ON GENERALIZED ALGEBRAS

BY

MIRELA BRÂNZEI

This paper is concerned with some properties of the generalized algebras; it is an account of the basic results of the homomorphisms and the normal chains of generalized algebras. We use the notations from the lattice theory [1].

§ 1. Definition 1. *A generalized algebra (genalgebra) is a system* $\mathcal{G} = < G, \Omega, A >$, *composed of two sets G and A, and a family* $\Omega = \{o_i \mid i \in I\}$ *(finite or not) of finitary operations, which are mappings:*

$$o_i : G^{m_i} \to A. \tag{1}$$

Here, m_i is the arity of o_i and the collection $\{m_i \mid i \in I,\ m_i \in N\}$ is the type of the genalgebra $\mathcal{G}$. F. M. Sioson has introduced in [3] this concept, in relation to the decompositions in direct and subdirect products.

$\mathcal{H} = < H, \Omega, B >$ is a *subgenalgebra* of $\mathcal{G}$, if we have $H \subseteq G$, $B \subseteq A$, and for every $i \in I$ and every $x_j \in H\ (j = 1, 2, \ldots, m_i)$, we have

$$o_i\,(x_1, x_2, \ldots, x_{m_i}) \in B.$$

The last condition takes the form:

$$\bigcup_{i \in I} o_i\,(H, H, \ldots, H) \subseteq B. \tag{2}$$

Definition 2. *Let* $\mathcal{G} = < G, \Omega, A >$ *and* $\mathcal{G}' = < G', \Omega' A' >$ *be two genalgebras of the same type. A pair of functions* $h : G \to G'$, $k : A \to A'$ *is a homomorphism between* $\mathcal{G}$ *and* $\mathcal{G}'$, *if for any* $i \in I$ *and any* $x_j \in G$, $j = 1, 2, \ldots, m_i$, *the following condition is satisfied:*

$$k\,(o_i(x_1, \ldots, x_{m_i})) = o_i'(h\,(x_1), \ldots, h\,(x_{m_i})). \tag{3}$$

If h and k are one-to-one, then (h, k) is an *isomorphism*.

Definition 3. *A congruence on the genalgebra $\mathcal{G}$ is a pair (θ, φ) of equivalence relations, θ on G, φ on A, so that for all $i \in I$, if $(x_j, y_j) \in \theta$, where $j = 1, 2, \ldots, m_i$, then:*

$$(o_i(x_1, \ldots, x_{m_i}),\ o_i(y_1, \ldots, y_{m_i})) \in \varphi. \tag{4}$$

Let (h, k) be a homomorphism between the genalgebras $\mathcal{G}$ and $\mathcal{G}'$ Then we can easily show that (θ, φ), where $\theta = h \circ h^{-1}$, $\varphi = k \circ k^{-1}$ is a congruence on $\mathcal{G}$, called the *kernel* of (h, k). Here we consider that if $h = \{(x, x') \mid x' = h(x),\ x \in G,\ x' \in G'\}$, then $h^{-1} = \{(x', x) \mid (x, x') \in h\}$ and $h \circ h^{-1} = \{(x_1, x_2) \mid (\exists x')(x' \in G'),\ (x_1, x') \in h \wedge (x', x_2) \in h^{-1}\}$ and the same for k. It is clear that $(x_1, x_2) \in h \circ h^{-1}$ if and only if $h(x_1) = h(x_2)$.

Thus, a homomorphism of $\mathcal{G}$ defines a congruence on $\mathcal{G}$, its kernel; conversely, if (θ, φ) is a congruence on $\mathcal{G}$, then there exists an unique genalgebra, $\mathcal{G}/(\theta, \varphi) = < G/\theta, \widetilde{\Omega}, A/\varphi >$ called *quotient genalgebra of $\mathcal{G}$ by* (θ, φ), so that the natural mapping nat $(\theta, \varphi) : \mathcal{G} \to \mathcal{G}/(\theta, \varphi)$ is a homomorphism. We put for all $\widetilde{o}_i \in \widetilde{\Omega}$:

$$\widetilde{o}_i(x_1/\theta, \ldots, x_{m_i}/\theta) = o_i(x_1, \ldots, x_{m_i})/\varphi, \tag{5}$$

where $o_i \in \Omega$, x_j/θ is a class modulo θ, all $i \in I$, $x_j \in G$, $j = 1, 2, \ldots, m_i$. The homomorphism $(h, k) = \text{nat}\ (\theta, \varphi)$ is defined by

$$\begin{cases} h(x) = x/\theta, & \text{all } x \in G, \\ k(a) = a/\varphi, & \text{all } a \in A \end{cases} \tag{6}$$

and ker (h, k) is the congruence (θ, φ).

We can now prove the isomorphism theorems of the genalgebras. First, we have a theorem about the induced homomorphism.

Theorem 1. *Let (h_1, k_1) and (h_2, k_2) be homomorphisms of a genalgebra, $\mathcal{G} = < G, \Omega, A >$ upon the genalgebras $\mathcal{G}_1 = < G_1, \Omega_1, A_1 >$ and $\mathcal{G}_2 = < G_2, \Omega_2, A_2 >$ respectively, so that* ker $(h_1, k_1) \subseteq$ ker (h_2, k_2). *Then there exists an unique homomorphism (f, g) of $\mathcal{G}_1$ upon $\mathcal{G}_2$ so that $(f, g) \circ (h_1, k_1) = (h_2, k_2)$.*

Proof. Let x and a be elements of G and A respectively, so that $h_1(x) = x_1$, $k_1(a) = a_1$, $x_1 \in G_1$, $a_1 \in A_1$. We define $f(x_1) = h_2(x)$ and $g(a_1) = k_2(a)$. These mappings are single-valued, for if $h_1(y) = x_1$, $y \in G$, or $k_1(b) = a_1$, $b \in A$, we have $(x, y) \in \ker h$ and $(a, b) \in \ker k$, so that $h_2(x) = h_2(y)$ and $k_2(b) = k_2(a)$. It is clear, by definition, that $(f, g) \circ (h_1, k_1) = (h_2, k_2)$ and the assertion that (f, g) is a homomorphism is deduced from the following calculus:

$$\begin{aligned} g(o_i(h_1(x_1), h_1(x_2), \ldots, h_1(x_{m_i}))) &= g(k_1(o_i(x_1, \ldots, x_{m_i}))) = \\ = (g \circ k_1)(o_i(x_1, \ldots, x_{m_i})) &= k_2(o_i(x_1, \ldots, x_{m_i})) = o_i(h_2(x_1), \ldots, h_2(x_{m_i})) \\ = o_i((f \circ h_1)(x_1), \ldots, (f \circ h_1)(x_{m_i})) &= o_i(f(h_1(x_1)), \ldots, f(h_1(x_{m_i}))) \end{aligned}$$

where o_i is an arbitrary element of Ω.

The uniqueness of (f, g) is obvious; for, if (f, g) is to satisfy the relation $(f, g) \circ (h_1, k_1) = (h_2, k_2)$, we are compelled to define (f, g) as above.

Theorem 2. *Let (h, k) be a homomorphism between $\mathcal{G}_1$ and $\mathcal{G}_2$, with kernel (θ, φ). Then there exists a decomposition of (h, k) in a product of three homomorphisms:*

$$(h, k) = (i_1, i_2) \circ (h', k') \circ \text{nat}\ (\theta, \varphi). \tag{7}$$

where $(i_1, i_2) : \text{Im}\ (h, k) \to \mathcal{G}_2$ is the inclusion mapping, nat $(\theta, \varphi) : \mathcal{G}_1 \to \mathcal{G}_1/(\theta, \varphi)$ *is the natural homomorphism and $(h', k') : \mathcal{G}_1/(\theta, \varphi) \to \text{Im}\ (h, k)$ is an isomorphism.*

Proof. The relation (7) may be written as a diagram:

$$\begin{array}{ccc} \mathcal{G}_1 & \xrightarrow{(h,\, k)} & \mathcal{G}_2 \\ \text{nat}\ (\theta, \varphi) \downarrow & & \uparrow (i_1, i_2) \\ \mathcal{G}_1/(\theta, \varphi) & \xrightarrow{(h',\, k')} & \text{Im}\ (h, k) \end{array} \tag{7'}$$

We must define the homomorphism (h', k') and show that the above diagram is commutative. We have then $h' : G_1/\theta \to \text{Im}\ h$, $h'(x/\theta) = h(x) \in \text{Im}\ h$ and $k' : A_1/\varphi \to \text{Im}\ k$, $k'(a/\varphi) = k(a) \in \text{Im}\ k$, and these definitions depend only upon the classes modulo θ and φ, respectively. Indeed, if $x' \in x/\theta$, this means that $h(x') = h(x)$ and that $h'(x'/\theta) = h'(x/\theta)$, and the same happens for k. One can easily show that the mapping (h', k') is a isomorphism.

Corollary 1. *Any homomorphism of genalgebras $(h, k) : \mathcal{G} \to \mathcal{G}'$, can be factored uniquely by* nat (θ, φ):

$$(h, k) = (\bar{h}, \bar{k}) \circ \text{nat}\ (\theta, \varphi) \tag{8}$$

where (θ, φ) is a congruence on $\mathcal{G}$ which is contained in the kernel of (h, k).

Proof. We must define $(\bar{h}, \bar{k})$ so that the diagram (8′) be commutative:

$$\begin{array}{ccc} \mathcal{G} & \xrightarrow{\text{nat}\ (\theta, \varphi)} & \mathcal{G}/(\theta, \varphi) \\ & {\scriptstyle (h,\, k)} \searrow \quad \swarrow {\scriptstyle \bar{h},\, \bar{k}} & \\ & \mathcal{G}' & \end{array} \tag{8'}$$

We may take $(\bar{h}, \bar{k})$ so that: $\bar{h}(x/\theta) = h(x)$ and $\bar{k}(a/\varphi) = k(a)$ and this is a homomorphism which depends only on the classes modulo θ and φ. This can be deduced from the definition of the kernel of (h, k) and from the hypotheses that $(\theta, \varphi) \subseteq$ ker (h, k).

Theorem 3. *Let $\mathcal{G}$ be a genalgebra, $\mathcal{G}_1 = < G_1, \Omega, A_1 >$ subgenalgebra of $\mathcal{G}$ and (θ, φ) a congruence on $\mathcal{G}$. Then $\mathcal{G}_1^{(\theta, \varphi)}$ is a subgenalgebra of $\mathcal{G}$, $(\theta_1, \varphi_1) = (\theta, \varphi) \cap \mathcal{G}_1^2$ is a congruence on $\mathcal{G}_1$ and*

$$\mathcal{G}_1/(\theta_1, \varphi_1) = \mathcal{G}_1^{(\theta, \varphi)}/(\theta, \varphi). \tag{9}$$

Proof. If $(\varepsilon_1, \varepsilon_2)$ is equal to nat $(\theta, \varphi) : \mathcal{G} \to \mathcal{G}/(\theta, \varphi)$, then its restriction to $\mathcal{G}_1$, $(\varepsilon_1, \varepsilon_2)$, is a homomorphism $\mathcal{G}_1 \to \mathcal{G}/(\theta, \varphi)$ and its image is

$\mathfrak{G}_1^{(0,\varphi)}/(0, \varphi)$. It is therefore a subgenalgebra of $\mathfrak{G}/(0, \varphi)$. In order to find out the kernel of $(\varepsilon_1', \varepsilon_1')$, we intersect the kernel of $(\varepsilon_1, \varepsilon_2)$ with $\mathfrak{G}_1^2$. Hence this kernel is just $(0_1, \varphi_1)$. Theorem 3, leads to:

Definition 4. *If a congruence $(0, \varphi)$ on the genalgebra $\mathfrak{G}$ satisfies the relation:*

$$(0, \varphi) \cap \mathfrak{G}_1^2 = \Delta_{\mathfrak{G}_1} = (\Delta_{G_1}, \Delta_{A_1}). \tag{10}$$

where $\mathfrak{G}_1$ is a subgenalgebra of $\mathfrak{G}$, it is said that $(0, \varphi)$ is a separating congruence on $\mathfrak{G}_1$, or that $(0, \varphi)$ separates $\mathfrak{G}_1$.

Corollary 2. *If $(0, \varphi)$ separates $\mathfrak{G}_1$, then*

$$\mathfrak{G}_1 \simeq \mathfrak{G}_1^{(0,\varphi)}/(0, \varphi). \tag{11}$$

Indeed, $\mathfrak{G}_1/\Delta_{\mathfrak{G}_1} \simeq \mathfrak{G}_1$, and we can write the relation (9) in this case.

Theorem 3 is the second isomorphism theorem for genalgebras; it states that any subgenalgebras of a quotient genalgebra of $\mathfrak{G}$ is isomorphic to a factor of $\mathfrak{G}$ (i.e. a quotient of a subgenalgebra of $\mathfrak{G}$). The converse is not true; we have some examples of this in the group theory.

In the same way, we can obtain the third isomorphism theorem for genalgebras.

§ 2. Now, we are going to study some special properties of the set of congruences on a genalgebra $\mathfrak{G}$. F. M. Sioson has shown in his paper [3], that this set is a lattice $\mathfrak{L}(\mathfrak{G})$, which is a complete sublattice of the direct product $\mathfrak{L}(G) \times \mathfrak{L}(A)$. Here $\mathfrak{L}(G)$ and $\mathfrak{L}(A)$ are the lattices of equivalences on G and A. He has also shown that for any congruence (θ, φ) on a genalgebra $\mathfrak{G}$, there is a lattice-isomorphism, f, between $\mathfrak{L}(\mathfrak{G}/(\theta, \varphi))$ and the lattice $\mathfrak{L}(\mathfrak{G}(\theta, \varphi))$ of all congruences (ζ, η) on $\mathfrak{G}$, which contain $(0, \varphi)$. Moreover, if $f(\zeta, \eta) = (\rho, \sigma)$, then:

$$\mathfrak{G}/(\zeta, \eta) \simeq (\mathfrak{G}/(\theta, \varphi))/(\rho, \sigma). \tag{12}$$

For any equivalences on $\mathfrak{G}$, (θ_1, φ_1) and (θ_2, φ_2), we define a composition:

$$(\theta_1 \circ \theta_2, \varphi_1 \circ \varphi_2) = \{((x_1, x_2), (a_1, a_2)) \mid x_i \in G, a_i \in A \text{ there exists } x \in G, \tag{13}$$
$$a \in A \text{ so that } (x_1, x) \in \theta_1 \wedge (x, x_2) \in \theta_2, (a_1, a) \in \varphi_1 \wedge (a, a_2) \in \varphi_2\}$$

We may ask when this composition is also an equivalence on $\mathfrak{G}$. The answer is given by:

Theorem 4. *The composition of two equivalences (congruences) on a genalgebra $\mathfrak{G}$ is also an equivalence (congruence) on $\mathfrak{G}$, if and only if these equivalences (congruences) commute:*

$$(\theta_1, \varphi_1) \circ (\theta_2, \varphi_2) = (\theta_2, \varphi_2) \circ (\theta_1, \varphi_1). \tag{14}$$

In this case,

$$(\theta_1, \varphi_1) \circ (\theta_2, \varphi_2) = (\theta_1, \varphi_1) \vee (\theta_2, \varphi_2). \tag{15}$$

Proof. Evidently, we have for any equivalences: $\theta_1 \cup \theta_2 \subseteq \theta_1 \circ \theta_2 \subseteq \subseteq \theta_1 \vee \theta_2$, so that, if $\theta_1 \circ \theta_2$ is an equivalence, we obtain $\theta_1 \circ \theta_2 = \theta_1 \cup \theta_2$ and $\theta_1 \circ \theta_2 = (\theta_1 \circ \theta_2)^{-1} = \theta_2^{-1} \circ \theta_1^{-1} = \theta_2 \circ \theta_1$.

Conversely, if $\theta_1 \circ \theta_2 = \theta_2 \circ \theta_1$ one can verify easily that $\theta_1 \circ \theta_2$ is an equivalence. Similarly, $\varphi_1 \circ \varphi_2$ is an equivalence. We have also: $(\theta_1, \varphi_1) \cup \cup (\theta_2, \varphi_2) \subseteq (\theta_1, \varphi_1) \circ (\theta_2, \varphi_2) \subseteq (\theta_1, \varphi_1) \vee (\theta_2, \varphi_2)$.

One may find the definition for the operation „$\vee$" between two relations in the book of Cohn ([2], p. 89).

Theorem 5. *If all congruences on the genalgebra $\mathfrak{G}$ commute, then the lattice $\mathfrak{L}(\mathfrak{G})$ is modular.*

Proof. We must show that, if $(\theta_2, \varphi_2) \subseteq (\theta_3, \varphi_3)$, then fore very (θ_1, φ_1) we have: $[(\theta_1, \varphi_1) \circ (\theta_2, \varphi_2)] \cap (\theta_3, \varphi_3) \subseteq [(\theta_1, \varphi_1) \cap (\theta_3, \varphi_3)] \circ (\theta_2, \varphi_2)$. Let $((x_1, x_2), (a_1, a_2))$ be in $[(\theta_1, \varphi_1) \circ (\theta_2, \varphi_2)] \cap (\theta_3, \varphi_3)$; then there are two elements $x \in G$ and $a \in A$, so that $((x_1, x), (a_1, a)) \in (\theta_1, \varphi_1)$ and $((x, x_2), (a, a_2)) \in (\theta_2, \varphi_2)$; that is $(x_1, x) \in \theta_1$, $(a_1, a) \in \varphi_1$, $(x, x_2) \in \theta_2$, $(a, a_2) \in \in \varphi_2$. But $(\theta_2, \varphi_2) \subseteq (\theta_3, \varphi_3)$; therefore $(x_1, x) \in \theta_1 \cap \theta_3$ and $(a_1, a) \in \in \varphi_1 \cap \varphi_3$, hence $((x_1, x_2), (a_1, a_2)) \in [(\theta_1, \varphi_1) \cap (\theta_3, \varphi_3)] \circ (\theta_2, \varphi_2)$.

By using the commutativity of congruences and the theorems 4 and 5, we can give an extension of the second isomorphism theorem, which generalizes the known Zassenhaus lemma:

Theorem 6. *Let $\mathfrak{G}$ be a genalgebra, $\mathfrak{G}_i (i = 1, 2)$ a subgenalgebra of $\mathfrak{G}$, (θ_i, φ_i) $(i = 1, 2)$ a congruence on $\mathfrak{G}_i$. If all congruences on $\mathfrak{B} = \mathfrak{G}_1 \cap \mathfrak{G}_2$ commute, then $(\alpha_i, \beta_i) = (\theta_i, \varphi_i) \circ (\theta_j, \varphi_j) \circ (\theta_i, \varphi_i)$ is an element of $\mathfrak{L}(\mathfrak{B}^{(\theta_i, \varphi_i)})$ or $i \neq j$, $i, j = 1, 2$ and there exists an isomorphism:*

$$\mathfrak{B}^{(\theta_1, \varphi_1)}/(\alpha_1, \beta_1) \simeq \mathfrak{B}^{(\theta_2, \varphi_2)}/(\alpha_2, \beta_2). \tag{15}$$

Proof. We put $(\theta_i', \varphi_i') = (\theta_i, \varphi_i) \cap \mathfrak{G}_j^2$, $i \neq j$, $i, j = 1, 2$. Then (α_i, β_i) is equal to $(\theta_i, \varphi_i) \circ \theta_j', \varphi_j') \circ (\theta_i, \varphi_i)$ and it is transitive, as we have: $(\alpha_i, \beta_i) \circ (\alpha_i, \beta_i) = (\alpha_i, \beta_i)$. We use (θ_i', φ_i') as congruences on $\mathfrak{B}$, hence they commute. Clearly, (α_i, β_i) is also reflexive and symmetrical. Moreover, (α_i, β_i) admits the operator domain Ω, thus (α_i, β_i) is a congruence on $\mathfrak{G}_i (i = 1, 2)$ and, similarly, $(\theta_i', \varphi_i') \circ (\theta_2', \varphi_2')$ is a congruence on $\mathfrak{B}$. One can immediately verify that:

$$(\theta_i', \varphi_i') \circ (\theta_j', \varphi_j') = (\alpha_i, \beta_i) \cap \mathfrak{B}^2,$$

since the lattice $\mathfrak{L}(\mathfrak{B})$ is modular (theorem 4).

From the theorem 2, we have

$$\mathfrak{B}^{(\theta_i, \varphi_i)}/(\alpha_i, \beta_i) \simeq \mathfrak{B}/(\theta_i', \varphi_i') \circ (\theta_j', \varphi_j'), \quad i \neq j.$$

But $(\theta_1', \varphi_1') \circ (\theta_2', \varphi_2') = (\theta_2', \varphi_2') \circ (\theta_1', \varphi_1')$ as congruences on $\mathfrak{B}$, hence we have (15).

§ 3. Now we are able to make some deductions from the isomorphism theorems, constituting a generalization of the Schreier and Jordan-Hölder theorems in the case of genalgebras.

Definition 5. *Let $\mathcal{G}$ be a genalgebra and $\mathcal{H}$ be a subgenalgebra of $\mathcal{G}$. A normal chain from $\mathcal{H}$ to $\mathcal{G}$ is a finite chain of subgenalgebras of $\mathcal{G}$:*

$$\mathcal{H} = \mathcal{G}_0 \subseteq \mathcal{G}_1 \subseteq \ldots \subseteq \mathcal{G}_m = \mathcal{G}, \tag{16}$$

with the congruences $(0_i, \varphi_i)$ on $\mathcal{G}_i$, so that $\mathcal{G}_{i-1}$ coincides with a $(0_i, \varphi_i)$-class.

(17) If
$$\mathcal{H} = \mathcal{H}_0 \subseteq \mathcal{H}_1 \subseteq \ldots \subseteq \mathcal{H}_n = \mathcal{G}$$

with the congruences (λ_i, μ_i) is another normal chain, we say that these chains are *isomorphic*, if $m = n$ and if there exists a permutation π of $1, 2, \ldots, n$ so that

$$\mathcal{G}_i/(0_i, \varphi_i) \simeq \mathcal{H}_{\pi(i)}/(\lambda_{\pi(i)}, \mu_{\pi(i)}). \tag{18}$$

Definition 6. *We call refinement of the normal chain (16) a new normal chain in which each interval $\mathcal{G}_{i-1}$, $\mathcal{G}_i$ is replaced by a normal chain from $\mathcal{G}_{i-1}$ to $\mathcal{G}_i$.*

Theorem 7. *If $\mathcal{G}$ is a genalgebra with a subgenalgebra $\mathcal{H}$ so that all congruences on any subgenalgebra of $\mathcal{G}$ commute, then every two normal chains from $\mathcal{H}$ to $\mathcal{G}$ have isomorphic refinements.*

Proof. In our proof, we use the refinements whose factors for the chains (16) and (17) are respectively:

$$\mathfrak{M}_{ij} = (\mathcal{G}_i \cap \mathcal{H}_j)^{(0_i, \varphi_i)}/(0_i, \varphi_i) \circ (\lambda_j, \mu_j) \circ (0_i, \varphi_i) \tag{19}$$
$$\text{for } i = 1, 2, \ldots, m$$

and

$$\mathfrak{K}_{ij} = (\mathcal{G}_i \cap \mathcal{H}_j)^{(\lambda_j, \mu_j)}/(\lambda_j, \mu_j) \circ (0_i, \varphi_i) \circ (\lambda_j, \mu_j) \tag{20}$$
$$\text{for } j = 1, 2, \ldots, n.$$

From the theorem 6, we have $\mathfrak{M}_{ij} \simeq \mathfrak{K}_{ij}$. Similarly as in the proof of Schreier refinement theorem for the lattices ([2], p. 70), one shows that the chain (16) can be refined to a normal chain with factors $\mathfrak{M}_{11}, \mathfrak{M}_{12}, \ldots, \mathfrak{M}_{1n}$ $\mathfrak{M}_{21}, \ldots, \mathfrak{M}_{mn}$, and the same for (17) and $\mathfrak{K}_{ij}$. We have obtained in this way two isomorphic refinements for the given normal chains (16) and (17).

Definition 7. *If a normal chain (16) has no proper refinement, it is called a composition series from $\mathcal{H}$ to $\mathcal{G}$.*

Clearly, a normal chain is a composition series if and only if all its factors have no proper congruence (that is, its factors are simple genalgebras).

With the same condition for $\mathcal{G}$ and $\mathcal{H}$, using the theorem 7, we obtain:

Corollary 4. *Any two composition series from $\mathcal{H}$ to $\mathcal{G}$ are isomorphic,* and

Corollary 5. *If there exists a composition series from $\mathcal{H}$ to $\mathcal{G}$, then any normal chain from $\mathcal{H}$ to $\mathcal{G}$ can be refined to a composition series.*

These theorems can be used in the representation theory of a genalgebra $\mathcal{G}$ as a finite direct or subdirect product of its subgenalgebras. We have also found a simple proof for some theorems proved by S i o s o n [3].

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ASUPRA ALGEBRELOR GENERALIZATE

Rezumat

În articol se stabilesc cîteva proprietăți ale algebrelor generalizate, introduse de S i o s o n [3]. Sînt demonstrate teoremele de omomorfism și izomorfism ale algebrelor generalizate, precum și teoreme asupra lanțurilor normale de algebre generalizate.