SAINT-VENANT'S PROBLEM FOR ORTHOTROPIC NATURALLY SLIGHTLY CURVED BEAMS

BY

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We will consider a beam limited by two planes $x_3 = 0$, $x_3 = h$ and a surface $\mathcal{F}$ given by the equation

$$f(x_1 + k\theta,\ x_2) = 0, \tag{1}$$

where $\theta = \theta(x_3)$ is a given function of x_3 and k is a small parameter such that the square and higher powers of it can be neglected.

We will call „naturally slightly curved beam" the beam described above.

The extension by a longitudinal force, the bending due to couples, the torsion by a torque and the bending by a transverse load will be studied in this paper.

In this purpose we will suppose, in turn, that on the end $x_3 = h$ there are applied tractions which are equivalent to a longitudinal force, to a couple, to a torque and to a transverse force. On the other end are applied tractions in order to equilibrate the tractions applied on the end $x_3 = h$.

First, such a beam was studied by Riz for the case when $\theta = (1/2)\, x_3^2$, the material of the beam being isotropic [7].

Many results are known in connection with the isotropic and anisotropic naturally slightly curved beams:

On the one hand we know many results when $\theta = (1/2)\, x_3^2$ [2, 3, 4, 7] etc. and on the other hand it was studied the case when $\theta = (1/n)\, x_3^n$ [1, 2].

Here, we will generalize the results contained in [1].

1. General equations. We will denote by $\mathcal{V}$ the region occupied by the beam.

We suppose that the surface $\mathcal{F}$ of the beam is free from tractions and that there are no body forces.

It follows that the stress components σ_{ij} must satisfy the equilibrium equations

$$\sigma_{ij,j} = 0 \quad \text{in } \mathcal{V} \tag{2}$$

and the boundary conditions

$$\sigma_{ij}\, n_j = 0 \quad \text{on } \mathcal{F}, \tag{3}$$

where n_i are the direction cosines of the exterior normal to the surface $\mathcal{F}$. The index j after comma indicates partial differentiation with respect to x_j. We use also the summation convention over the repeated indices.

At the end the boundary conditions are dependent of the problem which we study.

The material of the beam being orthotropic, the Hooke's law may be written in the form

$$\begin{cases} \sigma_{11} = A\ \gamma_{11} + H\ \gamma_{22} + G\ \gamma_{33}, \\ \sigma_{22} = H\ \gamma_{11} + B\ \gamma_{22} + F\ \gamma_{33}, \\ \sigma_{33} = G\ \gamma_{11} + F\ \gamma_{22} + C\ \gamma_{33}; \end{cases} \tag{4 a}$$

$$\sigma_{23} = L\gamma_{23}, \quad \sigma_{31} = M\gamma_{31}, \quad \sigma_{12} = D\gamma_{12}, \tag{4 b}$$

where $A, B, \ldots, L, M$ are moduli of elasticity.

The strain components γ_{ij} are given in terms of stress components by

$$\begin{cases} \gamma_{11} = \dfrac{1}{E}(\nu_{11}\,\sigma_{11} + \nu_{12}\,\sigma_{22} - \nu_1\,\sigma_{33}), \\ \gamma_{22} = \dfrac{1}{E}(\nu_{12}\,\sigma_{11} + \nu_{22}\,\sigma_{22} - \nu_2\,\sigma_{33}), \\ \gamma_{33} = \dfrac{1}{E}(-\nu_1\,\sigma_{11} - \nu_2\,\sigma_{22} + \sigma_{33}); \end{cases} \tag{5 a}$$

$$\gamma_{23} = \frac{1}{L}\sigma_{23}, \quad \gamma_{31} = \frac{1}{M}\sigma_{31}, \quad \gamma_{12} = \frac{1}{D}\sigma_{12}; \tag{5 b}$$

where the coefficients of strain E, ν_{ii}, ν_i may be expressed in terms of moduli of elasticity [2].

Let u_i be the components of displacement. The components of strain γ_{ij} are given by

$$\gamma_{ii} = u_{i,i} \ (\text{not summed}),$$
$$\gamma_{ij} = u_{i,j} + u_{j,i} \quad (i \neq j),$$

and they must satisfy the conditions of compatibility

$$\gamma_{11,22} + \gamma_{22,11} = \gamma_{12,12} \ldots, \tag{6 a}$$

$$(-\gamma_{23,1} + \gamma_{31,2} + \gamma_{12,3})_{,1} = 2\gamma_{11,23} \ldots \tag{6 b}$$

If we make the transformation

$$\xi = x_1 + k\theta, \quad \eta = x_2, \quad \zeta = x_3, \tag{7}$$

the surface (1) becomes

$$f(\xi, \eta) = 0, \tag{8}$$

which is a cylindrical surface $\mathcal{F}_1$ in the space ξ, η, ζ.

We will denote by S the domain and also the area of the cross-section of the cylindrical beam (8) and by Γ the boundary of S.

It is easy to prove the following formulae:

$$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial x_2} = \frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial x_3} = \frac{\partial}{\partial \zeta} + k\theta' \frac{\partial}{\partial \xi} \tag{9}$$

and

$$n_1 = \cos\alpha, \quad n_2 = \cos\beta, \quad n_3 = k\theta' \cos\alpha, \tag{10}$$

where n_i are the direction cosines of the exterior normal to the surface $\mathcal{F}$, $\cos\alpha$ and $\cos\beta$ are the direction cosines of the exterior normal to the curve Γ and θ' is the derivative of the function θ with respect to ζ.

The axes of ξ, η, ζ will be taken in the following way: the axis of ζ is the central-line of the beam (8) and the axes of ξ and η are the principal axes of inertia of the end $\zeta = 0$. In this conditions we will have

$$\iint_S \xi\, d\sigma = 0, \quad \iint_S \eta\, d\sigma = 0, \quad \iint_S \xi\eta\, d\sigma = 0.$$

We will try to solve the mentioned problems supposing that the displacements u_i are of the form

$$u_i = \tilde{u}_i + k u_i^0, \tag{11}$$

where $\tilde{u}_i$ are the displacements which correspond to the cylindrical beam (8) in the studied problem and u_i^0 are additional unknown displacements.

Let τ_{ij} be the stresses corresponding to the displacements u_i^0. From the equations of equilibrium we will find that these stresses satisfy equations of the form

$$\tau_{ij,j} + r_i = 0, \tag{12}$$

where r_i are known functions of ξ, η, ζ and the indices 1, 2, 3 after comma indicate partial differentiation with respect to ξ, η, ζ.

We will say that the stresses $\overset{*}{\tau}_{ij}$ represent a τ-solution for the equations (12) if they verify the equations (12) and the corresponding strains $\overset{*}{\gamma}_{ij}$ verify the conditions of compatibility. If we put down

$$\tau_{ij} = \overset{*}{\tau}_{ij} + \tilde{\tau}_{ij}, \tag{13}$$

we will get that the new stresses $\tilde{\tau}_{ij}$ must satisfy the equations

$$\tilde{\tau}_{ij,j} = 0. \tag{14}$$

From (3) we can get the boundary conditions which the stresses $\tilde{\tau}_{ij}$ must verify on the surface $\mathcal{F}_1$.

The solutions given here will satisfy all equations and boundary conditions excepting the end conditions which depend of the studied problem.

Therefore, it still remains to correct the end conditions by superposition of solutions of some adequate problems for the cylindrical beam (8) but we know how to solve all these additional problems [2, ch. 6,3 ch. 2.3.5].

In this way for each studied problem the finding of the stresses τ_{ij} is reduced to a certain generalized Almansi's problem *).

2. The extension by a longitudinal force. Let us suppose that the tractions applied on the end $x_3 = h$ are equivalent to a force F_3 acting in the line of ζ.

The displacements u_i are given by

$$\begin{cases} u_1 = -\gamma\nu_1 \xi + k u_1^0, \\ u_2 = -\gamma\nu_2 \eta + k u_2^0, \\ u_3 = \gamma\zeta + k u_3^0, \end{cases} \tag{15}$$

where γ is a constant which must be determined from the end conditions.

If we calculate the corresponding strains and stresses, finally, we will get that the stresses τ_{ij} must verify the equations

*) We vill call Almansi's generalized problem the problem which is defined by (14) with the following boundary conditions:

$$\tilde{\tau}_{i\alpha}\tilde{n}_\alpha = \theta_i P_i(\xi, \eta) \text{ on } \mathcal{F}_1 \text{ (not summed after } i = 1,2,3;\ \alpha = 1,2),$$

where $\tilde{n}_1 = \cos\alpha$, $\tilde{n}_2 = \cos\beta$, θ_i are given functions of ζ and P_i are given functions of ξ and η.

When θ_i are polinomials in ζ the problem becomes an Almansi's problem and we know how to solve it [5]. In the general case we may aproximate the functions θ_i by polinomials and in this way our general problem is reduced to an Almansi's problem.

$$\begin{cases} \dfrac{\partial\tau_{11}}{\partial\xi} + \dfrac{\partial\tau_{12}}{\partial\eta} + \dfrac{\partial\tau_{31}}{\partial\zeta} - \gamma M \nu_1 \theta'' = 0, \\ \dfrac{\partial\tau_{12}}{\partial\xi} + \dfrac{\partial\tau_{22}}{\partial\eta} + \dfrac{\partial\tau_{23}}{\partial\zeta} = 0, \quad \dfrac{\partial\tau_{31}}{\partial\xi} + \dfrac{\partial\tau_{23}}{\partial\eta} + \dfrac{\partial\tau_{33}}{\partial\zeta} = 0 \end{cases} \tag{16}$$

and if we take as τ-solution for these equations the stresses

$$\begin{gathered} \overset{*}{\tau}_{11} = 0, \quad \overset{*}{\tau}_{22} = 0, \quad \overset{*}{\tau}_{33} = 0, \\ \overset{*}{\tau}_{12} = 0, \quad \overset{*}{\tau}_{23} = 0, \quad \overset{*}{\tau}_{31} = \gamma M \nu_1 \theta'', \end{gathered} \tag{17}$$

then the boundary conditions for the corresponding generalized Almansi's problem are given by

$$\begin{cases} \tilde{\tau}_{11}\cos\alpha + \tilde{\tau}_{12}\cos\beta = 0, \\ \tilde{\tau}_{12}\cos\alpha + \tilde{\tau}_{22}\cos\beta = 0, \\ \tilde{\tau}_{31}\cos\alpha + \tilde{\tau}_{23}\cos\beta = -\gamma E \theta' \cos\alpha \quad \text{on } \mathcal{F}_1. \end{cases} \tag{18}$$

3. The bending due to couples. Let the tractions applied on the end $x_3 = h$ be equivalent to a couple paralel to the axis of η. In this case we have

$$\begin{cases} u_1 = \dfrac{1}{2} a(\nu_1 \xi^2 - \nu_2 \eta^2 + \zeta^2) + k u_1^0, \\ u_2 = a\nu_2 \xi\eta + k u_2^0, \\ u_3 = -a\xi\zeta + k u_3^0, \end{cases} \tag{19}$$

where a is a constant.

The corresponding stresses τ_{ij} are given by

$$\begin{cases} \sigma_{11} = -kaG\theta'\zeta + k\tau_{11}, \quad \sigma_{22} = -kaF\theta'\zeta + k\tau_{22}, \\ \sigma_{33} = -aE\xi - kaC\theta'\zeta + k\tau_{33}, \quad \sigma_{12} = k\tau_{12}, \\ \sigma_{23} = kaL\nu_2\eta\theta' + k\tau_{23}, \quad \sigma_{31} = kaM\nu_1\xi\theta' \end{cases} \tag{20}$$

and the equations (12) become

$$\begin{cases} \dfrac{\partial\tau_{11}}{\partial\xi} + \dfrac{\partial\tau_{12}}{\partial\eta} + \dfrac{\partial\tau_{31}}{\partial\zeta} + aM\nu_1\xi\theta'' = 0, \\ \dfrac{\partial\tau_{12}}{\partial\xi} + \dfrac{\partial\tau_{22}}{\partial\eta} + \dfrac{\partial\tau_{23}}{\partial\zeta} + aL\nu_2\eta\theta'' = 0 \\ \dfrac{\partial\tau_{31}}{\partial\xi} + \dfrac{\partial\tau_{23}}{\partial\eta} + \dfrac{\partial\tau_{33}}{\partial\zeta} + a(M\nu_1 + L\nu_2 - E)\theta' - aC(\theta' + \theta''\zeta) = 0. \end{cases} \tag{21}$$

The stresses

$$
(22)\qquad \begin{cases} \overset{*}{\tau}_{11} = aG\,\theta'\,\xi, \quad \overset{*}{\tau}_{22} = aF\,\theta'\,\xi, \quad \overset{*}{\tau}_{33} = aC\,\theta'\zeta + aE\,\theta, \\ \overset{*}{\tau}_{12} = aT\,\theta'\,\xi, \quad \overset{*}{\tau}_{23} = -\,aL\,\nu_2\,\eta\,\theta', \quad \overset{*}{\tau}_{31} = -\,aM\,\nu_1\,\xi\,\theta' \end{cases}
$$

represent a τ-solution for the equations (20).

Using (3), (20) and (22) we find that the stresses $\tilde{\tau}_{ij}$ verify the boundary conditions

$$
(23)\qquad \begin{cases} \tilde{\tau}_{11}\cos\alpha + \tilde{\tau}_{12}\cos\beta = 0, \\ \tilde{\tau}_{12}\cos\alpha + \tilde{\tau}_{22}\cos\beta = 0, \\ \tilde{\tau}_{31}\cos\alpha + \tilde{\tau}_{23}\cos\beta = aE\xi\,\theta'\cos\alpha \quad \text{on } \mathcal{F}_1. \end{cases}
$$

4. The torsion. Let the tractions applied on the end $x_3 = h$ be equivalent with a torque $\mathfrak{M}_3$.

We will try to find the solution supposing that

$$
(24)\qquad u_1 = -\,\tau\eta\zeta + ku_1^0, \quad u_2 = \tau\xi\zeta + ku_2^0, \quad u_3 = \tau\varphi(\xi,\eta) + ku_3^0,
$$

where τ is a constant and $\varphi(\xi, \eta)$ is the function of torsion of the cylindrical bar (8) defined by

$$
(25\text{ a})\qquad M\frac{\partial^2\varphi}{\partial\xi^2} + L\frac{\partial^2\varphi}{\partial\eta^2} = 0 \quad \text{in } S
$$

and

$$
(25\text{ b})\qquad \mathfrak{D}\varphi = M\eta\cos\alpha - L\xi\cos\beta \quad \text{on } \Gamma,
$$

the operator $\mathfrak{D}$ being defined by

$$
(26)\qquad \mathfrak{D} = M\cos\alpha\frac{\partial}{\partial\xi} + L\cos\beta\frac{\partial}{\partial\eta}.
$$

Using (24) and (4) we get

$$
\sigma_{11} = k\tau G\theta'\frac{\partial\varphi}{\partial\xi} + k\tau_{11}, \quad \sigma_{22} = k\tau F\theta'\frac{\partial\varphi}{\partial\xi}, \quad \sigma_{33} = k\tau\, C\,\theta'\frac{\partial\varphi}{\partial\xi},
$$

$$
(27)\qquad \sigma_{12} = k\tau\, T\theta'\frac{\partial\varphi}{\partial\xi}, \quad \sigma_{23} = \tau L\left[\left(\frac{\partial\varphi}{\partial\eta} + \xi\right) + k\theta'\zeta\right] + k\tau_{23},
$$

$$
\sigma_{31} = \tau\, M\left(\frac{\partial\varphi}{\partial\xi} - \eta\right) + k\tau_{31}
$$

and the equations (12) become

$$
(28)\qquad \begin{aligned} &\frac{\partial\tau_{11}}{\partial\xi} + \frac{\partial\tau_{12}}{\partial\eta} + \frac{\partial\tau_{31}}{\partial\zeta} + \tau(G + M)\,\theta'\frac{\partial^2\varphi}{\partial\xi^2} = 0, \\ &\frac{\partial\tau_{12}}{\partial\xi} + \frac{\partial\tau_{22}}{\partial\eta} + \frac{\partial\tau_{23}}{\partial\zeta} + \tau(F + L)\,\theta'\frac{\partial^2\varphi}{\partial\xi\partial\eta} + \tau L(2\theta' + \theta''\zeta) = 0, \\ &\frac{\partial\tau_{31}}{\partial\xi} + \frac{\partial\tau_{23}}{\partial\eta} + \frac{\partial\tau_{33}}{\partial\zeta} + \tau C\,\theta''\frac{\partial\varphi}{\partial\xi} = 0. \end{aligned}
$$

Let us take into account the following τ-solution of the equations (28):

$$
(29)\qquad \begin{aligned} &\overset{*}{\tau}_{11} = -\,\tau G\,\theta'\frac{\partial\varphi}{\partial\xi}, \quad \overset{*}{\tau}_{22} = -\,\tau F\,\theta'\frac{\partial\varphi}{\partial\xi}, \quad \overset{*}{\tau}_{33} = -\,\tau C\,\theta'\frac{\partial\varphi}{\partial\xi}, \\ &\overset{*}{\tau}_{12} = 0, \quad \overset{*}{\tau}_{23} = -\,\tau L\left[\left(\frac{\partial^2\varphi}{\partial\xi\partial\eta} + 1\right)\theta + \theta'\zeta\right], \quad \overset{*}{\tau}_{31} = -\,\tau M\,\theta\frac{\partial^2\varphi}{\partial\xi^2}. \end{aligned}
$$

From (3), (27) and (29) we find the following boundary conditions for the stresses $\tilde{\tau}_{ij}$:

$$
(30)\qquad \begin{aligned} &\tilde{\tau}_{11}\cos\alpha + \tilde{\tau}_{12}\cos\beta = -\,\tau M\left(\frac{\partial\varphi}{\partial\xi} - \eta\right)\theta'\cos\alpha, \\ &\tilde{\tau}_{12}\cos\alpha + \tilde{\tau}_{22}\cos\beta = -\,\tau L\left(\frac{\partial\varphi}{\partial\eta} + \xi\right)\theta'\cos\alpha, \\ &\tilde{\tau}_{31}\cos\alpha + \tilde{\tau}_{23}\cos\beta = \tau\theta\left[M\frac{\partial^2\varphi}{\partial\xi^2}\cos\alpha + L\left(\frac{\partial^2\varphi}{\partial\xi\partial\eta} + 1\right)\cos\beta\right] \quad \text{on } \mathcal{F}_1. \end{aligned}
$$

5. The bending by a transverse load. Let the tractions applied on the end $x_3 = h$ be equivalent with a force F_1 acting in the line of ξ.

We can solve our problem supposing that

$$
(31)\qquad \begin{aligned} &u_1 = -\,\tau\eta\zeta + a\left[\frac{1}{2}(h - \zeta)(\nu_1\xi^2 - \nu_2\eta^2) + \frac{1}{2}h\zeta^2 - \frac{1}{6}\zeta^3\right], \\ &u_2 = \tau\xi\zeta + a\,\nu_2(h - \zeta)\,\xi\eta, \\ &u_3 = \tau\varphi(\xi,\eta) - a\left[\chi(\xi,\eta) + \left(h - \frac{1}{2}\zeta\right)\xi\zeta\right], \end{aligned}
$$

where τ and a are constants which will be determined from the end conditions, φ and χ are the function of torsion and the function of flexion for the cylindrical beam (3). It follows that φ is defined by (25) and χ by

$$M\frac{\partial^2\chi}{\partial\xi^2}+L\frac{\partial^2\chi}{\partial\eta^2}+(M\nu_1+L\nu_2-E)\,\xi=0 \quad \text{in } S \tag{32 a}$$

and

$$\mathcal{D}\chi=-\frac{1}{2}M\,(\nu_1\xi^2-\nu_2\eta^2)\cos\alpha-L\nu_2\,\xi\eta\cos\beta \quad \text{on } \Gamma. \tag{32 b}$$

After some calculations easily to do we get for the stresses τ_{ij} the equations

$$\begin{aligned}
&\frac{\partial\tau_{11}}{\partial\xi}+\frac{\partial\tau_{12}}{\partial\eta}+\frac{\partial\tau_{31}}{\partial\zeta}+(G+M)\,\theta'\frac{\partial^2h_1}{\partial\xi^2}-a\,M\nu_1[2\theta'-(h-\zeta)\theta'']\,\xi=0,\\
&\frac{\partial\tau_{12}}{\partial\xi}+\frac{\partial\tau_{22}}{\partial\eta}+\frac{\partial\tau_{23}}{\partial\zeta}+(F+L)\,\theta'\frac{\partial^2h_1}{\partial\xi\partial\eta}+\tau L(2\,\theta'+\theta''\zeta)-\\
&-aL\nu_2[2\,\theta'-(h-\zeta)\,\theta'']\eta=0,\\
&\frac{\partial\tau_{31}}{\partial\xi}+\frac{\partial\tau_{23}}{\partial\eta}+\frac{\partial\tau_{33}}{\partial\zeta}+C\left[\frac{\partial h_1}{\partial\xi}-a(h-\frac{1}{2}\zeta)\,\zeta\right]\theta''+a\,(h-\zeta)\,(M\nu_1+\\
&+L\nu_2-E-C)\,\theta'=0,
\end{aligned}\tag{33}$$

where

$$h_1=\tau\varphi\,(\xi,\eta)-a\,\chi\,(\xi,\eta).$$

Let us take as τ-solution of the equations (33) the following stresses

$$\begin{aligned}
&\overset{*}{\tau}_{11}=-G\,\theta'\left[\frac{\partial h_1}{\partial\xi}-a\left(h-\frac{1}{2}\zeta\right)\zeta\right],\quad \overset{*}{\tau}_{22}=-F\,\theta'\left[\frac{\partial h_1}{\partial\xi}-a\left(h-\frac{1}{2}\zeta\right)\zeta\right],\\
&\overset{*}{\tau}_{33}=-C\,\theta'\left[\frac{\partial h_1}{\partial\xi}-a\left(h-\frac{1}{2}\zeta\right)\zeta\right]+aE\,(h-\zeta)\,\theta,\quad \overset{*}{\tau}_{12}=0,\\
&\overset{*}{\tau}_{23}=-L\left\{\frac{\partial^2h_1}{\partial\xi\partial\eta}\,\theta+\tau\,(\theta+\theta'\zeta)+a\,\nu_2[(h-\zeta)\,\theta'-\theta]\,\eta\right\},\\
&\overset{*}{\tau}_{31}=-M\left\{\frac{\partial^2h_1}{\partial\xi^2}\,\theta+a\,\nu_1\,[(h-\zeta)\,\theta'-\theta]\,\xi\right\}.
\end{aligned}\tag{34}$$

It results for the stresses $\tilde{\tau}_{ij}$ the following boundary conditions

$$\begin{aligned}
&\tilde{\tau}_{11}\cos\alpha+\tilde{\tau}_{12}\cos\beta=-M\left[\frac{\partial h_1}{\partial\xi}-\tau\eta-\frac{1}{2}a(\nu_1\xi^2-\nu_2\eta^2)\right]\theta'\cos\alpha,\\
&\tilde{\tau}_{12}\cos\alpha+\tilde{\tau}_{22}\cos\beta=-L\left[\frac{\partial h_1}{\partial\xi}+\tau\xi-a\nu_2\,\xi\eta\right]\theta'\cos\alpha,\\
&\tilde{\tau}_{31}\cos\alpha+\tilde{\tau}_{23}\cos\beta=\left[M\left(\frac{\partial^2h_1}{\partial\xi^2}-a\nu_1\,\xi\right)\theta+aE(h-\right.\\
&\left.-\zeta)\,\xi\,\theta'\right]\cos\alpha+L\left(\frac{\partial^2h_1}{\partial\xi\partial\eta}+\tau-a\nu_2\,\eta\right)\theta\cos\beta \text{ on } \mathcal{F}_1.
\end{aligned}\tag{35}$$

Some remarks : — When θ is a polinomial of x_3 in the equation (1) the boundary problems which we had get become Almansi's problems and in this case we know how to solve them [5].

— The problem of bending by a transverse load F_2, acting in the line of η, is quite different of that examined in this paper but it may be solved in the same way. We can do the same remark about the problem of bending by a couple paralel to the axis of ξ.

— The results which were obtained in this paper can be easily generalized to the anisotropic case with one plane of elastic symmetry.

— The given results can be generalized to the composite beams by many orthotropic materials.

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PROBLEMA LUI SAINT-VENANT PENTRU BARE ORTOTROPE UŞOR CURBATE

Rezumat

Se consideră o bară mărginită de două plane $x_3=0$, $x_3=h$ şi de o suprafaţă $\mathcal{F}$ a cărei ecuaţie este dată de (1). În lucrare se arată cum pot fi reduse la anumite probleme de tip Almansi următoarele probleme: întinderea de către o forţă longitudinală, încovoierea de către cupluri, torsiunea de către un moment şi încovoierea de către o forţă transversală.