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ŞIRURI GENERALIZATE DE FUNCŢII

Rezumat

Se consideră şirurile generalizate de funcţii:

$$\pi = \{f_a:\ X \to Z;\ a \in A\},$$

$$\pi = \{f_a:\ X \times Y \to E,\ a \in A\}$$

unde X şi Y sînt spaţii topologice, Z un spaţiu uniform, E un spaţiu vectorial topologic, $(A, \leqslant)$ o mulţime dirijată. Se găsesc condiţii necesare şi suficiente pentru ca limita să fie continuă sau bidimensional continuă.

A RESULT ON GENERALIZED HYPERGEOMETRIC FUNCTION AND GENERALIZED MEIJER FUNCTION OF TWO VARIABLES

BY

MANILAL SHAH

The object of the present paper is to evaluate an infinite integral involving product of hypergeometric function and generalized Meijer function of two variables with the help of known expansion and integral formulae. On specializing the parameters, the function $S\left[\begin{matrix}x\\ y\end{matrix}\right]$ incorporates not only Meijer's G-function, or the product of two G-functions, as its particular cases, but it also includes most of the commonly used functions in two arguments e.g. Kampé de Fériet's [5] double hypergeometric function F which in turn yields the Appell functions [2] F_1, F_2, F_3 and F_4, the Whittaker function of two variables etc. However, some interesting results are obtained as particular cases of the integral.

1. The integral. The integral to be evaluated is

$$(1.1)\quad \int_0^\infty x^{u+v}\,{}_mF_{n+1}\left[\begin{matrix}\alpha_m\\ \Delta(2,u+1),\ \beta_{n-1};\end{matrix}\ -4x^2yz^2\right] S\left[\begin{matrix}\alpha x^{2\rho}\\ \beta x^{2\sigma}\end{matrix}\right] dx = \frac{\rho^{u}}{(2z)^{u+v+1}}$$

$$\sum_{\sigma=0}^{\infty} \frac{(u/2+\sigma)\,\Gamma(u+\sigma)}{\sigma!}\ {}_{m+2}F_{n+1}\left[\begin{matrix}-\sigma,\ u+\sigma,\ \alpha_m;\ y\\ \Delta(2,u+1),\ \beta_{n-1}\end{matrix}\right]$$

$$S\left[\begin{array}{c|c|c} \begin{bmatrix} p+\rho, & 0 \\ A-p+\rho, & B\end{bmatrix} & \begin{array}{ll}\Delta(\rho, v/2+1/2(u+2\sigma)+1/2), & (a),\\ \Delta(\rho, v/2-1/2(u+2\sigma)+1/2); & (b)\end{array} & \alpha\left(\frac{\rho}{2z}\right)^{2\rho} \\ \begin{pmatrix} q\ , & r\\ C-q, & D-r\end{pmatrix} & \begin{array}{rl} & (c);\quad (d)\\ & (e);\quad (f)\end{array} & \beta\left(\frac{\rho}{2z}\right)^{2\rho} \\ \begin{pmatrix} k\ , & l\\ E-k, & F-l\end{pmatrix} & & \end{array}\right],$$

where ρ is a positive integer, $z>0$ and valid under the conditions:

$$\text{(i)}\quad \begin{cases} 2(p+q+r)>A+B+C+D,\ |\arg(\alpha)|<[p+q+r-1/2(A+B+\\ \qquad +C+D)]\,\pi,\\ 2(p+k+l)>A+B+E+F,\ |\arg(\beta)|<[p+k+l-1/2(A+B+\\ \qquad +E+F)]\,\pi,\\ -\operatorname{Re}(u+2\sigma)-3/2<\operatorname{Re}[v+2\rho(d_{h_1}+f_{h_2})]<-1/2,\\ \{h_1=1,2,\dots,r;\ h_2=1,2,\dots,l\} \end{cases}$$

or

$$\text{(ii)}\quad \begin{cases} A+C<B+D,\ A+E<B+F\\ \text{or else } A+C=B+D,\ A+E=B+F \text{ with } |\alpha|<1,\ |\beta|<1,\\ -\operatorname{Re}(u+2\sigma)-3/2<\operatorname{Re}[v+2\rho(d_{h_1}+f_{h_2})]<-1/2,\\ \{h_1=1,2,\dots,r;\ h_2=1,2,\dots,l\} \end{cases}$$

and where $S[:]$ is defined by Sharma in [6].

Proof: Firstly, in view of (1.3) and using Legendre's duplication formula [4, p. 5(18)]:

$$\Gamma(2z)=2^{2z-1}\pi^{-1/2}\,\Gamma(z)\,\Gamma(z+1/2),$$

we reduce the expansion formula [1]

$$\frac{\Gamma(\lambda+1)}{\Gamma(\mu+1)\,\Gamma(\gamma+1)}\ {}_pF_{q+1}\left[\begin{array}{l}\alpha_1,\alpha_2,\dots,\alpha_p\\ \mu+1,\ \gamma+1,\ \beta_1,\dots,\beta_{q-1}\end{array};4xy\right]=$$

$$=\sum_{n=0}^{\infty}\frac{(\lambda+2n)\,\Gamma(\lambda+n)}{n!}\,x^n R_n(\lambda+2n,\ \mu+n,\ \gamma+n,x).$$

$${}_{p+2}F_{q+1}\left[\begin{array}{l}-n,\ \lambda+n,\ \alpha_1,\alpha_2,\dots,\alpha_p\\ 1/2(\lambda+1),\ 1/2(\lambda+2),\ \beta_1,\dots,\beta_{q-1}\end{array};y\right]$$

with R defined in [1], to the following form

$$(1.2)\quad x^u{}_mF_{n+1}\left[\begin{array}{l}\alpha_m\\ \Delta(2,u+1),\ \beta_{n-1}\end{array};-4x^2yz^2\right]=(2z)^{-u}\sum_{\sigma=0}^{\infty}\frac{(u+2\sigma)\,\Gamma(u+\sigma)}{\sigma!}\,.$$

$$.\,J_{u+2\sigma}(4xz)\,{}_{m+2}F_{n+1}\left[\begin{array}{l}-\sigma,\ u+\sigma,\ \alpha_m\ ;y\\ \Delta(2,u+1),\ \beta_{n-1}\end{array}\right].$$

Multiplying both sides of (1.2) by $x^v S\left[\begin{array}{l}\alpha x^{2\rho}\\ \beta x^{2\rho}\end{array}\right]$ and integrating with respect to x over $(0,\infty)$, then changing the order of integration and summation on the right, we obtain

$$(1.3)\quad \int_0^{\infty} x^{u+v}{}_mF_{n+1}\left[\begin{array}{l}\alpha_m\\ \Delta(2,u+1),\ \beta_{n-1}\end{array};-4x^2yz^2\right]S\left[\begin{array}{l}\alpha x^{2\rho}\\ \beta x^{2\rho}\end{array}\right]dx=$$

$$(2z)^{-u}\sum_{\sigma=0}^{\infty}\frac{(u+2\sigma)\ \Gamma(u+\sigma)}{\sigma!}\ {}_{m+2}F_{n+1}\left[\begin{array}{l}-\sigma,\ u+\sigma,\ \alpha_m\ ;y\\ \Delta(2,u+1),\ \beta_{n-1}\end{array}\right]$$

$$\int_0^{\infty} x^v J_{u+2\sigma}(4xz)\,S\left[\begin{array}{l}\alpha x^{2\rho}\\ \beta x^{2\rho}\end{array}\right]dx.$$

Now making use of the known integral [7]:

$$(1.4)\quad \int_0^{\infty} x^{\mu} J_{\nu}(xy)\,S\left[\begin{array}{l}\alpha x^{2\rho}\\ \beta x^{2\rho}\end{array}\right]dx=(2\rho)^{\mu}y^{-\mu-1}$$

$$S\left[\begin{array}{c|c|c} \begin{bmatrix} p+\rho, & 0 \\ A-p+\rho, & B\end{bmatrix} & \begin{array}{ll}\Delta(\rho, \mu/2+1/2\nu+1/2, & (a)\\ \Delta(\rho, \mu/2-1/2\nu+1/2); & (b)\end{array} & \alpha\left(\frac{2\rho}{y}\right)^{2\rho} \\ \begin{pmatrix} q\ , & r\\ C-q, & D-r\end{pmatrix} & \begin{array}{rl} & (c);\quad (d)\\ & (e);\quad (f)\end{array} & \beta\left(\frac{2\rho}{y}\right)^{2\rho} \\ \begin{pmatrix} k\ , & l\\ E-k, & F-l\end{pmatrix} & & \end{array}\right]$$

where ρ is a positive integer, $y>0$ and valid under the following conditions:

$$
\text{(i)}\quad \begin{cases} 2(p+q+r) > A+B+C+D,\ |\arg(\alpha)| < [p+q+r-1/2(A+B+ \\ \qquad +C+D)]\,\pi, \\ 2(p+k+l) > A+B+E+F,\ |\arg(\beta)| < [p+k+l-1/2(A+B+ \\ \qquad +E+F)]\,\pi, \\ -\operatorname{Re}(\nu) - 3/2 < \operatorname{Re}[\mu + 2\rho(d_{h_1} + f_{h_2})] < -1/2, \\ \{h_1 = 1,2,\dots,r;\ h_2 = 1,2,\dots,l\}, \end{cases}
$$

or

$$
\text{(ii)}\quad \begin{cases} A+C < B+D,\ A+E < B+F, \\ \text{or else } A+C=B+D,\ A+E=B+F \text{ with} |\alpha| < 1, |\beta| < 1, \\ -\operatorname{Re}(\nu) - 3/2 < \operatorname{Re}[\mu + 2\rho(d_{h_1} + f_{h_2})] < -1/2, \\ \{h_1 = 1,2,\dots,r;\ h_2 = 1,2,\dots,l\}, \end{cases}
$$

on the right of (1.3), which completes the evaluation of (1.1).

The change of the order of integration and summation is justified due to Bromwich [3, p. 500] and under the conditions stated in (2.1) when.

$$
\text{(i)}\qquad \sum_{\sigma=0}^{\infty} \frac{(u+2\sigma)\,\Gamma(u+\sigma)}{\sigma!}\, J_{u+2\sigma}(4xz)\,{}_{m+2}F_{n+1}\left[\begin{matrix} -\sigma, u+\sigma, \alpha_m; y \\ \Delta(2, u+1), \beta_{n-1} \end{matrix}\right]
$$

converges uniformly for $0 < x \leqslant \xi$, where ξ is arbitrary,
(ii) generalized Meijer function in two variables is a continuous function of x and y for all finite values of $x \geqslant x_0 \geqslant 0$ and $y \geqslant y_0 \geqslant 0$,
(iii) the integral on the right converges under the conditions given in (1.4).

2. Applications. (A) Particular cases of (1.1) with proper choice of parameters of $S\left[\begin{matrix} \alpha x^{2\rho} \\ \beta x^{2\rho} \end{matrix}\right]$:

1. Setting $a = B = p = 0$, we obtain

$$
\text{(2.1)}\qquad \int_0^\infty x^{u+v}\,{}_mF_{n+1}\left[\begin{matrix} \alpha_m & ; & -4x^2yz^2 \\ \Delta(2, u+1), & \beta_{n-1} & \end{matrix}\right] G^{\,r,q}_{\,C,D}\left(\alpha x^{2\rho}\middle|\begin{matrix}(c)\\(d)\end{matrix}\right)
$$

$$
G^{\,l,k}_{\,E,F}\left(\beta x^{2\rho}\middle|\begin{matrix}(e)\\(f)\end{matrix}\right) dx = \frac{\rho^v}{(2z)^{u+v+1}} \sum_{\sigma=0}^{\infty} \frac{(u/2+\sigma)\,\Gamma(u+\sigma)}{\sigma!}\,{}_{m+2}F_{n+1}
$$

$$
\left[\begin{matrix} -\sigma, u+\sigma, \alpha_m; & y \\ \Delta(2, u+1), \beta_{n-1} & \end{matrix}\right]
$$

$$
S\left[\begin{matrix} \begin{bmatrix} \rho, 0 \\ \rho, 0 \end{bmatrix} \\ \begin{pmatrix} q, & r \\ C-q, & D-r \end{pmatrix} \\ \begin{pmatrix} k, & l \\ E-k, & F-l \end{pmatrix} \end{matrix}\ \middle|\ \begin{matrix} \Delta(\rho, v/2 \pm 1/2(u+2\sigma)+1/2; \dots \\ (c);\ (d) \\ (e);\ (f) \end{matrix}\ \middle|\ \begin{matrix} \alpha\left(\dfrac{\rho}{2z}\right)^{2\rho} \\ \beta\left(\dfrac{\rho}{2z}\right)^{2\rho} \end{matrix}\right]
$$

where G is Meijer's G-function [4, p. 207] and valid under the condition referred to (1.1) with $A = B = p = 0$.

2. Substituting $A = E = p = k = l - 1 = F - 1 = 0$ and replacing $A + C$ by A, $B + D$ by B, $A + q$ by s together with appropriate changes in the parameters etc., and then let $\beta \to 0$, we get

$$
\text{(2.2)}\qquad \int_0^\infty x^{u+v}\,{}_mF_{n+1}\left[\begin{matrix} \alpha_m & ; & -4x^2yz^2 \\ \Delta(2, u+1), & \beta_{n-1} & \end{matrix}\right] G^{\,r,s}_{\,A,B}\left(\alpha x^{2\rho}\middle|\begin{matrix}(a)\\(b)\end{matrix}\right) dx =
$$

$$
= \frac{\rho^v}{(2z)^{u+v+1}} \sum_{\sigma=0}^{\infty} \frac{(\sigma+u/2)\,\Gamma(u+\sigma)}{\sigma!}\,{}_{m+2}F_{n+1}\left[\begin{matrix} -\sigma, u+\sigma, \alpha_m; & y \\ \Delta(2, u+1), \beta_{n-1} & \end{matrix}\right]
$$

$$
G^{\,r,s+\rho}_{\,A+2\rho,B}\left(\alpha\left[\frac{\rho}{2z}\right]^{2\rho}\middle|\begin{matrix} \Delta(\rho, v/2+1/2(u+2\sigma)+1/2), (a), \\ \Delta(\rho, 1/2v - 1/2(u+2\sigma)+1/2 \\ (b) \end{matrix}\right)
$$

where ρ is a positive integer, $z > 0$ and

$$
\text{(i)}\quad \begin{cases} 2(r+s) > A+B,\ |\arg(\alpha)| < [r+s-1/2(A+B)]\pi, \\ -\operatorname{Re}(u+2\sigma) - 3/2 < \operatorname{Re}(v+2\rho b_j) < -1/2,\ j = 1,2,\dots,r \end{cases}
$$

$$
\text{(ii)}\quad \begin{cases} A < B \text{ or else } A = B \text{ with } |\alpha| < 1, \\ -\operatorname{Re}(u+2\sigma - 3/2 < \operatorname{Re}(v+2\rho b_j) < -1/2,\ j = 1,2,\dots,r. \end{cases}
$$

3. Taking $p = A = M$, $B = N$, $q = k = C = E = L$, $r = l = 1$, $D = F = p + 1$, $d_1 = f_1 = 0$ and replacing b_j, $1 - c_j$, $1 - d_j$, $1 - e_j$ and $1 - f_j$ by c_j, b_j', d_j, b_j' and d_j' respectively. Thus we obtain the result for Kampé de Fériet's double hypergeometric function:

$$(2.3)\quad \int_0^\infty x^{u+v}{}_m F_{n+1}\left[\begin{matrix}\alpha_m & ; -4x^2yz^2\\ \Delta(2,u+1), & \beta_{n-1}\end{matrix}\right] F\left(\begin{matrix}M\\L\\N\\P\end{matrix}\left|\begin{matrix}(a_M)\\(b_L);(b'_L)\\(c_N)\\(d_P);(d'_P)\end{matrix}\right|\begin{matrix}\alpha x^{2\rho}\\ \beta x^{2\rho}\end{matrix}\right) dx =$$

$$= \frac{\rho^v}{(2z)^{u+v+1}} \sum_{\sigma=0}^\infty \frac{(u/2+\sigma)\,\Gamma(u+\sigma)}{\sigma!}\; {}_{m+2}F_{n+1}\left[\begin{matrix}-\sigma, u+\sigma, \alpha_m; & y\\ \Delta(2,u+1), \beta_{n-1} & \end{matrix}\right] \prod_{i=0}^{\rho-1}$$

$$\Gamma\left(\frac{v/2 \pm 1/2(u+2\sigma)+1/2+i}{\rho}\right) F\left(\begin{matrix}M+2\rho\\L\\N\\P\end{matrix}\left|\begin{matrix}\Delta(\rho, v/2 \pm 1/2(u+2\sigma)+1/2), (a_M)\\ (b_L);\ (b'_L)\\ (c_N)\\ (d_P);\ (d'_P)\end{matrix}\right|\begin{matrix}\alpha\left(\frac{\rho}{2z}\right)^{2\rho}\\ \beta\left(\frac{\rho}{2z}\right)^{2\rho}\end{matrix}\right)$$

where ρ is a positive integer, $z > 0$ and valid if

$$(M+L) < (N+P+1),\ \{M+L = N+P+1:\ \text{then } |\alpha|<1,\ |\beta|<1\},$$

$-\operatorname{Re}(u+2\sigma) - 3/2 < \operatorname{Re}(v) < -1/2$, or if $M+L+1 > N+P$, then

$$|\arg(\beta)|,\quad |\arg(\alpha)| < (M+L-N-P)\frac{\pi}{2}.$$

(B) *Special cases of* (2.3): (*i*) By setting the parameters suitably the double hypergeometric function F can be reduced to Appell's functions. Thereby we can obtain several interesting results involving Appell's functions.

(ii) We know that the F-function reduces to the generalized hypergeometric function of one variable by taking $M=N$, $L=1$, $P=0$ and $\beta=\alpha$ etc. Thus we obtain

$$(2.4)\quad \int_0^\infty x^{u+v}{}_m F_{n-1}\left[\begin{matrix}\alpha_m & ; & -4x^2yz^2\\ \Delta(2,u+1), & \beta_{n-1} & \end{matrix}\right] {}_{p+1}F_p\left[\begin{matrix}a_p, b+b' & ; \alpha x^{2\rho}\\ c_p & \end{matrix}\right] dx =$$

$$= \frac{\rho^v}{(2z)^{u+v+1}} \sum_{\sigma=0}^\infty \frac{(1/2u+\sigma)\,\Gamma(u+\sigma)}{\sigma!}\; {}_{m+2}F_{n+1}\left[\begin{matrix}-\sigma, u+\sigma, \alpha_m & ; y\\ \Delta(2,u+1), \beta_{n-1} & \end{matrix}\right] \times$$

$$\prod_{i=0}^{\rho-1} \Gamma\left(\frac{v/2 \pm 1/2(u+2\sigma)+1/2+i}{\rho}\right) {}_{p+2\rho+1}F_p\left[\begin{matrix}a_p, \Delta(\rho, 1/2v + 1/2(u+2\sigma)+1/2), b+b' & ; \alpha\left(\frac{\rho}{2z}\right)^{2\rho}\\ c_p & \end{matrix}\right]$$

valid for ρ a positive integer and $-\operatorname{Re}(u+2\sigma) - 3/2 < \operatorname{Re}(v) < -1/2$.

Further setting $\rho = \alpha = 1$, $b+b' = -N$, $a_1 = N + \alpha + \beta + 1$, $c_1 = 1+\alpha$, $c_2 = 1/2$ and multiplying bot sides by $(1+\alpha)_N/N!$, we obtain

$$(2.5)\quad \int_0^\infty x^{u+v}{}_m F_{n+1}\left[\begin{matrix}\alpha_m & ; & -4x^2yz^2\\ \Delta(2,u+1), & \beta_{n-1} & \end{matrix}\right] f_N^{(\alpha,\beta)}\left(\begin{matrix}a_2,\dots,a_p\\ c_3,\dots,c_p\end{matrix}; x^2\right) dx =$$

$$= \frac{\rho^v}{(2z)^{u+v+1}} \sum_{\sigma=0}^\infty \frac{(u/2+\sigma)\,\Gamma(u+\sigma)}{\sigma!}\; {}_{m+2}F_{n+1}\left[\begin{matrix}-\sigma, u+\sigma, \alpha_m & ; y\\ \Delta(2,u+1), \beta_{n-1} & \end{matrix}\right]$$

$$\Gamma(1/2v \pm 1/2(u+2\sigma)+1/2)\, f_N^{(\alpha,\beta)}\left(\begin{matrix}a_2,\dots,a_p, & v/2 \pm 1/2(u+2\sigma)+1/2\\ c_3,\dots,c_p & \end{matrix}; \left[\frac{1}{2z}\right]^2\right)$$

where $-\operatorname{Re}(u+2\sigma) - 3/2 < \operatorname{Re}(v) < -1/2$ and

$$f_n^{\alpha,\beta}\left(\begin{matrix}a_2,\dots,a_p\\ b_3,\dots,b_q; & x\end{matrix}\right) = \frac{(1+\alpha)n}{n!}\, {}_{p+1}F_q\left[\begin{matrix}-n, n+\alpha+\beta+1, & a_2,\dots,a_p & ;\\ 1+\alpha, & 1/2, & b_3,\dots,b_q\end{matrix}\right]$$

the generalized Sister Celine polynomial [8].

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UN REZULTAT PRIVITOR LA FUNCŢII HIPERGEOMETRICE GENERALIZATE ŞI FUNCŢII MEIJER GENERALIZATE DE DOUĂ VARIABILE

Rezumat

Autorul evaluează unele integrale de tipul (1.1), prin funcţii de tip Meijer şi hipergeometrice generalizate. Cu ajutorul acestora, prin particularizarea parametrilor, se obţin diferite formule utile.

MEIJER-LAPLACE TRANSFORM OF TWO VARIABLES

BY

R. S. DAHIYA

1. Several generalizations of the classical Laplace-Stieltjes integrahave been given by Meijer [1], Bose [2], Varma [4, 5].

In the present paper I shall develop the theory of the Meijer-Laplace transform on parallel lines to that of Laplace transform of two variables. The Meijer-Laplace transform is defined as follows:

$$\Phi(s) = s\int_0^\infty e^{-wst}\, G_{pq}^{mn}\left(2st\,\middle|\,\begin{matrix}a_1, a_2,\ldots, a_p\\ b_1, b_2,\ldots, b_q\end{matrix}\right) d\alpha(t). \tag{1.1}$$

We shall use the short hand notation for the G-function

$$G_{pq}^{mn}\left(2st\,\middle|\,\begin{matrix}a_p\\ b_q\end{matrix}\right),$$

when no ambiguity can arise.

By taking $w = 1$ and with the help of the following identities, we can deduce the corresponding results for the transforms of Bose [2] and Varma [4] by giving particular values to the parameters of the G-function.

$$\text{(a)}\qquad e^{-st}\, G_{12}^{21}\left(2st\,\middle|\,\begin{matrix}1\\ 1/2-\nu,\ 1/2+\nu\end{matrix}\right) \equiv \Gamma(1/2\pm\nu)\left(\frac{2st}{\pi}\right)^{1/2} K_\nu(st),$$

and

$$\text{(b)}\qquad e^{-st}\, G_{12}^{21}\left(2st\,\middle|\,\begin{matrix}K+3/4\\ 1/4-\mu,\ \mu+1/4\end{matrix}\right) \equiv \Gamma(1/2\pm\mu-K)(2st)^{-1/4}\, W_{K,\mu}(2st).$$