A RULE IN m-DIMENSIONAL BILATERAL OPERATIONAL CALCULUS

BY

R. S. DAHIYA

1. The present paper deals with the operational calculus based on the $m = 2n$-dimensional two sided Laplace transformation.

(1.1) $$f(P_1, P_2, \ldots, P_{2n}) =$$

$$= P_1 P_2 \ldots P_{2n} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \exp\left(-\sum P_i x_i\right) \quad h(x_1, \ldots, x_{2n})\, dx_1 \ldots dx_{2n},$$

where h is the „original'' and f, its „image'' [1]; f being the operational representation of h, expressed symbolically as $f \underset{2n}{\subset} h$ or $h \underset{2n}{\supset} f$.

In this paper I shall obtain a bilateral relation in $2n$-dimensional operational calculus. This relation is next applied to evaluate certain operational results in several variables, which are of great interest and difficult to evaluate otherwise.

2. Rule: Let $F(P) \subset f(t)$, then

(2.1) $$\frac{4 \prod_{r=1}^{2n} P_r F(P_1^2 + \ldots + P_{2n}^2)}{P_1^2 + \ldots + P_{2n}^2} \underset{2n}{\subset} \frac{1}{\pi^n} \int_{-\infty}^{\infty} \exp\left[-\frac{u_1^2 + \ldots + u_{2n}^2}{t}\right] f(t) \frac{dt}{t^n}.$$

Proof: Let us consider

$$I = \frac{1}{\pi^n} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \exp\left(-p_1 u_1 \ldots - p_{2n} u_{2n}\right) \exp\left(-\frac{u_1^2 + \ldots + u_{2n}^2}{t}\right) f(t) \frac{dt}{t^n} =$$

$$= \frac{1}{\pi^n}\int_{-\infty}^{\infty} f(t)\frac{dt}{t^n}\left\{\int_{-\infty}^{\infty}\exp\left(-p_1u_1-\frac{u_1^2}{t}\right)du_1\right\}\dots$$

$$\dots\left\{\int_{-\infty}^{\infty}\exp\left(-p_{2n}u_{2n}-\frac{u_{2n}^2}{t}\right)du_{2n}\right\}=$$

$$=\frac{1}{\pi^n}\int_{-\infty}^{\infty} f(t)\frac{dt}{t^n}\left\{\int_{-\infty}^{\infty}\exp\left(-u_1^2-p_1u_1\sqrt{t}\right)\sqrt{t}\,du_1\right\}\dots(2n\ \mathit{integrals})=$$

$$=\int_{-\infty}^{\infty}\exp\left[(p_1^2+p_2^2+\dots+p_{2n}^2)\frac{t}{4}\right]f(t)dt=$$

$$=\frac{1}{p_1^2+\dots+p_{2n}^2}F\left(\frac{p_1^2+\dots+p_{2n}^2}{4}\right).$$

Therefore,

$$\frac{4\prod\limits_{r=1}^{2n} P_r\, F\left(\frac{p_1^2+\dots+p_{2n}^2}{4}\right)}{p_1^2+\dots\dots\dots\dots+p_{2n}^2}\underset{2n}{\subset}\frac{1}{\pi^n}\int_{-\infty}^{\infty}\exp\left(-\frac{u_1^2+\dots+u_{2n}^2}{t}\right)f(t)\frac{dt}{t^n}.$$

3. **Applications:** 1° Consider $f(t)=t^{\nu-1}e^{-t}\cup(t)\supset\frac{\Gamma(\nu)p}{(p+1)^\nu}\equiv F(p)$

$$-1<R(p)<\infty,\ R(\nu)>0.$$

Hence from (2.1), we get

(3.1)

$$\frac{p_1p_2\cdots p_{2n}}{(p_1^2+p_2^2+\dots+p_{2n}^2+4)^\nu}\underset{2n}{\subset}\frac{(u_1^2+\dots+u_{2n}^2)^{\frac{\nu-n}{2}}}{\pi^n\Gamma(\nu)\,2^{2\nu+1}}K_{n-\nu}\left[2\sqrt{u_1^2+\dots+u_{2n}^2}\right].$$

2° Consider $f(t)=t^{\nu-1}e^{-1/t}\cup(t)\supset 2p^{1-\nu/2}K_\nu(2\sqrt{p})=F(p)$.

Hence from (2.1), we get

$$\frac{p_1p_2\dots p_{2n}}{(p_1^2+p_2^2+\dots+p_{2n}^2)^{\nu/2}}K_\nu\left(\sqrt{p_1^2+p_2^2+\dots+p_{2n}^2}\right)\underset{2n}{\subset}\frac{\Gamma(n-\nu)}{2^{\nu+1}\pi^n}\times$$

$$\times(u_1^2+u_2^2+\dots+u_{2n}^2+1)^{\nu-n},\ R(n-\nu)>0.$$

4. **Particular case:** In particular, if $n=1$, we have

(4.1)
$$\frac{4p_1p_2}{p_1^2+p_2^2}F\frac{(p_1^2+p_2^2)}{4}\underset{2}{\subset}\frac{1}{\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{u^2+v^2}{t}\right)f(t)\frac{dt}{t}.$$

If we take $f(t)=J_n(t)\cup(t)$ in (4.1), we get

(4.2)
$$\frac{p_1p_2}{\sqrt{(p_1^2+p_2^2)^2+16}}\left[\sqrt{p_1^2+p_2^2+16}-(p_1^2+p_2^2)\right]^n\underset{2}{\subset}$$
$$\underset{2}{\subset}\frac{2^{2n-1}}{\pi}J_n\left[\sqrt{2(u^2+v^2)}\right]K_n\left[\sqrt{2(u^2+v^2)}\right].$$

REFERENCES

1. Vanderpol, B. and Bremmer, H. — *Operational Calculus Based on Two Sided Laplace Integral.* (1955).

Department of Mathematics Iowa State University

O FORMULĂ DIN CALCULUL OPERAŢIONAL BILATERAL ÎN m DIMENSIUNI

Rezumat

Pornind de la (1.1), se stabileşte formula (2.1), pentru care se dau diverse cazuri particulare.