8. Shah Manilal — *Certain integral involving the product of two generalized hypergeometric polynomials.* Proc. Nat. Acad. Sci., India, 37 A (I), (1967), pp. 79—96. MR 39—4454.

UN REZULTAT PRIVITOR LA FUNCŢII HIPERGEOMETRICE GENERALIZATE ŞI FUNCŢII MEIJER GENERALIZATE DE DOUĂ VARIABILE

Rezumat

Autorul evaluează unele integrale de tipul (1.1), prin funcţii de tip Meijer şi hipergeometrice generalizate. Cu ajutorul acestora, prin particularizarea parametrilor, se obţin diferite formule utile.

MEIJER-LAPLACE TRANSFORM OF TWO VARIABLES

BY

R. S. DAHIYA

1. Several generalizations of the classical Laplace-Stieltjes integra- have been given by Meijer [1], Bose [2], Varma [4, 5].

In the present paper I shall develop the theory of the Meijer-Laplace transform on parallel lines to that of Laplace transform of two variables. The Meijer-Laplace transform is defined as follows:

$$\Phi(s) = s\int_0^\infty e^{-wst}\, G^{mn}_{pq}\left(2st \,\middle|\, \begin{matrix} a_1, a_2, \dots, a_p \\ b_1, b_2, \dots, b_q \end{matrix}\right) d\alpha(t). \tag{1.1}$$

We shall use the short hand notation for the G-function

$$G^{mn}_{pq}\left(2st \,\middle|\, \begin{matrix} a_p \\ b_q \end{matrix}\right),$$

when no ambiguity can arise.

By taking $w = 1$ and with the help of the following identities, we can deduce the corresponding results for the transforms of Bose [2] and Varma [4] by giving particular values to the parameters of the G-function.

(a) $$e^{-st}\, G^{21}_{12}\left(2st \,\middle|\, \begin{matrix} 1 \\ 1/2 - \nu,\ 1/2 + \nu \end{matrix}\right) \equiv \Gamma(1/2 \pm \nu)\left(\frac{2st}{\pi}\right)^{1/2} K_\nu(st),$$

and

(b) $$e^{-st}\, G^{21}_{12}\left(2st \,\middle|\, \begin{matrix} K + 3/4 \\ 1/4 - \mu,\ \mu + 1/4 \end{matrix}\right) \equiv \Gamma(1/2 \pm \mu - K)(2st)^{-1/4}\, W_{K,\mu}(2st).$$

Again by taking $w=0$, we can deduce the corresponding results for transforms of Laplace-Stieltjes, Varma [5] and Meijer [1] with add the help of the following identities:

(c) $G_{12}^{20}\left(st\,\middle|\,\begin{matrix}\alpha\\ 0,\ \alpha\end{matrix}\right)\equiv e^{-st}$, (d $G_{12}^{20}\left(st\,\middle|\,\begin{matrix}1/2+\mu-K\\ 0,\qquad 2\mu\end{matrix}\right)\equiv (st)^{\mu-1/2}\,e^{-st/2}\,W_{K,\mu}(st)$

and

(e) $$G_{12}^{20}\left(st\,\middle|\,\begin{matrix}-2K\\ -\mu-K,\ \mu-K\end{matrix}\right)\equiv (st)^{-K-1/2}\,e^{-st/2}\,W_{K+1/2,\mu}(st).$$

We define the Meijer-Laplace transform of two variables as

(1.2) $$\Phi(p,q)=pq\int_0^\infty\int_0^\infty e^{-w(px+qy)}\,G_{p_1q_1}^{m_1n_1}\left(2px\,\middle|\,\begin{matrix}a_{p_1}\\ b_{q_1}\end{matrix}\right)G_{p_2q_2}^{m_2n_2}\left(2qy\,\middle|\,\begin{matrix}a_{p_2}\\ b_{q_2}\end{matrix}\right)f(x,y)\,dx\,dy,$$
$$R(p,q)>0,$$

which is the generalization of the well-known Laplace transform of two variables:

(1.3) $$\Phi(p,q)=pq\int_0^\infty\int_0^\infty e^{-px-qy}f(x,y)\,dx\,dy,\qquad R(p,q)>0$$

and as well as other transforms as shown above provided they are defined on parallel lines in two variables. (1.2) reduces to (1.3) by virtue of (c) when $w=0$. The rel tions (1.2) and (1.3) are symbolically denoted as

$$\Phi(p,q)\ \underset{q_1,\,q_2}{\overset{p_1,\,p_2}{\overleftarrow{\underrightarrow{\ w\ }}}}\ f(x,y),$$

and $\Phi(p,q)\ \overleftarrow{\underrightarrow{\ \ }}\ f(x,y)$ respectively.

2. Theorem 1. *If*

$$\Phi_1(p,q)\ \underset{q_1,\,q_2}{\overset{p_1,\,p_2}{\overleftarrow{\underrightarrow{\ w\ }}}}\ f_1(x,y)\quad\text{and}\quad \Phi_2(p,q)\ \underset{q_1,\,q_2}{\overset{p_1,\,p_2}{\overleftarrow{\underrightarrow{\ w\ }}}}\ f_2(x,y),$$

then

(2.1.1) $$\int_0^\infty\int_0^\infty \Phi_1(u,v)\,f_2(u,v)\,\frac{du}{u}\,\frac{dv}{v}=\int_0^\infty\int_0^\infty \Phi_2(s,t)\,f_1(s,t)\,\frac{ds}{s}\,\frac{dt}{t},$$

provided that the integrals involved are absolutely convergent.

Proof. We have

$$\int_0^\infty\int_0^\infty \Phi_1(u,v)\,f_2(u,v)\,\frac{du}{u}\,\frac{dv}{v}=$$

$$=\int_0^\infty\int_0^\infty\left\{uv\int_0^\infty\int_0^\infty e^{-w(us+vt)}\,G_{p_1q_1}^{m_1n_1}\left(2us\,\middle|\,\begin{matrix}a_{p_1}\\ b_{q_1}\end{matrix}\right)G_{p_2q_2}^{m_2n_2}\left(2vt\,\middle|\,\begin{matrix}a_{p_2}\\ b_{q_2}\end{matrix}\right)f_1(s,t)\,ds\,dt\right\}\times$$

$$\times f_2(u,v)\,\frac{du}{u}\,\frac{dv}{v}=\int_0^\infty\int_0^\infty f_1(s,t)\left\{st\int_0^\infty\int_0^\infty e^{-w(us+vt)}\,G_{p_1q_1}^{m_1n_1}\left(2us\,\middle|\,\begin{matrix}a_{p_1}\\ b_{q_1}\end{matrix}\right)\times\right.$$

$$\left.\times\, G_{p_2q_2}^{m_2n_2}\left(2vt\,\middle|\,\begin{matrix}a_{p_2}\\ b_{q_2}\end{matrix}\right)f_2(u,v)\,du\,dv\right\}\frac{ds}{s}\,\frac{dt}{t}=\int_0^\infty\int_0^\infty f_1(s,t)\,\Phi_2(s,t)\,\frac{ds}{s}\,\frac{dt}{t}.$$

The change of order of integration is justified if the double integrals in brackets above are absolutely and uniformly convergent. For this $R(n_1,\pm$ $\pm\frac{p_1}{q_1}+1)>0$, $R(n_2,\pm\frac{p_2}{q_2}+1)>0$, where

$$f_1(s,t)=\begin{cases}O(s^{n_1}) & \text{for small } s\\ O(t^{n_2}) & \text{for small } t\end{cases}\quad\text{and}\quad \begin{matrix}R\left(n_3\pm\dfrac{p_1}{q_1}+1\right)>0,\\[2ex] R\left(n_4\pm\dfrac{p_2}{q_2}+1\right)>0,\end{matrix}$$

$$f_2(u,v)=\begin{cases}O(u^{n_3}) & \text{for small } u\\ O(v^{n_4}) & \text{for small } v\end{cases}.$$

Theorem 2. *If* $f(x,y)=f_1(x)f_2(y)$ *and if*

$$\Phi_1(p)\ \underset{q_1}{\overset{p_1}{\overleftarrow{\underrightarrow{\ w\ }}}}\ f_1(x),\qquad \Phi_2(q)\ \underset{q_2}{\overset{p_2}{\overleftarrow{\underrightarrow{\ w\ }}}}\ f_2(y)\ \textit{then}$$

(2.2.1) $$f(x,y)\ \underset{q_1,\,q_2}{\overset{p_1,\,p_2}{\overleftarrow{\underrightarrow{\ w\ }}}}\ \Phi(p,q)=\Phi_1(p)\,\Phi_2(q),$$

provided that the integrals are absolutely convergent.

Proof: We have

$$\Phi(p, q) = pq \int_0^\infty \int_0^\infty e^{-w(px+qy)} G^{m_1, n_1}_{p_1, q_1}\left(2px \,\middle|\, \begin{matrix} a_{p_1} \\ b_{q_1} \end{matrix}\right) G^{m_2, n_2}_{p_2, q_2}\left(2qy \,\middle|\, \begin{matrix} a_{p_2} \\ b_{q_2} \end{matrix}\right) f_1(x) f_2(y) dx dy =$$

$$= \left\{ p \int_0^\infty e^{-wpx} G^{m_1, n_1}_{p_1, q_1}\left(2px \,\middle|\, \begin{matrix} a_{p_1} \\ b_{q_1} \end{matrix}\right) f_1(x)\, dx \right\} \left\{ q \int_0^\infty e^{-wqy} G^{m_2, n_2}_{p_2, q_2}\left(2qy \,\middle|\, \begin{matrix} a_{p_2} \\ b_{q_2} \end{matrix}\right) f_2(y)\, dy \right\} =$$

$$= \Phi_1(p)\, \Phi_2(q).$$

It can be shown that if

$$\Phi_1[p, q] \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} f_1(x, y),$$

$$\Phi_2[p, q] \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} f_2(x, y),$$

. . . .

. . . .

then

(2.2.2) $$\sum_{r=1}^{K} \lambda_r \Phi_r[p, q] \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} \sum_{r=1}^{K} \lambda_r f_r(x, y).$$

Theorem 3. *If* $\Phi(p, q) \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} f(x, y)$ *then*

(2.3.1) $$\Phi\left(\frac{p}{a}, \frac{q}{b}\right) \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} f(ax, by)$$

provided the integral converges absolutely.

The above theorem may be called *Similitude theorem* for the Meijer-Laplace transform of two variables.

Theorem 4. *If* $\Phi(p, q) \doteqdot f(x, y)$ *then*

$$\frac{\Phi(\log p, \log q)}{\log p \log q} \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{w}}}} \int_0^\infty \int_0^\infty (wx)^{s-1} (wy)^{t-1} G^{m_1, n_1+1}_{p_1+1, q_1}\left(\frac{2}{w} \,\middle|\, \begin{matrix} s, (a_{p_1}) \\ (b_{q_1}) \end{matrix}\right) \times$$

$$\times G^{m_2, n_2+1}_{p_2+1, q_2}\left(\frac{2}{w} \,\middle|\, \begin{matrix} t, (a_{p_2}) \\ (b_{q_2}) \end{matrix}\right) f(s, t) ds\, dt,$$

provided the integrals converge absolutely.

Theorem 5. *If* $\Phi(p, q) \underset{q_1, q_2}{\overset{p_1, p_2}{\underset{\longrightarrow}{\overset{\longleftarrow}{}}}} f(x, y)$ and

$$p^{-\lambda_1} q^{-\lambda_2} f(p^{-1/\mu_1}, q^{-1/\mu_2}) \underset{q_1', q_2'}{\overset{p_1', p_2'}{\underset{\longrightarrow}{\overset{\longleftarrow}{}}}} g(x, y) \text{ then}$$

(2.5.1) $$\Phi(p, q) = A p^{\mu_1(\lambda_1+1)} q^{\mu_2(\lambda_2+1)} \int_0^\infty \int_0^\infty G^{m_1+\mu_1 m_1', n_1+\mu_1 n_1'}_{p_1+\mu_1 p_1', q_1+\mu_1 q_1'} \times$$

$$\times \left(\frac{Sp^{\mu_2} 2^{\mu_1+1}}{(\mu_1)^{\mu_1(q_1'-p_1')}} \,\middle|\, \begin{matrix} \Delta_{\mu_1}[-\mu_1; \lambda_1\mu_1 - (a_{n_1'-1})], \Delta_{\mu_1}[-\mu_1; \lambda_1\mu_1 - 1], (a_{p_1}), \\ \Delta[\mu_1; (b_{m_1'}) - \mu_1(\lambda_1 + 1 - 1/\mu_1)], (b_{q_1}), \end{matrix} \right.$$

$$\left. \begin{matrix} \Delta[\mu_1; (a_{p'-n_1'}) - \mu_1(\lambda_1 + 1 - 1/\mu_1)] \\ \Delta \mu_1[-\mu_1; \lambda_1\mu_1 - (b_{q_1'-m_1'}) \end{matrix} \right) \times$$

$$\times G^{m_2+\mu_2 m_2', n_2+\mu_2 n_2'}_{p_2+\mu_2 p_2', q_2+\mu_2 q_2'} \left(\frac{tq^{\mu_1} (2)^{\mu_2+1}}{(\mu_2)^{\mu_2(q_2'-p_2')}} \,\middle|\, \begin{matrix} \Delta_{\mu_2}[-\mu_2; \lambda_2\mu_2 - (a_{n_2'-1})] \\ \Delta[\mu_2; (b_{m_2'}) - \mu_2(\lambda_2 + 1 - 1/\mu_2)], \end{matrix} \right.$$

$$\left. \begin{matrix} \Delta\mu_2[-\mu_2; \lambda_2\mu_2 - 1], (a_{p_2}), \Delta[\mu_2; (a_{p_2'-n_2'}) - \mu_2(\lambda_2 + 1 - 1/\mu_2)] \\ \Delta\mu_2[-\mu_2; \lambda_2\mu_2 - (b_{q_2'-m_2'})], (b_{q_2}) \end{matrix} \right) g(s, t) ds\, dt,$$

where

$$A = \Pi(2\pi)^{1/2(1-\mu_r)(2m_{\gamma'}+2n_{r'}-p_{r'}-q_{\gamma'})}.$$

$$.\, \Pi(\mu_\gamma)^{\mu_\gamma(\lambda_\gamma+1-1/\mu_r)(p_\gamma'-q_\gamma')-1/2(q_\gamma'-p_\gamma'+2)-\Sigma(a_{p_r'})-\Sigma(b_{q_r'})}$$

and the integrals converge absolutely with $w = 0$.

3. *Example* 1. Taking $f(x, y) = x^{\lambda_1} y^{\lambda_2} e^{-\alpha x - \beta y}$ and using (2.2.1), we find that

$$(3.1.1)\qquad x^{\lambda_1} y^{\lambda_2} e^{-\alpha x-\beta y} \xrightarrow[q_1,\, q_2]{\substack{p_1,\, p_2\\ w}} \frac{pq}{(wp+\alpha)^{\lambda_1+1}(wq+\beta)^{\lambda_2+1}} \cdot$$

$$\cdot\, G^{m_1, n_1+1}_{p_1+1, q_1}\left(\frac{2p}{(wp+\alpha)^{\lambda_1+1}}\middle|\begin{matrix}-\lambda_1, (a_{p_1})\\ (b_{q_1})\end{matrix}\right) G^{m_2, n_2+1}_{p_2+1, q_2}\left(\frac{2q}{(wq+\alpha)^{\lambda_2+1}}\middle|\begin{matrix}-\lambda_2, (a_{p_2})\\ (b_{q_2})\end{matrix}\right)$$

$$R(p, q) > 0,\ R(\alpha, \beta) > 0.$$

Example 2. Taking

$$f(x, y) = x^{-\frac{\lambda_1}{N_1}-1} y^{-\frac{\lambda_2}{N_2}-1} K_{2\mu_1}\left(x^{-\frac{1}{2N_1}}\right) K_{2\nu_1}\left(x^{-\frac{1}{2N_1}}\right) K_{2\mu_2}\left(y^{-\frac{1}{2N_2}}\right) K_{2\nu_2}\left(y^{-\frac{1}{2N_2}}\right)$$

and using (2.2.1) we find that

$$x^{-\frac{\lambda_1}{N_1}-1-\frac{\lambda_2}{N_2}} K_{2\mu_1}\left(x^{-\frac{1}{2N_1}}\right) K_{2\nu_1}\left(x^{-\frac{1}{2N_1}}\right) K_{2\mu_2}\left(y^{-\frac{1}{2N_2}}\right) K_{2\nu_2}\left(y^{-\frac{1}{2N_2}}\right)$$

$$(3.2.1)\qquad \xrightarrow[q_1,\, q_2]{p_1,\, p_2} \frac{\pi^{3-N_1-N_2}}{2^{N_1+N_2}} (N_1)^{2\lambda_1-1/2} (N_2)^{2\lambda_2-1/2}\, G^{m_1+4N_1, n_1}_{p_1+2N_1, q_1+4N_1}\left(\frac{p}{N_1^{2N_1}}\middle|\begin{matrix}(a_{p_1}), \Delta_{2N_1}[2N_1; 2\lambda_1]\\ \Delta_{N_1}[N_1; \lambda_1+\mu_1+\nu_1], \Delta_{N_1}[N_1; \lambda_1-\mu_1+\nu_1], \Delta_{2N_1}[2N_1; \lambda_1+\mu_1-\nu_1], (b_{q_1})\end{matrix}\right)$$

$$G^{m_2+4N_2, n_2}_{p_2+2N_2, q_2+4N_2}\left(\frac{q}{N_2^{2N_2}}\middle|\begin{matrix}(a_{p_2}), \Delta_{2N_2}[2N_2; 2\lambda_2]\\ \Delta_{N_2}[N_2; \lambda_2+\mu_2+\nu_2], \Delta_{N_2}[N_2; \lambda_2-\mu_2+\nu_2],\\ \Delta_{2N_2}[2N_2; \lambda_2+\mu_2-\nu_2], (b_{q_2})\end{matrix}\right).$$

The conditions of validity are

(i) $p_1+q_1 < 2(m_1+n_1)$, $|\arg p| < \left(m_1+n_1-\frac{1}{2}p_1-\frac{1}{2}q_1\right)\pi$,

(ii) $R(\lambda_1 \pm \mu_1 \pm \nu_1 + N(1-a_j)) > 0$, $j = 1, 2, 3, \ldots, n_1$,

(iii) $p_2+q_2 < 2(m_2+n_2)$, $|\arg q| < \left(m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2\right)\pi$,

(iv) $R(\lambda_2 \pm \mu_2 \pm \nu_2 + N(1-a_i)) > 0$, $i = 1, 2, 3, \ldots, n_2$.

Example 3. Taking

$$f(x, y) = x^{-\frac{\lambda_1}{N_1}-1} y^{-\frac{\lambda_2}{N_2}-1} W_{K_1,m_1}\left(x^{-\frac{1}{2N_1}}\right) W_{-K_1,m_1}\left(x^{-\frac{1}{2N_1}}\right) W_{K_2,m_2}\left(y^{-\frac{1}{2N_2}}\right) W_{-K_2,m_2}\left(y^{-\frac{1}{2N_2}}\right)$$

and using (2.2.1) we find that

$$(3.3.1)\qquad x^{-\frac{\lambda_1}{N_1}-1} y^{-\frac{\lambda_2}{N_2}-1} W_{K_1,m_1}\left(x^{-\frac{1}{2N_1}}\right) W_{-K_1,m_1}\left(x^{-\frac{1}{2N_1}}\right) W_{K_2,m_2}\left(y^{-\frac{1}{2N_2}}\right)$$

$$W_{-K_2,m_2}\left(y^{-\frac{1}{2N_2}}\right) \xrightarrow[p_2,\, q_2]{p_1,\, q_1} (2\pi)^{1/2-N_1} (2\pi)^{1/2-N_2} (2N_1)^{2\lambda_1+1/2} (2N_2)^{2\lambda_2+1/2} \times$$

$$\times\, G^{m_1+4N_1, n_1}_{p_1+2N_1, q_1+4N_1}\left(\frac{p}{(2N_1)^{2N_1}}\middle|\begin{matrix}(a_{p_1}), \Delta_{N_1}[N_1, \lambda_1+K_1+1],\\ \Delta_{2N_1}[2N_1; 2\lambda_1+1], \Delta_{N_1}[N_1; \lambda_1+m_1+1/2],\\ \Delta_{N_1}[N_1; \lambda_1-K_1+1]\\ \Delta_{N_1}[N_1; -\lambda_1-m_1+1/2], (b_{q_1})\end{matrix}\right) \times$$

$$\times\, G^{m_2+4N_2, n_2}_{p_2+2N_2, q_2+4N_2}\left(\frac{q}{(2N_2)^{2N_2}}\middle|\begin{matrix}(a_{p_2}), \Delta_{N_2}[N_2, \lambda_2+K_2+1],\\ \Delta_{2N_2}[2N_2; 2\lambda_2+1], \Delta_{N_2}[N_2; \lambda_2+m_2+1/2],\\ \Delta_{N_2}[N_2; -\lambda_2-K_2+1]\\ \Delta_{N_2}[N_2; \lambda_2-m_2+1/2], (b_{q_2})\end{matrix}\right).$$

The condition of validity are

(i) $p_1+q_1 < 2(m_1+n_1)$ and $|\arg p| < \pi\left(m_1+n_1-\frac{1}{2}p_1-\frac{1}{2}q_1\right)$,

(ii) $R\left[\lambda_1 \pm m_1 - \frac{1}{2}(1-a_j)\right] > 0$, $R[\lambda_1 + N_1(1-a_j)] > 0$, $j = 1, 2, \ldots, n_2$

(iii) $p_2+q_2 < 2(m_2+n_2)$ and $|\arg q| < \pi\left(m_2+n_2-\frac{1}{2}p_2-\frac{1}{2}q_2\right)$,

(iv) $R\left[\lambda_2 \pm m_2 - \frac{1}{2}(1-a_i)\right] > 0$, $R[\lambda_2 + N_2(1-a_i)] > 0$, $i = 1, 2, \ldots, n_2$.

Example 4. Taking

$$f(x, y) = x^{-\frac{\lambda_1}{N_1}-1} y^{-\frac{\lambda_2}{N_2}-1} \psi_1\left(x^{-\frac{1}{2N_1}}\right) \psi_2\left(y^{-\frac{1}{2N_2}}\right)$$

where $\psi_r(t) = \dfrac{1}{2\pi i} \displaystyle\int_{c-i\infty}^{c+i\infty} \dfrac{\Gamma[(a'_n)+s]\,\Gamma[(b''_n)-s]}{\Gamma[(c'_m)+s]\,\Gamma[(d''_m)-s]}\, t^{-s}\, ds$

and using (2.2.1), we find that

$$x^{-\frac{\lambda_1}{N_1}-1} y^{-\frac{\lambda_2}{N_2}-1} \psi_1\left(x^{-\frac{1}{2N_1}}\right) \psi_2\left(y^{-\frac{1}{2N_2}}\right) \underset{q_1, q_2}{\overset{p_1, p_2}{\rightleftharpoons}} (2\pi)^{(1/2-N_1-N_2)(n'+n''-m'-m'')} \times$$

(3.4.1)

$$\times \prod_{i=1}^{2} (2N_i)^{\sum_{r=1}^{n'}(a_r)+\sum_{r=1}^{n''}(b_r)-\sum_{r=1}^{m'}(c_r)-\sum_{r=1}^{m''}(d_r)+2\lambda_i(n'-n''-m'-m'')} \prod_{i=1}^{2} (2N_i)^{1-1/2(n'-n''-m'-m'')} \times$$

$$\times \prod_{i=1}^{2} G^{m_i+2N_i n', n_i+2N_i n''}_{2N_i n''+p_i+2N_i m', 2N_i n'+q_i+2N_i m''}\left(\frac{p}{(2N_i)^{-2N_i(n''-n'+m'-m'')}} \middle|\, \begin{matrix} \Delta_{2N_i}[-2N_i; (b''_n)-2\lambda_1-2N_1], (a_{p_i}), \Delta_{2N_i}[2N_i; (c'_m)+2\lambda_i] \\ \Delta_{2N_i}[2N_i; (a'_n)+2\lambda_i], (b_{q_i}), \Delta_{2N_i}[-2N_i; (d'_m)-2\lambda_i-2N_i)] \end{matrix}\right).$$

4. *Particular cases:* (i) When $w = 0$, $m_1 = m_2 = 2$, $n_1 = n_2 = 0$, $p_1 = p_2 = 0$, $q_1 = q_2 = 2$, $b_1 = 0$, $b_2 = a_1$; results (2.1.1) to (2.3.1) and (3.1.1) to (3.4.1) give the corresponding well known results in Laplace transforms of two variables. Result (2.4.1) will be reduced to a known result due to Bose [2].

(ii) When $w = 1$, $m_1 = m_2 = 2$, $n_1 = n_2 = p_1 = p_2 = 1$, $q_1 = q_2 = 2$, $a_1 = 1$, $b_1 = 1/2 - v$, $b_2 = 1/2 + v$; results (2.1.1) to (2.4.1) and (3.1.1) give the known results due to Rathie [6]. Quite a few results can be reduced to known results in generalized Laplace transforms of two variables given by Varma, Saxena and by other authors.

REFERENCES

1. Meijer C. S. — *Über eine Erweiterung der Laplace-transformation.* Proc. Kon. Neder-Acad. van Wet., Amsterdam, 43 (1940), pp. 599—608 and 702—711.
2. Bose S. K. — *On Laplace transform of two variables.* Bull. Cal. Math. Soc., 41 (1949), pp. 137—178.
3. Erdélyi A. — *Tables of integral transforms*, vol. I, II, McGraw-Hill, New York, 1954.
4. Varma R. S. — *A generalization of Laplace transform.* Current science (Bangalore), 16, (1947), pp. 17—18.

5. Varma R. S. — *On a generalization of Laplace Integral.* Proc. Nat. Acad. Sciences, India, A, 20 (1951), pp. 209—216.
6. Rathie C. B. — *Meijer's Bessel Transform of two variables.* Annales de la Soc. scient. de Bruxelles, T. 79, I (1965), pp. 41—46.
7. Saxena R. K. — *Some theorems on generalized Laplace transform*, I. Proc. Nat. Inst. Sci., India, 26 A (1960), pp. 400—413.

Department of Mathematics
Iowa State University

TRANSFORMĂRI MEIJER-LAPLACE CU DOUĂ VARIABILE

Rezumat

Pornind de la definiția (1.2) a transformării Meijer-Laplace cu două variabile, se stabilesc egalitățile din teoremele 1, 2, 3, 4, 5 și se aplică rezultatele la exemplele 1—4.