BENDING OF MICROPOLAR ELASTIC BEAMS BY TERMINAL COUPLES

BY

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In this paper we consider the problem of bending of homogeneous and isotropic beams by terminal couples in the linear theory of micropolar elasticity.

1. Basic equations. Throughout this paper a rectangular coordinate system Ox_k $(k = 1, 2, 3)$ is employed.

Let V be a region of space occupied by a micropolar elastic material whose boundary is S.

We assume that the body forces and the body couples are absent.

The basic equations in the static theory of homogeneous and isotropic micropolar elastic solids are [1], [2] :

equilibrium equations

$$t_{ji,j} = 0, \qquad m_{ji,j} + \epsilon_{ijk} t_{jk} = 0, \tag{1.1}$$

constitutive equations

$$t_{ij} = \lambda \varepsilon_{rr} \delta_{ij} + (\mu + \varkappa) \varepsilon_{ij} + \mu \varepsilon_{ji}, \qquad m_{ij} = \alpha \varphi_{r,r} \delta_{ij} + \beta \varphi_{i,j} + \gamma \varphi_{j,i}, \tag{1.2}$$

geometrical equations

$$\varepsilon_{ij} = u_{j,i} + \epsilon_{jim} \varphi_m . \tag{1.3}$$

In these relations we have used the following notations : t_{ij} — components of the stress tensor, m_{ij}-components of the couple-stress tensor, u_i-components of the displacement vector, φ_i-components of the microrotation vector, ε_{ij}-components of the micropolar strain tensor, δ_{ij}-Kronecker's delta,

ϵ_{ijk}-alternating symbol, λ, μ, $\varkappa$, α, β, γ-characteristic constants of the material, the comma denotes partial derivation with respect to the variables x_i.

The surface tractions and surface couples acting at a point $x(x_k)$ on the surface S are given by

$$t_i = t_{ji}\, n_j, \qquad m_i = m_{ji}\, n_j, \tag{1.4}$$

where n_j are the direction cosines of the outward normal to S at x.

2. Bending of beams by terminal couples. We consider a cylindrical beam of homogeneous and isotropic micropolar elastic material bounded by plane ends perpendicular to the generators.

The cross-section Σ is assumed to be a simply-connected region bounded by a closed Liapunov curve L.

Throughout this paper the axis Ox_3 of our coordinate system will be directed parallel to the generators of the beam. The beam is assumed to be of length l, and one of its bases is taken to lie in the x_1Ox_2-plane, while the other is in the plane $x_3 = l$.

We assume that the body forces and the body couples are absent, the lateral surface is free of applied forces and couples, and that the load of the beam is distributed over its bases, $x_3 = 0$ and $x_3 = l$, in a way which fulfills the equilibrium conditions of a rigid body.

Let the loading applied to the plane $x_3 = 0$ be statically equivalent to a force $P\,(0,\ 0,\ 0)$ and a couple $M\,(M_1,\ M_2,\ 0)$.

On the plane $x_3 = 0$ we have the following conditions

$$\int_\Sigma t_{31}\, d\sigma = 0, \qquad \int_\Sigma t_{32}\, d\sigma = 0, \tag{2.1}$$

$$\int_\Sigma t_{33}\, d\sigma = 0, \tag{2.2}$$

$$\int_\Sigma (x_2 t_{33} + m_{31})\, d\sigma = -M_1, \qquad \int_\Sigma (x_1 t_{33} - m_{32})\, d\sigma = M_2, \tag{2.3}$$

$$\int_\Sigma (x_1\, t_{32} - x_2\, t_{31} + m_{33})\, d\sigma = 0. \tag{2.4}$$

The resultant forces and moments calculated across each cross section satisfy the conditions of equilibrium, so that the conditions (2.1)—(2.4) must be satisfied for $x_3 = h\ (0 \leqslant h \leqslant l)$.

On the lateral surface of the beam we have the following conditions

$$t_{\alpha k}\, n_\alpha = 0, \qquad m_{\alpha k}\, n_\alpha = 0, \qquad (k = 1, 2, 3;\ \alpha = 1, 2). \tag{2.5}$$

The problem consists in the solving of the equations (1.1)—(1.3) with the conditions (2.1)—(2.5).

We try to solve the problem assuming that

$$\begin{aligned}
u_1 &= -\frac{1}{2}\, a_1[x_3^2 + \nu(x_1^2 - x_2^2)] - a_2 \nu x_1 x_2 - b\nu x_1 + a_1 v_1^{(1)}(x_1, x_2) + a_2 v_1^{(2)}(x_1, x_2),\\
u_2 &= -a_1 \nu x_1 x_2 - \frac{1}{2}\, a_2[x_3^2 - \nu(x_1^2 - x_2^2)] - b\nu x_2 + a_1 v_2^{(1)}(x_1, x_2) + a_2 v_2^{(2)}(x_1, x_2),\\
u_3 &= (a_1 x_1 + a_2 x_2 + b) x_3,\\
\varphi_1 &= a_2\, x_3, \qquad \varphi_2 = -a_1\, x_3,\\
\varphi_3 &= -a_1\, \nu x_2 + a_2 \nu x_1 + a_1 w^{(1)}(x_1, x_2) + a_2 w^{(2)}(x_1, x_2).
\end{aligned} \tag{2.6}$$

where $v_\alpha^{(\beta)}(x_1, x_2)$, $w^{(\beta)}(x_1, x_2)$, $(\alpha, \beta = 1, 2)$ are unknown functions and the constants $a_\alpha (\alpha = 1, 2)$ and b will be determined in the following.

In (2.6) we have used the notation

$$\nu = \frac{\lambda}{2\lambda + 2\mu + \varkappa}. \tag{2.7}$$

From (1.3) and (2.6) we obtain

$$\begin{aligned}
\varepsilon_{\alpha\beta} &= -\nu(a_1\, x_1 + a_2\, x_2 + b)\delta_{\alpha\beta} + a_1\, \varepsilon_{\alpha\beta}^{(1)} + a_2\, \varepsilon_{\alpha\beta}^{(2)},\\
\varepsilon_{33} &= a_1\, x_1 + a_2\, x_2 + b, \qquad \varepsilon_{\alpha 3} = \varepsilon_{3\alpha} = 0, \qquad (\alpha, \beta = 1, 2),
\end{aligned} \tag{2.8}$$

where

$$\varepsilon_{\alpha\beta}^{(\rho)} = v_{\beta,\alpha}^{(\rho)} + \epsilon_{\beta\alpha 3} w^{(\rho)}, \qquad (\alpha,\ \beta,\ \rho = 1,\ 2). \tag{2.9}$$

Using (2.8), from (1.2) we get

$$\begin{aligned}
t_{\alpha\beta} &= a_1\, \sigma_{\alpha\beta}^{(1)} + a_2\, \sigma_{\alpha\beta}^{(2)}, \qquad t_{\alpha 3} = t_{3\alpha} = 0,\\
t_{33} &= E(a_1\, x_1 + a_2\, x_2 + b) + \lambda(a_1\, \varepsilon_{\alpha\alpha}^{(1)} + a_2\, \varepsilon_{\alpha\alpha}^{(2)}), \quad (\alpha, \beta = 1, 2),
\end{aligned} \tag{2.10}$$

where

$$\sigma_{\alpha\beta}^{(\rho)} = \lambda v_{\eta,\eta}^{(\rho)}\, \delta_{\alpha\beta} + (\mu + \varkappa)\varepsilon_{\alpha\beta}^{(\rho)} + \mu\varepsilon_{\beta\alpha}^{(\rho)}, \qquad (\alpha,\ \beta,\ \eta,\ \rho = 1,\ 2), \tag{2.11}$$

and in which we have used the notation

(2.12) $$E = \frac{(2\mu + \varkappa)(3\lambda + 2\mu + \varkappa)}{2\lambda + 2\mu + \varkappa}.$$

From (1.2) and (2.6) we obtain

$$m_{\alpha\beta} = 0, \qquad m_{33} = 0, \qquad (\alpha, \beta = 1, 2),$$

(2.13)
$$\begin{aligned}
m_{13} &= a_2(\beta + \gamma\nu) + a_1\,\mu_{13}^{(1)} + a_2\,\mu_{13}^{(2)},\\
m_{31} &= a_2(\beta\nu + \gamma) + a_1\,\mu_{31}^{(1)} + a_2\,\mu_{31}^{(2)},\\
m_{23} &= -\,a_1(\beta + \gamma\nu) + a_1\,\mu_{23}^{(1)} + a_2\,\mu_{23}^{(2)},\\
m_{32} &= -\,a_1(\beta\nu + \gamma) + a_1\,\mu_{32}^{(1)} + a_2\,\mu_{32}^{(2)},
\end{aligned}$$

where

(2.14) $$\mu_{\alpha 3}^{(\rho)} = \gamma w^{(\rho)}_{,\alpha}, \qquad \mu_{3\alpha}^{(\rho)} = \beta w^{(\rho)}_{,\alpha}, \qquad (\alpha, \rho = 1, 2).$$

Taking into account the relations (.10), (2.13), the equilibrium equations (1.1) are reduced to

2.15) $$t_{\alpha\beta,\alpha} = 0, \qquad m_{\alpha 3,\alpha} + t_{12} - t_{21} = 0,$$

and the boundary conditions (2.5) become

2.16) $$t_{\alpha\beta}\, n_\alpha = 0, \qquad m_{\alpha 3}\, n_\alpha = 0, \qquad \text{on } L.$$

The equilibrium equations (2.15) and the boundary conditions (2.16) will be satisfied if the following equations

(2.17) $$\sigma^{(\rho)}_{\alpha\beta,\alpha} = 0 \qquad \mu^{(\rho)}_{\alpha 3,\alpha} + \sigma^{(\rho)}_{12} - \sigma^{(\rho)}_{21} = 0, \qquad (\alpha, \beta, \rho = 1, 2), \text{ in } \Sigma,$$

and the boundary conditions

(2.18)
$$\begin{aligned}
&\sigma^{(1)}_{\alpha\beta}\, n_\alpha = 0, \qquad \sigma^{(2)}_{\alpha\beta}\, n_\alpha = 0,\\
&\mu^{(1)}_{\alpha 3}\, n_\alpha = (\beta + \gamma\nu) n_2, \qquad \mu^{(2)}_{\alpha 3}\, n_\alpha = -\,(\beta + \gamma\nu) n_1, \text{ on } L
\end{aligned}$$

are satisfied.

Thus,the functions $v^{(\rho)}_\alpha(x_1, x_2)$, $w^{(\rho)}(x_1, x_2)$ are the solutions of the plane micropolar strain problems (2.9), (2.11), (2.14), (2.17), (2.18).

The problem of the plane micropolar strain was studied in [5]. The necessary and sufficient conditions to solve the problems (2.17), (2.18) are [5]

(2.19) $$\int_L (\beta + \gamma\nu) n_\alpha\, ds = 0, \qquad (\alpha = 1, 2).$$

Obviously, these conditions are satisfied.

In what follows we assume that the functions $v^{(\rho)}_\alpha(x_1, x_2)$, $w^{(\rho)}(x_1, x_2)$ are known.

The conditions (2.1) and (2.4) are identically satisfied on the basis of the relations (2.10), (2.13).

From (2.2) and (2.10) we obtain

(2.20) $$b = -\,a_1 x_1^0 - a_2 x_2^0,$$

where

(2.21) $$x_1^0 = \frac{1}{EA}\int_\Sigma (E x_1 + \lambda \varepsilon^{(1)}_{\alpha\alpha}) d\sigma, \qquad x_2^0 = \frac{1}{EA}\int_\Sigma (E x_2 + \lambda \varepsilon^{(2)}_{\alpha\alpha}) d\sigma,$$

A being the area of the cross-section.

Therefore from (2.10) and (2.20) we get

(2.22) $$t_{33} = a_1[E(x_1 - x_1^0) + \lambda\varepsilon^{(1)}_{\alpha\alpha}] + a_2[E(x_2 - x_2^0) + \lambda\varepsilon^{(2)}_{\alpha\alpha}].$$

From (2.3) and (2.13), (2.22) we obtain

(2.23)
$$\begin{aligned}
L_{11}\, a_1 + L_{12}\, a_2 &= M_2,\\
L_{21}\, a_1 + L_{22}\, a_2 &= -\,M_1,
\end{aligned}$$

where

(2.24)
$$\begin{aligned}
L_{11} &= \int_\Sigma \{x_1[E(x_1 - x_1^0) + \lambda\varepsilon^{(1)}_{\alpha\alpha}] + \beta\nu + \gamma - \beta w^{(1)}_{,2}\} d\sigma,\\
L_{12} &= \int_\Sigma \{x_1[E(x_2 - x_2^0) + \lambda\varepsilon^{(2)}_{\alpha\alpha}] - \beta w^{(2)}_{,2}\} d\sigma,\\
L_{21} &= \int_\Sigma \{x_2[E(x_1 - x_1^0) + \lambda\varepsilon^{(1)}_{\alpha\alpha}] + \beta w^{(1)}_{,1}\} d\sigma,\\
L_{22} &= \int_\Sigma \{x_2[E(x_2 - x_2^0) + \lambda\varepsilon^{(2)}_{\alpha\alpha}] + \beta\nu + \gamma + \beta w^{(2)}_{,1}\} d\sigma.
\end{aligned}$$

Let us show that the linear equations (2.23) determine the constants a_1 and a_2.

We assume that the internal energy density [2]

(2.25)
$$\begin{aligned}
2U(u, \varphi) = t_{ij}\,\varepsilon_{ij} + m_{ij}\,\varphi_{j,i} &= \lambda\varepsilon_{kk}\,\varepsilon_{ll} + (\mu + \varkappa)\,\varepsilon_{kl}\,\varepsilon_{kl} +\\
&+ \mu\varepsilon_{kl}\,\varepsilon_{lk} + \alpha\,\varphi_{k,k}\,\varphi_{l,l} + \beta\varphi_{k,l}\,\varphi_{l,k} + \gamma\varphi_{k,l}\,\varphi_{k,l},
\end{aligned}$$

is a positive definite quadratic form.

Let us consider two elastic configurations $u'_i, \varphi'_i, \varepsilon'_{ij}, t'_{ij}, m'_{ij}$ and $u''_i, \varphi''_i, \varepsilon''_{ij}, t''_{ij}, m''_{ij}$. If we note

$$2U(u', \varphi'; u'', \varphi'') = t'_{ij}\varepsilon''_{ij} + m'_{ij}\varphi''_{j,i}, \tag{2.26}$$

we have

$$\begin{aligned} &U(u', \varphi'; u'', \varphi'') = U(u'', \varphi''; u', \varphi'), \\ &U(u, \varphi; u, \varphi) = U(u, \varphi). \end{aligned} \tag{2.27}$$

It is known [6], [7] that

$$\begin{aligned} &\int_S (t'_i u''_i + m'_i \varphi''_i)d\sigma = \int_S (t''_i u'_i + m''_i \varphi'_i)d\sigma = 2\int_V U(u', \varphi'; u'', \varphi'')d\tau, \\ &\int_S (t_i u_i + m_i \varphi_i)d\sigma = 2\int_V U(u, \varphi)d\tau, \end{aligned} \tag{2.28}$$

where t_i and m_i are given by (1.4).

The relations (2.6), (2.10), (2.13), (2.22) can be written in the form

$$\begin{aligned} &u_i = a_1 u_i^{(1)} + a_2 u_i^{(2)}, \qquad \varphi_i = a_1 \varphi_i^{(1)} + a_2 \varphi_i^{(2)}, \\ &t_{ij} = a_1 t_{ij}^{(1)} + a_2 t_{ij}^{(2)}, \qquad m_{ij} = a_1 m_{ij}^{(1)} + a_2 m_{ij}^{(2)}, \end{aligned} \tag{2.29}$$

where

$$u_1^{(1)} = -\frac{1}{2}[x_3^2 + \nu(x_1^2 - x_2^2)] + \nu x_1^0 x_1 + \tau_1^{(1)}, \quad u_2^{(1)} = -\nu x_2(x_1 - x_1^0) + \tau_2^{(1)},$$

$$u_3^{(1)} = (x_1 - x_1^0)x_3, \quad u_1^{(2)} = -\nu x_1(x_2 - x_2^0) + \tau_1^{(2)},$$

$$u_2^{(2)} = -\frac{1}{2}[x_3^2 - \nu(x_1^2 - x_2^2)] + \nu x_2^0 x_2 + \tau_2^{(2)}, \quad u_3^{(2)} = (x_2 - x_2^0)x_3,$$

$$\begin{aligned} &\varphi_1^{(1)} = 0, \ \varphi_2^{(1)} = -x_3, \ \varphi_3^{(1)} = -\nu x_2 + w^{(1)}, \ \varphi_1^{(2)} = x_3, \ \varphi_2^{(2)} = 0, \\ &\varphi_3^{(2)} = \nu x_1 + w^{(2)}, \ t_{\alpha\beta}^{(1)} = \sigma_{\alpha\beta}^{(1)}, \ t_{\alpha\beta}^{(2)} = \sigma_{\alpha\beta}^{(2)}, \ t_{\alpha 3}^{(1)} = t_{3\alpha}^{(1)} = 0, \\ &t_{33}^{(1)} = E(x_1 - x_1^0) + \lambda\varepsilon_{\alpha\alpha}^{(1)}, \ t_{33}^{(2)} = E(x_2 - x_2^0) + \lambda\varepsilon_{\alpha\alpha}^{(2)}, \ t_{\alpha 3}^{(2)} = t_{3\alpha}^{(2)} = 0, \\ &m_{\alpha\beta}^{(1)} = m_{\alpha\beta}^{(2)} = m_{33}^{(1)} = m_{33}^{(2)} = 0, \\ &m_{13}^{(1)} = \gamma w_{,1}^{(1)}, \ m_{31}^{(1)} = \beta w_{,1}^{(1)}, \ m_{23}^{(1)} = \gamma w_{,2}^{(1)} - \beta - \gamma\nu, \ m_{32}^{(1)} = \beta w_{,2}^{(1)} - \beta\nu - \gamma, \\ &m_{13}^{(2)} = \gamma w_{,1}^{(2)} + \beta + \gamma\nu, \ m_{31}^{(2)} = \beta w_{,1}^{(2)} + \beta\nu + \gamma, \ m_{23}^{(2)} = \gamma w_{,2}^{(2)}, \ m_{32}^{(2)} = \beta w_{,2}^{(2)}. \end{aligned} \tag{2.30}$$

It is easy to show that

$$2U(u, \varphi) = a_1^2 U(u^{(1)}, \varphi^{(1)}) + 2a_1 a_2 U(u^{(1)}, \varphi^{(1)}; u^{(2)}, \varphi^{(2)}) + a_2^2 U(u^{(2)}, \varphi^{(2)}). \tag{2.31}$$

The internal energy is

$$W = \int_V U(u, \varphi)d\tau = a_1^2 W_{11} + 2a_1 a_2 W_{12} + a_2^2 W_{22}, \tag{2.32}$$

where

$$\begin{aligned} &W_{11} = \int_V U(u^{(1)}, \varphi^{(1)})d\tau, \ W_{12} = \int_V U(u^{(1)}, \varphi^{(1)}; u^{(2)}, \varphi^{(2)})d\tau, \ W_{22} = \\ &= \int_V U(u^{(2)}, \varphi^{(2)})d\tau. \end{aligned} \tag{2.33}$$

The internal energy is a positive definite quadratic form so that

$$W_{11} W_{22} - W_{12}^2 > 0. \tag{2.34}$$

Let us apply the formula (2.28) for the configurations $u_i^{(\alpha)}, \ldots, m_{ij}^{(\alpha)}$, $(\alpha = 1, 2)$. On the surface $x_3 = l$, we have

$$\begin{aligned} &t_1^{(\alpha)} = t_2^{(\alpha)} = 0, \ t_3^{(\alpha)} = t_{33}^{(\alpha)}, \ m_1^{(\alpha)} = m_{31}^{(\alpha)}, \ m_2^{(\alpha)} = m_{32}^{(\alpha)}, \ m_3^{(\alpha)} = 0, \\ &u_3^{(1)} = l(x_1 - x_1^0), \ \varphi_1^{(1)} = 0, \ \varphi_2^{(1)} = -l, \ u_3^{(2)} = l(x_2 - x_2^0), \ \varphi_1^{(2)} = l, \\ &\varphi_2^{(2)} = 0, \ (\alpha = 1, 2). \end{aligned} \tag{2.35}$$

and on $x_3 = 0$, we have

$$t_1^{(\alpha)} = t_2^{(\alpha)} = 0, \ u_3^{(\alpha)} = 0, \ \varphi_1^{(\alpha)} = \varphi_2^{(\alpha)} = 0, \ m_3^{(\alpha)} = 0, \ (\alpha = 1, 2). \tag{2.36}$$

Taking into account the boundary conditions (2.5) and the relations (2.30), (2.35), (2.36), from $(2.28)_2$, (2.33) we obtain

$$\begin{aligned} &2W_{11} = l\int_\Sigma \{(x_1 - x_1^0)[E(x_1 - x_1^0) + \lambda\varepsilon_{\alpha\alpha}^{(1)}] + \beta\nu + \gamma - \beta w_{,2}^{(1)}\}d\sigma = H_{11}, \\ &2W_{22} = l\int_\Sigma \{(x_2 - x_2^0)[E(x_2 - x_2^0) + \lambda\varepsilon_{\alpha\alpha}^{(2)}] + \beta\nu + \gamma + \beta w_{,1}^{(2)}\}d\sigma = H_{22}, \end{aligned} \tag{2.37}$$

on the basis of relations (2.21).

If we apply the formula $(2.28)_1$ for $u_i = u_i^{(1)}$, $\varphi_i = \varphi_i^{(1)}$ and $u'_i = u_i^{(2)}$, $\varphi'_i = \varphi_i^{(2)}$ and use (2.30), (2.33), (2.21) we get

$$2W_{12} = lL_{12} = lL_{21}. \tag{2.38}$$

From (2.34), (2.37) and (2.38), it follows that

$$d = L_{11}L_{22} - L_{12}^2 \neq 0, \tag{2.39}$$

so that from (2.23) we obtain

$$a_1 = \frac{1}{d}(L_{22}M_2 + L_{12}M_1), \quad a_2 = -\frac{1}{d}(L_{11}M_1 + L_{12}M_2). \tag{2.40}$$

Thus, the solution of our problem has the form (2.6) where $v_\alpha^{(\rho)}(x_1, x_2)$, $w^{(\rho)}(x_1, x_2)$, $(\alpha, \rho = 1, 2)$ are the solutions of the plane micropolar strain problems (2.9), (2.11), (2.14), (2.17), (2.18) and the constants a_1, a_2 are given by (2.40).

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ÎNCOVOIEREA PURĂ A BARELOR ELASTICE MICROPOLARE

Rezumat

Se rezolvă problema încovoierii de către cupluri a barelor omogene şi izotrope în teoria liniară a elasticităţii micropolare.

MHD COUETTE FLOW BETWEEN TWO NON-CONDUCTING PARALLEL WALLS WITH VARIABLE SUCTION (I)

BY

SHISHIR KUMAR DUBE

Introduction. Recently Messiha [1] has investigated the Couette flow of a viscous incompressible and electrically conducting fluid between two infinite parallel walls in the presence of a uniform transverse magnetic field when the suction velocity as well as the motion of one wall varies periodically with time about a non-zero constant mean. He has however, neglected the induced magnetic field assuming that the magnetic Reynolds number is small. He has also assumed that the electric field $\vec{E} = 0$. In view of these simplified assumptions, Maxwell's equations become redundant and the problem becomes simple.

In this problem the above assumptions are not made and the induced magnetic field is taken into consideration in the particular case when the magnetic Reynolds number is equal to the viscous Reynolds number. Tables showing the variations of amplitude, phase lag at the fixed wall and the phase lead at the moving wall of the skin friction fluctuations are made.

Similar types of problems have been studied by Muhuri [2], Soundalgekar [3], Dube [4, 5] and some others.

Analysis. Let the x'-axis be taken along the fixed wall and the y'-axis normal to it and the moving wall is at $y' = h/R$, R being the suction Reynolds number. The equations governing the unsteady flow of a viscous incompressible and electrically conducting fluid between two non-conducting and nonmagnetic infinite porous flat plates subjected to periodic suction in the presence of a uniform transverse magnetic field of strength H_0, are given by

$$v' = V_0 (1 + \varepsilon A e^{i\omega' t'}) \tag{1}$$