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ON QUASI-CLASS A OPERATORS

BY

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Abstract. Let H be a separable infinite dimensional complex Hilbert space, and let $B(H)$ denote the algebra of all bounded linear operators on H. Let A, B be operators in $B(H)$. In this paper we prove that if A is quasi-class A and B^* is invertible quasi-class A and $AX = XB$, for some $X \in C_2$ (the class of Hilbert-Schmidt operators on H), then $A^*X = XB^*$. We also prove that if A is a quasi-class A operator and f is an analytic function on a neighborhood of the spectrum of A, then $f(A)$ satisfies generalized Weyl's theorem. Other related results are also given.

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1. Introduction

Let H be a separable infinite dimensional complex Hilbert space, and let $B(H)$ denote the algebra of all bounded linear operators on H. Let $A \in B(H)$. Set as usual, $|A| = (A^*A)^{\frac{1}{2}}$ and $[A^*, A] = A^*A - AA^* = |A|^2 - |A^*|^2$ (the self commutator of A), and consider the following standard definitions: A is normal if $A^*A = AA^*$, hyponormal if $A^*A - AA^* \geq 0$, p-hyponormal $(0 < p \leq 1)$ if $(|A|^{2p} - |A^*|^{2p} \geq 0)$. An operator $A \in B(H)$ is said to be paranormal if $\|Ax\|^2 \leq \|A^2x\| \, \|x\|$, for all $x \in H$. In general hyponormal $\subset$ $p-$hyponormal $\subset$ paranormal. A is said to be log-hyponormal if A is invertible and satisfies the following inequality $\log(A^*A) \geq \log(AA^*)$. It is known that invertible p-hyponormal operators are log-hyponormal operators but the converse is not true [27]. However it is very interesting that we may regard log-hyponormal operators are 0-hyponormal operators [27, 28]. The idea of log-hyponormal operator is due to Ando ([2]) and the first paper in which log-hyponormality appeared is [16]. See [27, 28, 29] for properties of log-hyponormal operators.

We say that an operator $A \in B(H)$ belongs to the class $\mathcal{A}$ if $|A^2| \geq |A|^2$. Class $\mathcal{A}$ was first introduced by Furuta-Ito-Yamazaki ([15]) as a subclass of paranormal operators which includes the classes of p-hyponormal and log-hyponormal operators. The following theorem is one of the results associated with class $\mathcal{A}$.

Theorem 1.1 ([15])**.** (1) *Every* log-*hyponormal operator is a class* $\mathcal{A}$.
(2) *Every class* $\mathcal{A}$ *operator is a paranormal operator.*

A is said to be p-quasihyponormal if $A^*((A^*A)^p - (AA^*)^p) \geq 0$, $(0 < p \leq 1)$, quasi-class A if $A^*|A^2|A \geq A^*|A|^2A \geq 0$. A is said to be normaloid if $\|A\| = r(A)$ (the spectral radius of A). Let (pH), (HN), $Q(p)$, QA, $\mathcal{A}$ and (NL) denote the classes consisting of hyponormal, p-hyponormal, p-quasihyponormal, quasi-class A, class $\mathcal{A}$ and normaloid operators. these classes are related by proper inclusion $(HN) \subset (pH) \subset (Q(p)) \subset QA$ and $(HN) \subset (pH) \subset \mathcal{A} \subset QA$. It is known that hyponormal, p-hyponormal, and p-quasihyponormal are normaloid. But quasi-class A operator is not normaloid [20].

Fuglede-Putnam theorem is given in [11, 15, 17] as follows:

Theorem 1.2. *If A and B are normal operators and if X is an operator such that $AX = XB$, then $A^*X = XB^*$.*

Berbebian ([3]) relaxes the hypothesis on A and B in Theorem 1.2 at the cost of requiring X to be Hilbert-Schmidt class. Cha ([12]) showed that the hyponormality in the result of Berberian ([3]) can be replaced by the quasi-hyponormality of A and B^* under some additional conditions. Recently Lee ([22]) proved that if A is p-quasihyponormal operator and B^* is an invertible p-quasihyponormal operator such that $AX = XB$ for some $X \in C_2(H)$ and $\||A|^{1-p}\|.\||B^{-1}|^{1-p}\| \leq 1$, then $A^*X = XB^*$. In this paper we prove that the above result remains true for quasi-class A operators without the additional condition $\||A|^{1-p}\|.\||B^{-1}|^{1-p}\| \leq 1$ as it is proved in ([21, Theorem 4]).

2. Main results

Lemma 2.1 ([20])**.** *Let A be a quasi-class A operator on Hilbert space H. if $0 \neq \lambda \in \mathbb{C}, x \in H$ and $Ax = \lambda x$, then $A^*x = \overline{\lambda}x$.*

Lemma 2.2 ([20]). *Let $A, B \in B(H)$. A and B are quasi-class A operators if and only if $A \otimes B$ is also a quasi-class A operator.*

Let $C_2(H)$ denote the class of Hilbert-Schmidt operators on H.

Corollary 2.1. *Let $A, B \in B(H)$. If A and B are quasi-class A operators, then the operator $\mathcal{K} : C_2(H) \mapsto C_2(H)$ defined by $\mathcal{K}X = AXB^*$ is a quasi-class A operator.*

Proof. It is known that $\mathcal{K}X$ can be identified with $A \otimes B$ (see [1]). □

Theorem 2.1. *Let $A, B \in B(H)$. If A is quasi-class A operator and B^* is an invertible quasi-class A operator such that $AX = XB$, for some $X \in C_2(H)$, then $A^*X = XB^*$.*

Proof. Let $\mathcal{K} : C_2(H) \mapsto C_2(H)$ be defined by $\mathcal{K}Y = AYB^{-1}$. Since B is quasi-class A, B^{-1} is a quasi-class A (see [21]). Then it follows from Corollary 2.1 that $\mathcal{K}$ is a quasi-class A operator, furthermore, $\mathcal{K}X = AXB^{-1} = X$ and so, X is an eigenvector of $\mathcal{K}$. Now by applying Lemma 2.1 we get $\mathcal{K}^*X = A^*X(B^{-1})^* = X$, that is, $A^*X = XB^*$ and the proof is achieved.□

Remark 2.1. In [30], Uchiyama presented an example of non-reducing eigenspace of a paranormal operator. Thus Lemma 2.1 does not hold for paranormal operator. Since the proof of Theorem 2.1 is thoroughly dependent on Lemma 2.1, Theorem 2.1 does not hold for paranormal operator.

3. Generalized Weyl's theorem

Let $K(H)$ denote, respectively, the algebra of bounded linear operators and the ideal of compact operators acting on infinite dimensional separable Hilbert space H. If $A \in B(H)$ we shall write $N(A)$ and $R(T)$ for the null space and the range of A, respectively. Also, let $\alpha(A) := \dim N(A)$, $\beta(A) := \dim(A^*)$, and let $\sigma(A)$, $\sigma_a(A)$ and $\pi_0(A)$ denote the spectrum, approximate point spectrum and point spectrum of A, respectively.

An operator $A \in B(H)$ is called Fredholm if it has closed range, finite dimensional null space, and its range has finite co-dimension.

The index of a Fredholm operator is given by $I(A) = \alpha(A) - \beta(A)$. An operator $A \in B(H)$ is called Weyl if it is a Fredholm of index zero, and Browder if it is Fredholm of finite ascent and descent, equivalently [19, Theorem 7.9.3] if A is Fredholm and $A - \lambda$ is invertible for sufficiently small

$|\lambda| > 0$, $\lambda \in \mathbb{C}$. The essential spectrum $\sigma_e(A)$, the Weyl spectrum $\sigma_w(A)$ and the Browder spectrum $\sigma_b(A)$ of A are defined by [18, 19]

$$\sigma_e(A) = \{\lambda \in \mathbb{C} : A - \lambda \text{ is not Weyl}\},$$
$$\sigma_w(A) = \{\lambda \in \mathbb{C} : A - \lambda \text{ is not Fredholm}\},$$
$$\sigma_b(A) = \{\lambda \in \mathbb{C} : A - \lambda \text{ is not Browder}\},$$

respectively. Evidently

$$\sigma_e A \subseteq \sigma_w(A) \subseteq \sigma_b A = \sigma_e(A) \cup acc\sigma(A),$$

where we write $accK$ for the accumulation points of $K \subseteq \mathbb{C}$. If we write $isoK = K \setminus accK$, then we let

$$\pi_{00}(A) := \{\lambda \in iso\sigma(A) : 0 < \alpha(A - \lambda) < \infty\},$$
$$p_{00}(A) := \sigma(A) \setminus \sigma_b(A).$$

We say that Weyl's theorem holds for A if $\sigma(A) \setminus \sigma_w(A) = \pi_{00}(A)$. The operator A is said to be B-Fredholm if there exists a natural number n such that $A^n(H)$ is closed and the induced operator $A_n = A|_{A^n(H)}$ is Fredholm, A is B-Weyl if it is B-Fredholm of index 0, and A satisfies generalized Weyl's theorem if $\sigma(A) \setminus \sigma_{Bw}(A) = E(A)$, where $\sigma_{Bw}(A)$ is the B-Weyl spectrum of A, i.e., the set of complex numbers λ for which $A - \lambda I$ fails to be B-Weyl and $E(A)$ is the set of isolated eigenvalues of A.

Note that if the generalized Weyl's theorem holds for A, then so does Weyl's theorem [6]. Recently in [7], Berkani showed that if A is a hyponormal operator, then A satisfies Weyl's theorem $\sigma_{Bw}(A) = \sigma(A) \setminus E(A)$, and the B-weyl spectrum $\sigma_{Bw}(A)$ of A satisfies the spectral mapping theorem. In [31], Weyl proved that Weyl's theorem holds for hermtian operators. Weyl's theorem has been extended from hermitian operators to hyponormal operators and Toeplitz operators [10], and to several classes of operators including semi-normal operators ([4, 5]). Curto and Han ([9]) have extended Lee's results to algebraically paranormal operators. In [13] the authors showed that Weyl's theorem holds for algebraically p-hyponormal operators. Mecheri ([23, 24, 25, 26]) showed that Weyl's and generalized Weyl's theorem hold for algebraically (p, k)-quasihyponormal operators, class $\mathcal{A}$ operators and class $H(q)$ operators. Recently in [14] the authors showed that Weyl's theorem holds for quasi-class $\mathcal{A}$ operator. In this paper we show that generalized Weyl's theorem holds for quasi-class A operators.

4. Results

Before proving the following lemma, we need a notation and a definition.

We say that $A \in B(H)$ has the single valued extension property (SVEP) if for every open set $U \subseteq \mathbb{C}$ the only analytic function $f : U \mapsto H$ which satisfies the equation $(A - \lambda)f(\lambda) = 0$ is the constant function $f \equiv 0$.

Lemma 4.1 ([20]). *Let A be a quasi-class A operator. Then A has SVEP.*

Lemma 4.2 ([20]). *Let A be a quasi-class A operator and $\lambda \in \mathbb{C}$. If $\sigma(A - \lambda) = \{0\}$, then $A - \lambda = 0$.*

It is shown in [9] that a quasinilpotent algebraically paranormal operator A is nilpotent. By the same way we prove that this result remains hold for a quasi-class A operator.

Lemma 4.3. *Let A be a quasinilpotent algebraically quasi-class A operator. Then A is nilpotent.*

Proof. Assume that $p(A)$ is quasi-class A for some nonconstant polynomial p. Since $\sigma(p(A)) = p(\sigma(A))$, the operator $p(A)-p(0)$ is quasinilpotent. Thus Lemma 4.2 would imply that $CA^m(A-\lambda_1)...(A-\lambda_n) \equiv p(A)-p(0) = 0$, where $m \geq 1$. Since $A - \lambda_i$ is invertible for every $\lambda_i \neq 0$, we must have $A^m = 0$. $\square$

Lemma 4.4. *Let A be an algebraically quasi-class A operator. Then A is isoloid.*

Proof. Let $\lambda \in iso\sigma(A)$ and let

$$P := \frac{1}{2\pi i} \int_{\partial D} (\mu - A)^{-1} d\mu$$

be the associated Riesz idempotent, where D is a closed disk centered at λ which contains no other points of $\sigma(A)$. We can then represent A as the direct sum

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix},$$

where $\sigma(A_1) = \{\lambda\}$ and $\sigma(A_2) = \sigma(A) \setminus \{\lambda\}$. Since A is algebraically quasi-class A, $p(A)$ is quasi-class A for some nonconstant polynomial p. Since $\sigma(A_1) = \{\lambda\}$, we must have $\sigma(p(A_1)) = p(\sigma(A_1)) = \{p(\lambda)\}$.

Therefore $p(A_1) - p(\lambda)$ is quasinilpotent. Since $p(A_1)$ is quasi-class A, it follows from Lemma 4.2 that $p(A_1) - p(\lambda) = 0$. Put $q(z) := p(z) - p(\lambda)$. Then $q(A_1) = 0$, so A_1 is algebraically quasi-class A, it follows from Lemma 4.3 that $A_1 - \lambda$ is nilpotent. Therefore $\lambda \in \pi_0(A_1)$, and hence $\lambda \in \pi_0(A)$. This shows that A is isoloid. $\square$

Theorem 4.1. *Let A be an algebraically quasi-class A operator. Then generalized Weyl's theorem holds for A.*

Proof. We will show that $\sigma(A) \setminus \sigma_{Bw}(A) \subset E(A)$. For this assume that $\lambda \in \sigma(A) \setminus \sigma_{Bw}(A)$. Then $A - \lambda I$ is a B- Fredholm operator of index zero and there exists a direct sum decomposition $H = H_1 \oplus H_2$ such that $A_1 = (A - \lambda I) \mid_{H_1}$ is a Fredholm operator of index zero, $A_2 = (A - \lambda I) \mid_{H_2}$ is nilpotent and $A - \lambda I = A_1 \oplus A_2$ [8, Lemma 4.1]. We have two possibilities: either $\lambda \in \sigma(A \mid_{H_1})$ or $\lambda \notin \sigma(A \mid_{H_1})$.

Assume that $\lambda \in \sigma(A \mid_{H_1})$. Since A is algebraically quasi-class A, $A \mid_{H_1}$ is also algebraically quasi-class A. Hence [14] implies $A \mid_{H_1}$ satisfies Weyl's theorem. Therefore if $\lambda \in \sigma(A \mid_{H_1})$, then $\lambda \in \pi_{00}(A \mid_{H_1})$. Hence $\lambda \in iso\sigma(A \mid_{H_1})$. Now since $A - \lambda I = (A \mid_{H_1} - \lambda I) \oplus A_2$, and A_2 is nilpotent, we have $\sigma(A_1) \setminus \{0\} = \sigma(A - \lambda I) \setminus \{0\}$ and $\lambda \in iso\sigma(A)$. This implies that $\lambda \in \pi_{00}(A) \subset E(A)$. Now assume that $\lambda \notin \sigma(A \mid_{H_1})$. Then we deduce from $A - \lambda I = (A \mid_{H_1} - \lambda I) \oplus A_2$ that λ is isolated in $\sigma(A)$. Since $A - \lambda I$ is not invertible, $\lambda \in E(A)$. Conversely, let $\lambda \in E(A)$, i.e., an isolated point of the spectrum of A which is an eigenvalue. Let $P = P_\lambda$ be the spectral projection with respect to λ. Then $H = PH \oplus (I - P)H = H_1 \oplus H_2$ and $\sigma(A \mid_{H_1}) = \{\lambda\}, \sigma(A \mid_{H_2}) = \sigma(A) \setminus \{\lambda\}$. Then Lemma 4.2 implies $(A - \lambda I) \mid_{H_1} = 0$. Hence $A - \lambda I = 0 \oplus (A - \lambda I) \mid_{H_2}$ is invertible, it implies $\lambda \in \sigma(A) \setminus \sigma_{Bw}(A)$. $\square$

Corollary 4.1. (1) *Every algebraically class A operator satisfies generalized Weyl's theorem. In particular Weyl's theorem holds for algebraically class A operators.*

(2) *Every algebraically* log-*hyponormal operator satisfies generalized Weyl's theorem. In particular Weyl's theorem holds for algebraically* log-*hyponormal operators.*

(3) *Every algebraically p-hyponormal operator satisfies generalized Weyl's theorem. In particular Weyl's theorem holds for algebarically p-hyponormal operators.*

(4) *Every algebraically p-quasihyponormal operator satisfies generalized Weyl's theorem. In particular generalized Weyl's theorem holds for p-quasi-hyponormal operators.*

Theorem 4.2. *Let A be an algebraically quasi-class A operator. Then generalized Weyl's theorem for $f(A)$ for every function f analytic on a neighborhood of $\sigma(A)$.*

Proof. Since A is isoloid by Lemma 4.4, has the SVEP and satisfies generalized Weyl's theorem, it follows from ([32, Theorem 2.2]) that $f(A)$ satisfies generalized Weyl's theorem. □

Corollary 4.2. *Let $A \in B(H)$. Then the generalized Weyl's theorem holds for $f(A)$ for every function f analytic in a neighborhood of $\sigma(A)$ under either of the following hypothesis*

(1) *A is algebraically quasi-class A operator.*
(2) *A is algebraically class A operator.*
(3) *A is an algebraically* log*-hyponormal operator.*
(4) *A is an algebraically p-hyponormal operator.*
(5) *A is an algebraically quasihyponormal operator.*
(6) *A is an algebraically p-quasihyponormal operator.*

Theorem 4.3. *Let $A \in B(H)$ be a quasi-class A operator and let $\sigma_w(A) = 0$. Then A is compact and normal.*

Proof. Since Weyl's theorem holds for A by the previous theorem and $\sigma_w(A) = 0$ and since a quasi-class A operator is normaloid, every non zero spectrum of A is an isolated normal eigenvalue with finite dimensional eigenspace, which reduces A. Hence $\sigma(A) \setminus \sigma_w(A)$ is a finite set or a countable infinity set whose accumulation point is only zero. Let $\sigma(A) \setminus \sigma_w(A) = \{\lambda_n\}$ with $|\lambda_1| \geq |\lambda_2| \geq ... \geq 0$ and let E_n be the orthogonal projection onto $\ker(A - \lambda_n)$. Then $AE_n = E_nA = \lambda_n E_n$ and $E_nE_m = 0$ if $n \neq m$. Put $E = \oplus_n E_n$. Then

$$A = \oplus_n \lambda_n E_n \oplus A \mid_{(1-E)H}$$

and $\sigma(A \mid_{(1-E)H}) = \{0\}$. Since $A \mid_{(1-E)H}$ is also a quasi-class operator because EH is a reducing subspace of A, $A \mid_{(1-E)H} = 0$. This implies that $A = \oplus_n \lambda_n E_n$ is normal. The compactness of A follows from the finiteness of the countability of $\{\lambda_n\}_n$ satisfying $|\lambda_n| \downarrow 0$ and each E_n is a finite rank projection. □

Corollary 4.3. *Let $A \in B(H)$. Then*

(1) *Every class A operator with $\sigma_w(A) = 0$ is compact and normal.*

(2) *Every* log-*hyponormal operator with $\sigma_w(A) = 0$ is compact and normal.*

(3) *Every p-hyponormal operator with $\sigma_w(A) = 0$ is compact and normal.*

(4) *Every p-quasihyponormal operator with $\sigma_w(A) = 0$ is compact and normal.*

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