X$ such that $x_0 \notin \cup_{i=1}^{n} A_i$. Since X is an open discrete subset of κX, x_0 is an isolated point of κX and hence $x_0 \notin \overline{(\cup_{i=1}^{n} A_i)} = \kappa X$, which is a contradiction. Consequently $\kappa X \setminus X$ contains far points. □

One may raise a very natural question: whether $\kappa X \setminus X$ contains proximate points, for any infinite discrete space X? In the next theorem we show that the existence of proximate points in $\kappa X \setminus X$ depends on the size of X.

Theorem 3.8. If X is an infinite discrete space, then $\kappa X \setminus X$ contains proximate points iff $|X| > \aleph_0$.

Proof. First, let X be a discrete space with $X > \aleph_0$. Let $A \subseteq X$ be such that $|A| = \aleph_0$. Since X is an open discrete subset of κX, A is open in κX. Since κX is H-closed and A is an open subset of κX, $\overline{A}$ is also H-closed (see [7]). Since every point of X is an isolated point of κX, no point of $X \setminus A$ can belong to $\overline{A}$. We claim that there exists some $x \in \kappa X \setminus X$

such that $x \in \overline{A}$. If not, then $\overline{A} = A$ and A becomes H-closed which cannot happen, as A is an infinite discrete set. Therefore, there is some $x \in \kappa X \setminus X$ such that $x \in \overline{A}$, where $A \subseteq X$ and $|A| = \aleph_0 < |X|$. This shows that x is a proximate point.

Conversely, let X be a discrete space with $|X| = \aleph_0$. Thus for any $A \subseteq X$, if $|A| < |X|$, then A is a finite set and hence A is closed in κX. Therefore, no point of $\kappa X \setminus X$ can belong to $\overline{A}$, which means that $\kappa X \setminus X$ contains no proximate point. This proves the theorem completely. □

The following two corollaries are easily obtained consequences of the last two theorems.

Corollary 3.9. *Every point of $\kappa\mathbb{N} \setminus \mathbb{N}$ is a far point.*

Corollary 3.10. *If X is a discrete space with $|X| > \aleph_0$, then $\kappa X \setminus X$ contains both far points and proximate points.*

As the final step, we now like to determine the tightness of the Katětov extensions of infinite discrete spaces, the definition of tightness being given by Juhász in the following way:

Definition 3.11 ([4]). Suppose X is a topological space. Let $p \in X$, $S \subseteq X$ and $p \in \overline{S}$. Then we define

$$a(p, S) = \min\{|M| : M \subseteq S \text{ and } p \in \overline{M}\}$$

and

$$t(p, X) = \sup\{a(p, S) : p \in \overline{S} \subseteq X\}.$$

Then $t(p, X)$ is called the tightness of X in p and $t(X) = \sup\{t(p, X) : p \in X\}$ is called the tightness of the space X.

The main result in the present connection, giving the tightness of κX for an infinite discrete space X, runs as follows.

Theorem 3.12. *If X is an infinite discrete space, then $t(\kappa X) = |X|$.*

Proof. We need the following two lemmas.

Lemma 3.13. *If p is an isolated point of a topological space Y, then $t(p, Y) = 1$.*

Proof. Suppose p is an isolated point of a space Y. Let $S \subseteq Y$ be such that $p \in \overline{S}$. Since p is isolated, $p \in S$ and consequently we have $p \in \overline{\{p\}} \subseteq S$. This shows that $a(p, S) = \min\{|M| : p \in \overline{M} \text{ and } M \subseteq S\} = 1$, for all $S \subseteq Y$ with $p \in \overline{S}$, and hence $t(p, Y) = \sup\{a(p, S) : p \in \overline{S} \subseteq Y\} = 1$. □

Lemma 3.14. If X is an infinite discrete space, then for each $p \in \kappa X \setminus X$, $t(p, \kappa X) \leq |X|$.

Proof. Suppose $p \in \kappa X \setminus X$ and $S \subseteq \kappa X$ be such that $p \in \overline{S}$. If $p \in S$, then $a(p, S) = 1$, by Lemma 3.13. So we assume that $p \notin S$. Let $A = S \cap X$ and $B = S \cap (\kappa X \setminus X)$. Since $\kappa X \setminus X$ is a closed, discrete subset of κX, B is a closed subset of κX with $p \notin B$ and hence $p \notin \overline{B}$. Now, $S = A \cup B$, i.e., $\overline{S} = \overline{A} \cup \overline{B}$. Since $p \in \overline{S}$ and $p \notin \overline{B}$, it follows that $p \in \overline{A}$. Since $a(p, S) = \min\{|M| : p \in \overline{M} \text{ and } M \subseteq S\}$ and $p \in \overline{A}$ with $A \subseteq X$, we infer that $a(p, S) \leq |A| \leq |X|$, and therefore, $t(p, \kappa X) = \sup\{a(p, S) : p \in \overline{S} \subseteq \kappa X\} \leq |X|$. □

Proof of the main theorem. Since X is an open, discrete subset of κX, every point of X is an isolated point of κX and then by Lemma 3.13, $t(p, \kappa X) = 1$, for each $p \in X$. Hence in view of Lemma 3.14, we have $t(\kappa X) = \sup\{t(p, X) : p \in \kappa X\} \leq |X|$. We are thus only to find a point p in κX such that $t(p, \kappa X) = |X|$. For this, let $p \in \kappa X \setminus X$ be a far point (the existence of such a far point is guaranteed by Theorem 3.7). Let $S \subseteq \kappa X$ be such that $p \in \overline{S}$. If $p \in S$, then $a(p, S) = 1$. So we assume that $p \notin S$. Then $p \in \overline{A}$, where $A = S \cap X$, as $p \notin B = S \cap (\kappa X \setminus X)$ and B is closed in κX. This proves that whenever $p \notin S$ then $p \in \overline{S}$ iff there is some $A \subseteq S$ with $A \subseteq X$ such that $p \in \overline{A}$. Since p is a far point and $A \subseteq X$ with $p \in \overline{A}$, we have $|A| = |X|$. Therefore, $a(p, S) = |X|$, if $p \notin S$ and p is a far point and hence $t(p, \kappa X) = \sup\{a(p, S) : p \in \overline{S} \subseteq \kappa X\} = |X|$ and the theorem is completely proved. □

Corollary 3.15. *For any infinite cardinal number α, there exists a Hausdorff space Y such that $t(Y) = \alpha$.*

Proof. Consider a discrete space X with $|X| = \alpha$. Then $Y = \kappa X$ serves the purpose. □

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