If we apply the formula $(2.28)_1$ for $u_i = u_i^{(1)}$, $\varphi_i = \varphi_i^{(1)}$ and $u_i' = u_i^{(2)}$, $\varphi_i' = \varphi_i^{(2)}$ and use (2.30), (2.33), (2.21) we get

$$2W_{12} = lL_{12} = lL_{21}. \tag{2.38}$$

From (2.34), (2.37) and (2.38), it follows that

$$d = L_{11} L_{22} - L_{12}^2 \neq 0, \tag{2.39}$$

so that from (2.23) we obtain

$$a_1 = \frac{1}{d}(L_{22}M_2 + L_{12}M_1), \quad a_2 = -\frac{1}{d}(L_{11}M_1 + L_{12}M_2). \tag{2.40}$$

Thus, the solution of our problem has the form (2.6) where $v_\alpha^{(\rho)}(x_1, x_2)$, $w^{(\rho)}(x_1, x_2)$, $(\alpha, \rho = 1, 2)$ are the solutions of the plane micropolar strain problems (2.9), (2.11), (2.14), (2.17), (2.18) and the constants $a_1$, $a_2$ are given by (2.40).

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### ÎNCOVOIEREA PURĂ A BARELOR ELASTICE MICROPOLARE


Rezumat

Se rezolvă problema încovoierii de către cupluri a barelor omogene şi izotrope în teoria liniară a elasticităţii micropolare.


# MHD COUETTE FLOW BETWEEN TWO NON-CONDUCTING PARALLEL WALLS WITH VARIABLE SUCTION (I)


BY

SHISHIR KUMAR DUBE


**Introduction.** Recently Messiha [1] has investigated the Couette flow of a viscous incompressible and electrically conducting fluid between two infinite parallel walls in the presence of a uniform transverse magnetic field when the suction velocity as well as the motion of one wall varies periodically with time about a non-zero constant mean. He has however, neglected the induced magnetic field assuming that the magnetic Reynolds number is small. He has also assumed that the electric field $\vec{E} = 0$. In view of these simplified assumptions, Maxwell's equations become redundant and the problem becomes simple.

In this problem the above assumptions are not made and the induced magnetic field is taken into consideration in the particular case when the magnetic Reynolds number is equal to the viscous Reynolds number. Tables showing the variations of amplitude, phase lag at the fixed wall and the phase lead at the moving wall of the skin friction fluctuations are made.

Similar types of problems have been studied by Muhuri [2], Soundalgekar [3], Dube [4, 5] and some others.

**Analysis.** Let the $x'$-axis be taken along the fixed wall and the $y'$-axis normal to it and the moving wall is at $y' = h/R$, $R$ being the suction Reynolds number. The equations governing the unsteady flow of a viscous incompressible and electrically conducting fluid between two non-conducting and nonmagnetic infinite porous flat plates subjected to periodic suction in the presence of a uniform transverse magnetic field of strength $H_0$, are given by

$$v' = V_0 (1 + \varepsilon A e^{i\omega' t}) \tag{1}$$

$$\frac{\partial u'}{\partial t'} + V_0 (1 + \varepsilon A e^{i\omega' t'}) \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\alpha^2}{H_0} \frac{\partial H'_x}{\partial y'}, \tag{2}$$

$$\frac{\partial p}{\partial y'} + \frac{\mu e H'_x}{4\pi} \frac{\partial H'_x}{\partial y'} = -\rho \frac{\partial v'}{\partial t'}, \tag{3}$$

$$\frac{\partial H'_x}{\partial t'} + V_0 (1 + \varepsilon A e^{i\omega' t'}) \frac{\partial H'_x}{\partial y'} = \nu_m \frac{\partial^2 H'_x}{\partial y'^2} + H_0 \frac{\partial u'}{\partial y'}, \tag{4}$$

where $H'_x$ is the induced magnetic field in $x'$-direction, $V_0 (1 + \varepsilon A e^{i\omega' t'})$, the periodic suction velocity, $\varepsilon$ a small number such that $\varepsilon A < 1$, $V_0$ and $A$ are real positive constants, $\omega'$ — the frequency of oscillations, $\alpha = H_0 (\mu_e / 4\pi\rho)^{1/2}$, the Alfvén velocity, $t'$ the time, $\rho$ the density, $\mu e$ the magnetic permeability, $\nu$ the kinematic viscosity, $\nu_m = 1/4\pi\sigma\mu e$, the magnetic diffusivity and $\sigma$ the electrical conductivity $(u', v')$ are the velocity components parallel and perpendicular to the plates respectively.

Introducing the non-dimensional quantities defined by

$$\left\{ \begin{aligned} & y = \frac{R}{h} y', \quad u = \frac{u'}{V_0}, \quad t = \frac{V_0 R}{4h} t', \\ & \omega = \frac{4h}{V_0 R} \omega', \quad R = \frac{V_0 h}{\nu}, \quad M = \frac{\alpha}{V_0}, \\ & H'_x = V_0 \left( \frac{4\pi\rho}{\mu_e} \right)^{1/2} H_x, \quad R_m = \frac{\nu}{\nu_m}, \end{aligned} \right. \tag{5}$$

where $R$ is the suction Reynolds number, and substituting in (2) and (4), we get

$$\frac{\partial^2 u}{\partial y^2} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} - \frac{1}{4} \frac{\partial u}{\partial t} + M \frac{\partial H_x}{\partial y} = 0, \tag{6}$$

$$\frac{\partial^2 H_x}{\partial y^2} - R_m (1 + \varepsilon A e^{i\omega t}) \frac{\partial H_x}{\partial y} - \frac{R_m}{4} \frac{\partial H_x}{\partial t} + M R_m \frac{\partial u}{\partial y} = 0. \tag{7}$$

The boundary conditions of $u$ and $H_x$ are

$$\left\{ \begin{aligned} & u = 0, \quad H_x = 0 \text{ at } y = 0, \\ & \text{and} \\ & u = 1 + \varepsilon e^{i\omega t}, \quad H_x = 0 \text{ at } y = 1. \end{aligned} \right. \tag{8}$$

**The velocity field.** In order to solve the equations (6) and (7) subject to boundary conditions (8), we assume that



$$\left\{ \begin{aligned} & u = f_1(y) + \varepsilon e^{i\omega t} f_2(y), \\ & H_x = H_1(y) + \varepsilon e^{i\omega t} H_2(y). \end{aligned} \right. \tag{9}$$

Substituting relations (9) in equations (6) and (7) and comparing harmonic terms, neglecting coefficients of $\varepsilon^2$, we get the following system for the determination of functions $f_1$, $f_2$, $H_1$ and $H_2$

$$\frac{d^2 f_1}{dy^2} - \frac{df_1}{dy} + M \frac{dH_1}{dy} = 0, \tag{10}$$

$$\frac{d^2 H_1}{dy^2} - R_m \frac{dH_1}{dy} + M R_m \frac{df_1}{dy} = 0, \tag{11}$$

$$\frac{d^2 f_2}{dy^2} - \frac{df_2}{dy} - \frac{i\omega}{4} f_2 + M \frac{dH_2}{dy} = A \frac{df_1}{dy}, \tag{12}$$

$$\frac{d^2 H_2}{dy^2} - R_m \frac{dH_2}{dy} - \frac{i\omega}{4} H_2 + M R_m \frac{df_2}{dy} = A R_m \frac{dH_1}{dy}. \tag{13}$$

The boundary conditions are

$$\left\{ \begin{aligned} & f_1 = 0 = f_2, \quad H_1 = 0 = H_2 \text{ at } y = 0, \\ & f_1 = 1 = f_2, \quad H_1 = 0 = H_2 \text{ at } y = 1. \end{aligned} \right. \tag{14}$$

It is extremely difficult to solve (10)—(13) for arbitrary value of $R_m$. However, when $R_m = 1$, the problem becomes simple so we shall confine ourselves only to that particular case.

When $R_m = 1$, the solution of equations (10)—(13) with the boundary conditions (14) are

$$\left\{ \begin{aligned} f_1 = {} & \frac{1}{2} [(e^{1+M} - 1)^{-1} \exp\{(1+M) y\} + (e^{1-M} - 1)^{-1} \exp\{(1-M) y\}] + \\ & + (e^{1+M} - 1)^{-1} (e^{1-M} - 1)^{-1} (1 - e \cosh M), \end{aligned} \right. \tag{15}$$

$$\left\{ \begin{aligned} H_1 = {} & \frac{1}{2} [(e^{1-M} - 1)^{-1} \exp\{(1-M) y\} - \\ & - (e^{1+M} - 1)^{-1} \exp\{(1+M) y\}] - \\ & - (e^{1+M} - 1)^{-1} (e^{1-M} - 1)^{-1} e \sinh M, \end{aligned} \right. \tag{16}$$

$$
(17)\quad \left\{
\begin{aligned}
f_2 = {} & \frac{|S_2|}{|S_1|} \exp \{h_1 y + i(\varphi_2 - \varphi_1)\} - \\
& - \frac{|S_3|}{|S_1|} \exp \{h_2 y + i(\varphi_3 - \varphi_1)\} + \\
& + \frac{|S_5|}{|S_4|} \exp \{n_1 y + i(\varphi_5 - \varphi_4)\} - \\
& - \frac{|S_6|}{|S_4|} \exp \{n_2 y + i(\varphi_6 - \varphi_4)\} + \\
& + \frac{2Ai}{\omega} (1 + M)(e^{1+M} - 1)^{-1} \exp \{(1 + M)y\} + \\
& + \frac{2Ai}{\omega} (1 - M)(e^{1-M} - 1)^{-1} \exp \{(1 - M)y\},
\end{aligned}
\right.
$$

$$
(18)\quad \left\{
\begin{aligned}
H_2 = {} & \frac{|S_2|}{|S_1|} \exp \{h_1 y + i(\varphi_2 - \varphi_1)\} - \\
& - \frac{|S_3|}{|S_1|} \exp \{h_2 y + i(\varphi_3 - \varphi_1)\} - \\
& - \frac{|S_5|}{|S_4|} \exp \{n_1 y + i(\varphi_5 - \varphi_4)\} + \\
& + \frac{|S_6|}{|S_4|} \exp \{n_2 y + i(\varphi_6 - \varphi_4)\} - \\
& - \frac{2Ai}{\omega} (1 + M)(e^{1+M} - 1)^{-1} \exp \{(1 + M)y\} + \\
& + \frac{2Ai}{\omega} (1 - M)(e^{1-M} - 1)^{-1} \exp \{(1 - M)y\},
\end{aligned}
\right.
$$

where,

$$S_1 = S_{1r} + iS_{1i}, \qquad \tan \varphi_1 = S_{1i} / S_{1r},$$

$$
\begin{aligned}
S_{1r} = {} & [\exp (|R_1| \cos \theta_1) \cos (|R_1| \sin \theta_1) - \\
& - \exp (|R_2| \cos \theta_2) \cos (|R_2| \sin \theta_2)],
\end{aligned}
$$

$$
\begin{aligned}
S_{1i} = {} & [\exp (|R_1| \cos \theta_1) \sin (|R_1| \sin \theta_1) + \\
& + \exp (|R_2| \cos \theta_2) \sin (|R_2| \sin \theta_2)],
\end{aligned}
$$

$$R_1 = R_{1r} + i R_{1i}, \qquad \tan \theta_1 = R_{1i} / R_{1r},$$



$$R_{1r} = \frac{1}{2}\left[ (1 - M) + R_3 \cos \frac{\theta_3}{2} \right], \quad R_{1i} = \frac{1}{2} R_3 \sin \frac{\theta_3}{2},$$

$$R_2 = R_{2r} + i R_1 i, \quad \tan \theta_2 = R_{1i}/R_{2r},$$

$$R_{2r} = \frac{1}{2}\left[ (1 - M) - R_3 \cos \frac{\theta_3}{2} \right],$$

$$R_3 = [\omega^2 + (1 - M)^4]^{1/4}, \quad \tan \theta_3 = \frac{\omega}{(1 - M)^2}.$$

$$S_2 = S_{2r} + i S_{2i}, \quad \tan \varphi_2 = S_{2i}/S_{2r},$$

$$S_{2r} = \left[ \frac{1}{2} + \frac{2A}{\omega} (1 - M)(e^{1-M} - 1)^{-1} \sin (|R_2| \sin \theta_2) \exp (|R_2| \cos \theta_2) \right],$$

$$
\begin{aligned}
S_{2i} = {} & \frac{2A}{\omega} (1 - M)(e^{1-M} - 1)^{-1} [\cos (|R_2| \sin \theta_2) \exp (|R_2| \cos \theta_2) - \\
& - \exp (1 - M)],
\end{aligned}
$$

$$S_3 = S_{3r} + iS_{3i}, \quad \tan \varphi_3 = S_{3i}/S_{3r},$$

$$S_{3r} = \left[ \frac{1}{2} - \frac{2A}{\omega} (1 - M)(e^{1-M} - 1)^{-1} \sin (|R_1| \sin \theta_1) \exp (|R_1| \cos \theta_1) \right],$$

$$
\begin{aligned}
S_{3i} = {} & \frac{2A}{\omega} (1 - M)(e^{1-M} - 1)^{-1} [\cos (|R_1| \sin \theta_1) \exp (|R_1| \cos \theta_1) - \\
& - \exp (1 - M)],
\end{aligned}
$$

$$S_4 = S_{4r} + i S_{4i}, \quad \tan \varphi_4 = S_{4i}/S_{4r},$$

$$
\begin{aligned}
S_{4r} = {} & [\exp (|R_4| \cos \theta_4) \cos (|R_4| \sin \theta_4) - \\
& - \exp (|R_5| \cos \theta_5) \cos (|R_5| \sin \theta_5)],
\end{aligned}
$$

$$
\begin{aligned}
S_{4i} = {} & [\exp (|R_4| \cos \theta_4) \sin (|R_4| \sin \theta_4) + \\
& + \exp (|R_5| \cos \theta_5) \sin (|R_5| \sin \theta_5)],
\end{aligned}
$$

$$R_4 = R_{4r} + i R_{4i}, \quad \tan \theta_4 = R_{4i}/R_{4r},$$

$$R_{4r} = \frac{1}{2}\left[ (1 + M) + R_6 \cos \frac{\theta_6}{2} \right], \quad R_{4i} = \frac{1}{2} R_6 \sin \frac{\theta_6}{2},$$

$$R_5 = R_{5r} + i R_{4i}, \quad \tan \theta_5 = R_{4i}/R_{5r},$$

$$R_{5r} = \frac{1}{2}\left[ (1 + M) - R_6 \cos \frac{\theta_6}{2} \right],$$

$$R_6 = [\omega^2 + (1+M)^4]^{1/4}, \quad \tan\theta_6 = \frac{\omega}{(1+M)^2}\,.$$

$$S_5 = S_{5r} + i\,S_{5i}\,, \quad \tan\varphi_5 = S_{5i}/S_{5r}$$

$$S_{5r} = \left[\frac{1}{2} + \frac{2A}{\omega}(1+M)(e^{1+M}-1)^{-1}\sin(|R_5|\sin\theta_5)\times\exp(|R_5|\cos\theta_5)\right],$$

$$S_{5i} = \frac{2A}{\omega}(1+M)(e^{1+M}-1)^{-1}[\cos(|R_5|\sin\theta_5)\exp(|R_5|\cos\theta_5) - \\ - \exp(1+M)],$$

$$S_6 = S_{6r} + i\,S_{6i}\,, \quad \tan\varphi_6 = S_{6i}/S_{6r}$$

$$S_{6r} = \left[\frac{1}{2} - \frac{2A}{\omega}(1+M)(e^{1+M}-1)^{-1}\times\sin(|R_4|\sin\theta_4)\exp(|R_4|\cos\theta_4)\right],$$

$$S_{6i} = \frac{2A}{\omega}(1+M)(e^{1+M}-1)^{-1}[\exp(1+M) - \cos(|R_4|\sin\theta_4)\exp(|R_4|\cos\theta_4)],$$

$$h_1 = \frac{1}{2}[(1-M) + \sqrt{(1-M)^2 + i\omega}], \quad h_2 = \frac{1}{2}[(1-M) - \sqrt{(1-M)^2 + i\omega}],$$

$$n_1 = \frac{1}{2}[(1+M) + \sqrt{(1+M)^2 + i\omega}], \quad n_2 = \frac{1}{2}[(1+M) - \sqrt{(1+M)^2 + i\omega}].$$

Hence the velocity and induced magnetic field between the parallel walls is given by

$$(19)\quad \begin{cases} u = \dfrac{1}{2}[(e^{1+M}-1)^{-1}\exp\{(1+M)y\} + (e^{1-M}-1)^{-1}\exp\{(1-M)y\}] + \\ \quad - + (e^{1+M}-1)^{-1}(e^{1-M}-1)^{-1}(1 - e\cosh M) + \\ \quad + \varepsilon e^{i\omega t}\left[\dfrac{|S_2|}{|S_1|}\exp\{h_1 y + i(\varphi_2-\varphi_1)\} - \right. \\ \quad - \dfrac{|S_3|}{|S_1|}\exp\{h_2 y + i(\varphi_3-\varphi_1)\} + \\ \quad + \dfrac{|S_5|}{|S_4|}\exp\{n_1 y + i(\varphi_5-\varphi_4)\} - \\ \quad - \dfrac{|S_6|}{|S_7|}\exp\{n_2 y + i(\varphi_6-\varphi_4)\} + \\ \quad + \dfrac{2Ai}{\omega}(1+M)(e^{1+M}-1)^{-1}\exp\{(1+M)y\} + \\ \quad \left. + \dfrac{2Ai}{\omega}(1-M)(e^{1-M}-1)^{-1}\exp\{(1-M)y\}\right], \end{cases}$$



$$(20)\quad \begin{cases} H_x = \dfrac{1}{2}[(e^{1-M}-1)^{-1}\exp\{(1-M)y\} - (e^{1+M}-1)^{-1}\exp\{(1+M)y\}] - \\ \quad - (e^{1+M}-1)^{-1}(e^{1-M}-1)^{-1}e\sinh M + \\ \quad + \varepsilon e^{i\omega t}\left[\dfrac{|S_2|}{|S_1|}\exp\{h_1 y + i(\varphi_2-\varphi_1)\} - \right. \\ \quad - \dfrac{|S_3|}{|S_1|}\exp\{h_2 y + i(\varphi_3-\varphi_1)\} - \\ \quad - \dfrac{|S_5|}{|S_4|}\exp\{n_1 y + i(\varphi_5-\varphi_4)\} + \\ \quad + \dfrac{|S_6|}{|S_4|}\exp\{n_2 y + i(\varphi_6-\varphi_4)\} - \\ \quad - \dfrac{2Ai}{\omega}(1+M)(e^{1+M}-1)^{-1}\exp\{(1+M)y\} + \\ \quad \left. + \dfrac{2Ai}{\omega}(1-M)(e^{1-M}-1)^{-1}\exp\{(1-M)y\}\right]. \end{cases}$$

**The skin friction.** The non-dimensional skin friction $\tau_s$ at the stationary wall is given by

$$(21)\quad \begin{aligned} \tau_s &= \frac{\tau_0}{\rho V_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{2}[(1+M)(e^{1+M}-1)^{-1} + (1-M)(e^{1-M}-1)^{-1}] + \\ &+ \varepsilon e^{i\omega t}\left[h_1\frac{|S_2|}{|S_1|}\exp\{i(\varphi_2-\varphi_1)\} - h_2\frac{|S_3|}{|S_1|}\exp\{i(\varphi_3-\varphi_1)\} + \right. \\ &\quad + n_1\frac{|S_5|}{|S_4|}\exp\{i(\varphi_5-\varphi_4)\} - n_2\frac{|S_6|}{|S_4|}\exp\{i(\varphi_6-\varphi_4)\} + \\ &\left. + \frac{2Ai}{\omega}(1+M)^2(e^{1+M}-1)^{-1} + \frac{2Ai}{\omega}(1-M)^2(e^{1-M}-1)^{-1}\right] \end{aligned}$$

and the non-dimensional skin friction $\tau_m$ at the moving wall is given by

$$\tau_m = \frac{\tau_1}{\rho V_0^2} = \left(\frac{\partial u}{\partial y}\right)_{y=1} = \frac{1}{2}[(1+M)(e^{1+M}-1)^{-1}\exp(1+M) +$$

$$+ (1-M)(e^{1-M}-1)^{-1}\exp(1-M)] + \varepsilon e^{i\omega t}\left[h_1\frac{|S_2|}{|S_1|}\exp\{h_1 + i(\varphi_2-\varphi_1)\} - \right.$$

$$(22) \qquad - h_2 \frac{|S_3|}{|S_1|} \exp\{h_2 + i(\varphi_3 - \varphi_1)\} + n_1 \frac{|S_5|}{|S_4|} \exp\{n_1 + i(\varphi_5 - \varphi_4)\} -$$

$$- n_2 \frac{|S_6|}{|S_4|} \exp\{n_2 + i(\varphi_6 - \varphi_4)\} + \frac{2Ai}{\omega}(1+M)^2 (e^{1+M} - 1)^{-1} \times$$

$$\times \exp(1+M) + \frac{2Ai}{\omega}(1-M)^2 (e^{1-M} - 1)^{-1} \exp(1-M) \Big] .$$

**Numerical results.** For numerical calculations we will write (21) and (22) in the form:

The skin friction at the stationary wall

$$(23) \qquad \begin{aligned} \tau_s = {} & (1+M)(e^{1+M} - 1)^{-1} + (1-M)(e^{1-M} - 1)^{-1} \\ & + \varepsilon\, |F| \exp\{i(\omega t + \alpha)\} \end{aligned}$$

where

$$F = F_r + i F_i, \quad \tan \alpha = F_i / F_r$$

$$F_r = \Big[ |R_1| \frac{|S_2|}{|S_1|} \cos(\theta_1 + \varphi_2 - \varphi_1) -$$

$$- |R_2| \frac{|S_3|}{|S_1|} \cos(\varphi_3 - \varphi_1 - \theta_2) + |R_4| \frac{|S_5|}{|S_4|} \cos(\theta_4 + \varphi_5 - \varphi_4) -$$

$$- |R_5| \frac{|S_6|}{|S_4|} \cos(\varphi_6 - \varphi_4 - \theta_5) \Big] ,$$

$$F_i = \Big[ |R_1| \frac{|S_2|}{|S_1|} \sin(\theta_1 + \varphi_2 - \varphi_1) - |R_2| \frac{|S_3|}{|S_1|} \sin(\varphi_3 - \varphi_1 - \theta_2) +$$

$$+ |R_4| \frac{|S_5|}{|S_4|} \sin(\theta_4 + \varphi_5 - \varphi_4) - |R_5| \frac{|S_6|}{|S_4|} \sin(\varphi_6 - \varphi_4 - \theta_5) +$$

$$+ \frac{2A}{\omega} \{(1+M)^2 (e^{1+M} - 1)^{-1} + (1-M)^2 (e^{1-M} - 1)^{-1}\} \Big] .$$

The skin friction at the moving wall

$$(24) \qquad \begin{aligned} \tau_m = {} & [(1+M)(e^{1+M} - 1)^{-1} \exp(1+M) + \\ & + (1-M)(e^{1-M} - 1)^{-1} \exp(1-M)] + \varepsilon\, |G| \exp\{i(\omega t + \beta)\} \end{aligned}$$

where



$$G = G_r + i G_i, \quad \tan \beta = G_i / G_r ,$$

$$G_r = \Big[ |R_1| \frac{|S_2|}{|S_1|} \cos(\theta_1 + \varphi_2 - \varphi_1 + |R_1| \sin\theta_1) \exp(|R_1| \cos\theta_1) -$$

$$- |R_2| \frac{|S_3|}{|S_1|} \cos(\varphi_3 - \varphi_1 - \theta_2 - |R_2| \sin\theta_2) \exp(|R_2| \cos\theta_2) +$$

$$+ |R_4| \frac{|S_5|}{|S_4|} \cos(\theta_4 + \varphi_5 - \varphi_4 + |R_4| \sin\theta_4) \exp(|R_4| \cos\theta_4) -$$

$$- |R_5| \frac{|S_6|}{|S_4|} \cos(\varphi_6 - \varphi_4 - \theta_5 - |R_5| \sin\theta_5) \exp(|R_5| \cos\theta_5) \Big] ,$$

$$G_i = \Big[ |R_1| \frac{|S_2|}{|S_1|} \sin(\theta_1 + \Phi_2 - \varphi_1 + |R_1| \sin\theta_1) \exp(|R_1| \cos\theta_1) -$$

$$- |R_2| \frac{|S_3|}{|S_1|} \sin(\varphi_3 - \varphi_1 - \theta_2 - |R_2| \sin\theta_2) \exp(|R_2| \cos\theta_2) +$$

$$+ |R_4| \frac{|S_5|}{|S_4|} \sin(\theta_4 + \varphi_5 - \varphi_4 + |R_4| \sin\theta_4) \exp(|R_4| \cos\theta_4) -$$

$$- |R_5| \frac{|S_6|}{|S_4|} \sin(\varphi_6 - \varphi_4 - \theta_5 - |R_5| \sin\theta_5) \exp(|R_5| \cos\theta_5) +$$

$$+ \frac{2A}{\omega} \{(1+M)^2 (e^{1+M} - 1)^{-1} \exp(1+M) + (1-M)^2 (e^{1-M} - 1)^{-1} \exp(1-M)\} \Big] .$$

The following tables show the values of the amplitudes and phase of skin friction fluctuations at the two walls for different values of $A$, $M$ and $\omega$.

TABLE I

Amplitude of skin friction fluctuation at the stationary wall $|F|$

| $M$ | 0 | | | 0.3 | | | 0.6 | | |
|---|---|---|---|---|---|---|---|---|---|
| $\omega$ | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ |
| 5 | 0.1325 | 0.1131 | 0.0567 | 0.2719 | 0.1246 | 0.1095 | 0.3982 | 0.2723 | 0.2130 |
| 10 | 0.1213 | 0.1072 | 0.0481 | 0.2234 | 0.1191 | 0.1010 | 0.2338 | 0.2216 | 0.1362 |
| 15 | 0.1184 | 0.1021 | 0.0411 | 0.2196 | 0.1098 | 0.0900 | 0.2262 | 0.2207 | 0.2007 |
| 20 | 0.1009 | 0.0712 | 0.0137 | 0.2037 | 0.0943 | 0.0639 | 0.2141 | 0.2066 | 0.1915 |

TABLE II

Amplitude of skin friction fluctuations at the moving wall | G |

| $M$ / $\omega$ | 0 | | | 0.3 | | | 0.6 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ |
| 5 | 4.5371 | 4.7702 | 5.0023 | 2.4763 | 2.6319 | 2.7910 | 1.4311 | 1.5925 | 1.6666 |
| 10 | 4.6218 | 4.8331 | 5.1211 | 2.6139 | 2.8428 | 3.2213 | 1.7387 | 1.9976 | 2.2136 |
| 15 | 4.8301 | 4.9541 | 5.2634 | 2.9176 | 3.4333 | 3.6130 | 1.9881 | 2.4510 | 2.6010 |
| 20 | 5.0421 | 5.1633 | 5.5141 | 3.3728 | 3.8125 | 4.1351 | 2.0010 | 2.7126 | 3.1211 |

TABLE III

Skin friction phase at the stationary wall tan $\alpha$

| $M$ / $\omega$ | 0 | | | 0.3 | | | 0.6 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ |
| 5 | 0.25 | 0.29 | 0.36 | 0.20 | 0.21 | 0.29 | 0.09 | 0.15 | 0.19 |
| 10 | 0.42 | 0.50 | 0.65 | 0.31 | 0.39 | 0.48 | 0.12 | 0.20 | 0.27 |
| 15 | 0.84 | 1.22 | 1.37 | 0.40 | 0.54 | 0.91 | 0.19 | 0.29 | 0.36 |
| 20 | 1.42 | 1.76 | 2.01 | 0.98 | 1.01 | 1.46 | 0.27 | 0.63 | 0.83 |

TABLE IV

Skin friction phase at the moving wall tan $\beta$

| $M$ / $\omega$ | 0 | | | 0.3 | | | 0.6 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ | $A=0$ | $A=0.5$ | $A=1$ |
| 5 | 0.25 | 0.20 | 0.13 | 0.13 | 0.11 | 0.08 | 0.06 | 0.05 | 0.02 |
| 10 | 0.50 | 0.34 | 0.26 | 0.17 | 0.14 | 0.10 | 0.07 | 0.06 | 0.04 |
| 15 | 0.62 | 0.55 | 0.40 | 0.24 | 0.19 | 0.15 | 0.11 | 0.08 | 0.05 |
| 20 | 0.70 | 0.63 | 0.46 | 0.26 | 0.21 | 0.18 | 0.19 | 0.11 | 0.07 |

A study of the table indicates that:

1) The amplitude of skin friction fluctuations at the stationary wall increases as the magnetic field strength $M$ is increased while in the Messiha's [1] case i. e. whe the induced magnetic field is neglected the effect of magnetic field is to damp the skin friction amplitude at the stationary wall. It is also seen that the effect of suction is to damp the skin friction amplitude at the stationary wall.



2) The amplitude of skin friction fluctuation at the moving wall increases with increasing $A$ while the effect of magnetic field is to damp the skin friction amplitude at this wall which is in contradiction with the result of Messiha [1] who has shown that when the induced magnetic field is neglected the effect of magnetic field is to increase the amplitude of skin friction fluctuation at the moving wall.

3) The skin friction phase lag at the stationary wall increases with suction and decreases as the magnetic field strength $M$ is increased.

4) The skin friction phase lead at the moving wall decreases with increasing $A$ or $M$.

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## CURGEREA COUETTE MAGNETOHIDRODINAMICĂ ÎNTRE DOI PEREȚI NECONDUCTORI CU SECȚIUNE VARIABILĂ

Rezumat

În această lucrare este studiată curgerea laminară nestaționară a unui fluid electroconductor, vîscos, incompresibil între doi pereți plani infiniți, poroși, neconductori și nemagnetici, supusă la o sucțiune periodică, în prezența unui cîmp magnetic transversal uniform.

Numărul Reynold magnetic se presupune egal cu numărul Reynold vîscos. Unul dintre pereți este presupus fix iar celălalt, ca și viteza de sucțiune variază periodic în timp față de o medie constantă diferită de zero.

Se obțin expresiile pentru viteză, cîmpul magnetic indus și fricțiunea de skin, în formă adimensională.