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ON THE POSITIVITY OF THE RICCI CURVATURE, OF THE SECTIONAL CURVATURE AND OF THE CURVATURE OPERATOR

BY

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Abstract. We show that the positivity of the the Ricci curvature Ric of an n-dimensional Riemannian manifold (M, g) is equivalent to the positivity of the higher scalar curvatures $scal_k : M \to \mathbb{R}$, $k = 1, 2, \ldots, n$. Then we introduce higher Ricci curvatures $Ric_k : S(TM) \to \mathbb{R}$, $k = 1, 2, \ldots, n-1$ which are homogeneous symmetric forms of order $2k$ on the tangent unit shere bundle, and show that the positivity of the sectional curvature K is equivalent to the positivity of the Ric_k. At last we show that the curvature operator O is positive definite if and only if certain scalar operator curvature invariants $\sigma_k(O) : M \to \mathbb{R}$ are positive for all $k = 1, 2, \ldots, \binom{n}{2}$.

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Key words: positive curvature, Ricci curvature, scalar curvature, sectional curvature, curvature operator, symmetric function, elementary invariant.