ON THE THEORY OF LINEAR AUTONOMOUS SEQUENTIAL NETWORKS

BY

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A linear autonomous network contains the following three elements:

1) The elements of delay,

2) The summators over $GF(p)$,

3) The elements, which perform the scalar multiplication by modulus p.

Let us consider a linear autonomous sequential network with n elements of delay. The network works at discrete moments. In any moment of time the internal state of the network can be determined by the state $(X_1, X_2, \ldots, X_n)$ of inputs of the delay elements. The values $(X'_1, X'_2, \ldots, X'_n)$ of inputs of the delay elements become — after a unit of time — the values of their outputs. The values of inputs X'_i can be represented by linear combinations with the coefficients over $GF(p)$ of various values of outputs of delay elements at the present moment. Therefore the state $(X'_1, X'_2, \ldots, X'_n)$ at the following moment can be expressed through the state at the present moment by means of the linear system:

$$X'_i = \sum_{j=1}^{n} a_{ij} X_j,\ i = 1, 2, \ldots, n,$$

with $a_{ij} \in GF(p)$, $1 \leqslant i, j \leqslant n$.

Let us denote $X' = \begin{pmatrix} X'_1 \\ \vdots \\ X'_n \end{pmatrix}$, $X = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$, $T = (a_{ij})$.

We have $X' = TX$. If X is an initial state of the network, then the sequence of the network states in the following moments of time is:

$$TX,\ T^2X,\ T^3X \ldots$$

If the matrix T is nonsingular, then to a given state, it corresponds a unique preceding state. In this case the network is said to be undegenerated. The behaviour of networks along the sequence of the states can be described by the diagram (graph) of the states, where the circlets will represent the states, while an arrow will lead from X to Y, whenever $Y = TX$. The number of distinct states in the diagram for the networks over $GF(p)$ with n elements of delay is p^n. The considered networks are determined in the sense that the state at the present moment determines uniquely the state at the next moment. The behaviour of networks can be investigated by means of the matrix T. E l s p a s [1] studied the behaviour of the undegenerated networks. Our aim is to study the networks in the general case, including both the undegenerated and degenerated case.

I. Case of the undegenerated networks. Let $i_1(x), i_2(x), \ldots, i_t(x)$ be the invariant factors of the matrix T corresponding to the given undegenerated network. We have:

$$i_1(x) = [\psi_1(x)]^{e_{11}}\,[\psi_2(x)]^{e_{12}} \ldots [\psi_k(x)]^{e_{1k}},$$

$$i_2(x) = [\psi_1(x)]^{e_{21}}\,[\psi_2(x)]^{e_{22}} \ldots [\psi_k(x)]^{e_{2k}},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$i_t(x) = [\psi_1(x)]^{e_{t1}}\,[\psi_2(x)]^{e_{t2}} \ldots [\psi_k(x)]^{e_{tk}},$$

where $e_{1j} \geqslant e_{2j} \geqslant \ldots \geqslant e_{tj}$, with $j = 1, 2, \ldots, k$ and $[\psi_j(x)]^{e_{ij}}$ are the elementary divisors of T.

Lemma 1. [2] *If T and T' are similar, their state graphs are isomorphic.*

Because the network is undegenerated, all the irreducible polynomials $\psi_j(x)$, with $j = 1, 2, \ldots k$, are different from the polynomial $t(x) = x$.

Let $\mathcal{T}$ be the linear operator corresponding to the matrix T in any basis of the linear space $V_n(p)$. It is known from linear algebra that the linear space $V_n(p)$ is decomposable in a direct sum of cyclic spaces:

$$V_n(p) = I_{11} \dotplus I_{12} + \ldots \dotplus I_{1k} + \ldots + I_{t1} \dotplus I_{t2} \dotplus \ldots \dotplus I_{tk}.$$

The minimal polynomial of the cyclic subspaces I_{ij} is $[\psi_j(x)]^{e_{ij}}$, for $i = 1, 2, \ldots, t$, $j = 1, 2, \ldots, k$. The subspace I_{ij} is the zero space of the operator $[\psi_j(\mathcal{T})]^{e_{ij}}$, I_{ij} is invariant under the operator $\mathcal{T}$. Let k_j be the degree of the polynomial ψ_j, $j = 1, 2, \ldots, k$. The space I_{ij} has the dimension $k_j e_{ij}$. The restriction of the operator on the subspace I_{ij} will have — as characteristic polynomial — just $[\psi_j(x)]^{e_{ij}}$.

Definition. *The cycle structure of $[\psi_j(x)]^{e_{ij}}$ is the cycle structure induced by the companion matrix M_{ij}. The cycle structure of M_{ij} is denoted by*

$$[\mu_1(k_1), \ldots, \mu_s(k_s)],$$

if M_{ij} induces μ_i cycles of length i, $i = 1, 2, \ldots, s$.

Difinition [2]. *Let $[\mu_1(k_1), \ldots, \mu_s(k_s)]$ and $[\eta_1(h_1), \ldots, \eta_u(h_u)]$ be cycle structures. The cross operation is defined by:*

$$[\mu_1(k_1), \ldots, \mu_s(k_s)] \times [\eta_1(h_1), \ldots, \eta_u(h_u)] = [\gamma_1(l_1), \ldots, \gamma_r(l_r)],$$

where $\gamma_i(l_i) = \mu_m \times \eta_u \times (k_m, h_u) \times [k_m, h_u]$, $[k_m, h_u]$ is the least common multiple of k_m and h_u and (k_m, h_u) is their greatest common divisor.

Since the space $V_n(p)$ is the direct sum of the cyclic spaces I_{ij}, his cycle structure is the cross operation of the cycle structures of $[\psi_j(x)]^{e_{ij}}$, $j = 1, 2, \ldots, k$, $i = 1, 2, \ldots, t$.

II. The case of the degenerated networks. Let T be the matrix assigned to the given network, with n elements of delay. Let T be singular matrix and r the rank of T, then $r < n$. We regard the columns of the matrix T as vectors from $V_n(p)$. There are r linearly independent columns. Let C_i be these vectors ($i = 1, 2, \ldots, r$). Then all column vectors from the matrix T will be expressed linearly in C_i:

$$C_j = \sum_{i=1}^{r} k_{ij}\, C_j \qquad j = 1, 2, \ldots, n, \quad k_{ij} \in GF(p).$$

The state X of the network at the present moment and the state X' at the next moment satisfy the relation $TX = X'$.

Let X' be a given state; it will admit preceding states iff the vector X' is a linear combination of the column vectors from T:

$$X' = \sum_{i=1}^{n} h_i\, C_i = \sum_{j=1}^{r} k_j C_j, \quad h_i,\ k_j \in GF(p), \tag{1}$$

$$i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, r.$$

If a state X allows preceding states, then the number of these states is p^{n-r}. The number of the nonzero states, which admit the preceding states is $p^r - 1$. The zero state denoted $\overline{O}$ admits p^{n-r} preceding states. That is why the number of the states, which admit the preceding states is p^r. It follows that a state of the network admits no preceding state or admits the preceding states in number of p^{n-r}. The matrix T of the network being singular, it follows that the characteristic polynomial $\varphi(x)$ of the matrix T will contain as irreducible factors over $GF(p)$ the powers of the polynomial x, namely of the form x^l. Therefore the minimal polynomial of T, namely $i_1(x)$, will contain a power of x, that is x^{λ_1}. We consider the elementary divisors of T, which are the power of x:

$$x^{\lambda_1},\ x^{\lambda_2}, \ldots, x^{\lambda_k}, \quad \text{with } \lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_k \geqslant 1. \tag{2}$$

As in case I, the space $V_n(p)$ is decomposable in the direct sum of the cycle subspaces, corresponding to the elementary divisors of T. We consider the subspaces corresponding to the elementary divisors others than (2). The direct sum of these subspaces will give a subspace E_0. The cyclic structure of the subspace E_0 is known from the case of the undegenerated networks. Hence the cyclic structure of the subspace E_0 will consist of cycles of certain lengths.

Let E_i be the cyclic subspace corresponding to the elementary divisor x^{λ_i}, and X_i the generating vector of the cyclic subspace $E_i (i = 1, 2 \dots k)$. We have:

$$V_n(p) = E_0 \dotplus E_1 \dotplus E_2 \dotplus \dots \dotplus E_k.$$

Let us denote $E' = E_1 \dotplus E_2 \dotplus \dots \dotplus E_k$. The vectors from $V_n(p)$, which are obtained by adding a vector from E_0 and the zero vector of every subspace E_i (for $i = 1, 2, \dots k$), will be placed in the diagram of states in the cyclic structure of E_0. The vectors from $V_n(p)$, which are obtained by adding the zero vector of E_0 and a vector from E' will be placed in the diagram of states in configuration of the zero state, because for all $Y \in E'$, $T^{\lambda_1} Y = \bar{O}$.

The vectors $X_i, TX_i, \dots T^{\lambda_i - 1} X_i$ form a basis in the subspace E_i, $i = 1, 2, \dots k$. A basis in E' will be formed of the vectors $X_1, TX_1, \dots T^{\lambda_1 - 1} X_1, X_2, TX_2, \dots T^{\lambda_1 - 1} X_2, \dots, X_k, TX_k, \dots, T^{\lambda_k - 1} X_k$.

A vector $Y \in E'$ will be written:

$$Y = \sum_{i=1}^{k} \sum_{j=1}^{\lambda_i} a_{ij} T^{j-1} X_j, \tag{3}$$

where $a_{ij} \in GF(p)$, $i = 1, 2, \dots k$, $j = 1, 2, \dots \lambda_i$. The number of the non-zero vectors from E' is $p^{\sum_1^k \lambda_i} - 1$. For the vector $Y \in E'$ we suppose that there exists Z, such that $TZ = Y$. We have:

$$Z = \sum_{i=1}^{k} \sum_{j=1}^{\lambda_i} b_{ij} T^{j-1} X_i, \quad TZ = \sum_{i=1}^{k} \sum_{j=1}^{\lambda_i - 1} b_{ij} T^j X_i.$$

It follows:

$$a_{ij} = b_{i(j-1)}, \ j = 2, \dots \lambda_i, \ i = 1, 2, \dots k, \ a_{i1} = 0, \ i = 1, 2, \dots k. \tag{4}$$

Consequently, from all the states of the diagram of the subspace E', only the states for which $a_{i1} = 0$, $i = 1, 2, \dots k$, have preceding states. Those states for which at least $a_{i1} \neq 0$, with $i \in \{1, 2, \dots k\}$ will have no preceding state. From (4) we have:

$$b_{ij} = a_{ij+1}, \ j = 1, 2, \dots \lambda_i - 1, \ i = 1, 2, \dots k.$$

It follows from (3) that if Y admits preceding states, then their number is p^k. It is known that $n - r = k$, therefore this number (p^k) is the same with p^{n-r}, which was established above.

III. The analysis of the set of states. Definition. *The set of all states X, for which there is i, such that $T^i X = \bar{O}$ form the null tree.*

Definition. *A state X is said to be on the level i of a tree if i is the smallest exponent, for which $T^i X = \bar{O}$.* The states on the level λ_h are characterized by the following conditions:

$$a_{i(j_i - \lambda_h)} = O \text{ for all } i = 1, 2 \dots h - 1, \ j_i = 1, 2 \dots \lambda_i \tag{5}$$

and at least $a_{i(\lambda_i - \lambda_h + 1)} \neq 0$ for any $i \in \{1, 2, \dots h - 1\}$ *or* $a_{h1} \neq 0$.

Indeed, let Y be the vector with $a_{i(\lambda_i - \lambda_h + 1)} \neq 0$, for i fixed. We have:

$$T^{\lambda_h} Y = T^{\lambda_h}(a_{i(\lambda_i - \lambda_h + 1)} T^{\lambda_i - \lambda_h} X_i) = a_{i(\lambda_i - \lambda_h + 1)} T^{\lambda_i} X_i = \bar{O}$$
$$T^l Y = T^l (a_{i(\lambda_i - \lambda_h + 1)} T^{\lambda_i - \lambda_h} X_i) = a_{i(\lambda_i - \lambda_h + 1)} T^{\lambda_i - \lambda_h + l} X_i$$

for $1 \leqslant l < \lambda_h$. If it exists $a_{i(\lambda_i - \lambda_h + 1)} \neq 0$, then $T^l Y \neq \bar{O}$, for $1 \leqslant l < \lambda_h$, because the vectors $T^{\lambda_i - \lambda_h + l} X_i$ are linearly independent for $l < \lambda_h$. We denote by d_{λ_h} the number of states, belonging to the level λ_h. According to the characterization (5) of these states, we obtain that:

$$d_{\lambda_h} = (p^h - 1) p^{\sum_{i=1}^{k} \lambda_i - (\lambda_1 - \lambda_h + \lambda_2 - \lambda_h + \dots + \lambda_{h-1} - \lambda_h) - h} =$$
$$= (p^h - 1) p^{h(\lambda_h - 1) + \sum_{i=h+1}^{k} \lambda_i} \text{ for } 1 \leqslant h \leqslant k.$$

We will denote by d'_{λ_h} number of the states, which are on the level λ_h and admit anterior states, and by d''_{λ_h} the number of states on level λ_h, which do not admit an anterior state. For the first ones we have:

$$a_{h1} \neq 0 \text{ or } a_{(h+1)1} \neq 0 \text{ or } \dots \text{ or } a_{k1} \neq 0$$

$$d'_{\lambda_h} = (p - 1) p^{\sum_{i=1}^{k} \lambda_i - 1 - (\lambda_1 + \dots + \lambda_{h-1} - (h-1)\lambda_h)} +$$

$$+ (p^{k-h} - 1)(p^{h-1} - 1) p^{\sum_{i=1}^{k} \lambda_i - (k-h+1) - (h-1) - (\lambda_1 + \lambda_2 + \dots + \lambda_{h-1} - (h-1)\lambda_h)} =$$

$$= p^{h\lambda_h + \sum_{h+1}^{k} \lambda_i - k} (p^k - p^{h-1} - p^{k-h} + 1).$$

$$d''_{\lambda_h} = (p^h - 1) p^{h(\lambda_h - 1) + \sum_{h+1}^{k} \lambda_i} - (p^k - p^{h-1} - p^{k-h} + 1) p^{h\lambda_h + \sum_{h+1}^{k} \lambda_i - k} =$$

$$= (p^{h-1} - 1) p^{h\lambda_h + \sum_{h+1}^{k} \lambda_i - k}.$$

Similarly, we find :

$$d_{\lambda_h+j} = (p^{h-1} - 1)p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i + (j-1)(h-1)},$$

for $1 \leqslant j \leqslant \lambda_{h-1} - \lambda_h, \quad 2 \leqslant h \leqslant k, \quad \lambda_h > \lambda_{h-1}$.

$$d'_{\lambda_h+j} = (p^{k-h+1} - 1)\ (p^{h-1} - 1)p^{k\lambda_h + \sum_{i=h+1}^{k} \lambda_i + (j-1)(h-1) - (k-h+1)}$$

for $1 \leqslant j \leqslant \lambda_{h-1} - \lambda_h, \quad 2 \leqslant h \leqslant k, \quad \lambda_h > \lambda_{h-1},$

$$d''_{\lambda_h+j} = (p^{h-1} - 1)p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i + (j-1)(h-1) - (k-h+1)},$$

for $1 \leqslant j \leqslant \lambda_{h-1} - \lambda_h, \quad 2 \leqslant h \leqslant k, \quad \lambda_h > \lambda_{h-1},$

$$\sum_{j=1}^{k} d_j = (p^k - 1)\ (1 + p^k + \ldots + p^{(\lambda_k-1)k}) = p^{k\lambda_k} - 1$$

$$S = \sum_{\lambda_h+1}^{h-1} d_s = (p^{h-1} - 1)\, p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i} (1 + p^{h-1} + \ldots + p^{(h-1)(\lambda_{h-1}-\lambda_h-1)}) =$$

$$= (p^{(h-1)(\lambda_h-1-\lambda_h)} - 1)\ p^{h\lambda_h + \sum_{h+1}^{k} \lambda_i} = p^{(h-1)\lambda_{h-1} + \sum_{i=h}^{k} \lambda_i} -$$

$$- p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i}.$$

$$r = \sum_{\lambda_k+1}^{\lambda_1} d_r = \sum_{h=2}^{k} \left(\sum_{s=\lambda_h+1}^{\lambda_h-1} d_s \right) = \sum_{h=2}^{k} \left(p^{(h-1)\lambda_{h-1} + \sum_{i=h}^{k} \lambda_i} - p^{h\lambda_h + \sum_{h+1}^{k} \lambda_i} \right) =$$

$$= p^{\sum_{i=1}^{k} \lambda_i} - p^{k\lambda_k}.$$

Therefore the number of states of the null tree is :

$$\sum_{v=1}^{\lambda_1} d_v + 1 = (p^{\sum_{i=1}^{k} \lambda_i} - p^{k\lambda_k}) + (p^{k\lambda_k} - 1) + 1 = p^{\sum_{i=1}^{k} \lambda_i}.$$

This is exactly the number of vectors from E'.

a) When

$$\lambda_h = \lambda_{h-1} = \ldots = \lambda_{h-j+1}, \text{ with } 1 \leqslant j \leqslant h,$$

one obtains :

$$d_{\lambda_h} = (p^h - 1)\, p^{h(\lambda_h-1) + \sum_{i=h+1}^{k} \lambda_i}, \text{ for } 1 \leqslant h \leqslant k,$$

$$d'_{\lambda_h} = (p^k - p^{k-j} - p^{k-h} + 1)p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i - k}.$$

It follows that :

$$d''_{\lambda_h} = (p^{h-j} - 1)\, p^{h\lambda_h + \sum_{i=h+1}^{k} \lambda_i - k} = (p^{h-j} - 1)p^{(h-j)\lambda_h + \sum_{i=h-j+1}^{k} \lambda_i - k}.$$

b) 1° When

$$\lambda_k > 1, \text{ then } \lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_k > 1.$$

Let Y be the vector, which admits anterior states. Let Z_1, Z_2 be two anterior states of Y, i. e. $TZ_1 = Y$, $TZ_2 = Y$.

Lemma 2. *If Z_1 admits anterior states, then Z_2 admits anterior states.*

Proof. We have :

$$Z_1 = \sum_{i=1}^{k} \sum_{j=1}^{\lambda i-1} a_{ij+1}\, T^{j-1} X_i + \sum_{h=1}^{k} b^1_{h\lambda_h}\, T^{\lambda_h-1} X_h,$$

$$Z_2 = \sum_{i=1}^{k} \sum_{j=1}^{\lambda i-1} a_{ij+1}\, T^{j-1} X_i + \sum_{h=1}^{k} b^2_{h\lambda_h}\, T^{\lambda_h-1} X_h.$$

Since $\lambda_k > 1$, it follows that $\lambda_i > 1$, for $i = 1, 2, \ldots k - 1$. Then in Z_1 and Z_2 the coefficients for X_i are the same, namely a_{i2}, for all $i = 1, 2, \ldots k$. If $a_{i2} = 0$ for $i = 1, 2, \ldots k$, then the states Z_1 and Z_2 admit anterior states. If $a_{i2} \neq 0$ for at least one $i \in \{1, 2, \ldots k\}$, Z_1 and Z_2 do not admit the perceding states.

$$2°.\ \lambda_k = \lambda_{k-1} = \ldots = \lambda_{k-j+1} = 1, \ \lambda_{k-j} > 1, \text{ where } 1 \leqslant j \leqslant k$$

Lemma 3. *If the state Y admits anterior states, then from these p^k anterior states of its, a number of $(p^j - 1)p^{k-j}$ do not admit anterior state, while a number of p^{k-j} admit preceding states.*

Thus, the configuration of the null tree is completely elucidated, up to an isomorphism ([2], p. 176).

IV. The general case. We have $V_n(p) = E_0 + E'$. The vectors from E' form the null tree. The nonzero vectors from E_0 are placed in the cycles [1]. Let us study the diagram of state vectors from $V_n(p)$, which are obtained as the sum of a nonzero vector from E_0 and a nonzero vector from E'.

Definition. *X will be said to be on level h with respect to Y, if h is the least integer such that $T^h X = Y$.* Let $X \in E_0$, $X \neq \bar{O}$. The vector X belongs to a cycle C_X of length k, that is $T^k X = X$, k being the least one with this property. Let $Y \in E'$, $Y \neq \overline{O}$. Assume that Y is on level h in the null tree. We obviously have the following Lemmas:

Lemma 4. *The vector $X + Y$ is on the level h, with respect to $T^h X \in C_X$.*

Lemma 5. *There is $Z \in C_X$, such that $Z + Y$ is on level h, with respect to X.*

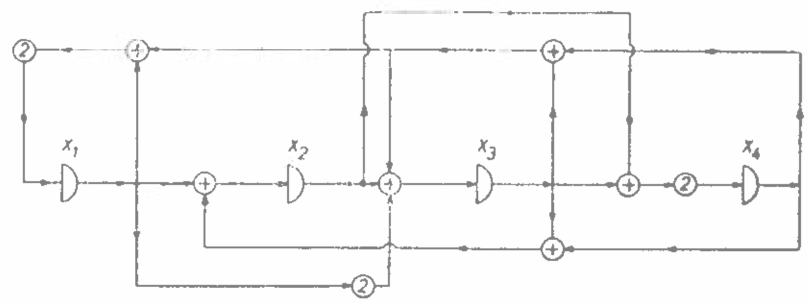

Fig. 1

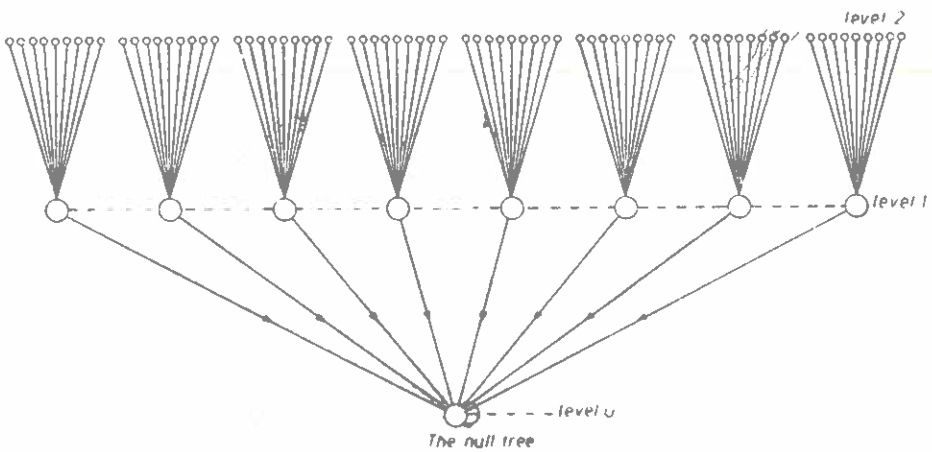

Fig. 2

Lemma 6. *Let Z be on level h, with respect to X. If $Z = X_1 + Y'$, where $X_1 \in E_0$, $Y' \in E'$, then Y' is on level h in the null tree and $X_1 \in C_X$. If Y' admits anterior states, then Z admits preceding states.*

Lemma 7. *If Y has not anterior state, then $X + Y$ has not anterior state.*

From Lemmas 4–7 it results:

Theorem [2]. *The state diagram of the given network is obtained, up to an isomorphism, by attaching the null tree (without the state $\overline{O}$) to each state in the state diagram for E_0.*

Let us consider two examples. For the network of Fig. 1. we have: $\varphi(x) = x^4$, $i_1(x) = x^2$, $i_2(x) = x^2$, $i_3(x) = i_4(x) = 1$, $\lambda_1 = \lambda_2 = 2$, $k = 2$, $p = 3$, $n = 4$, $d_0 = 1$, $d_1 = 8$, $d_2 = 72$.

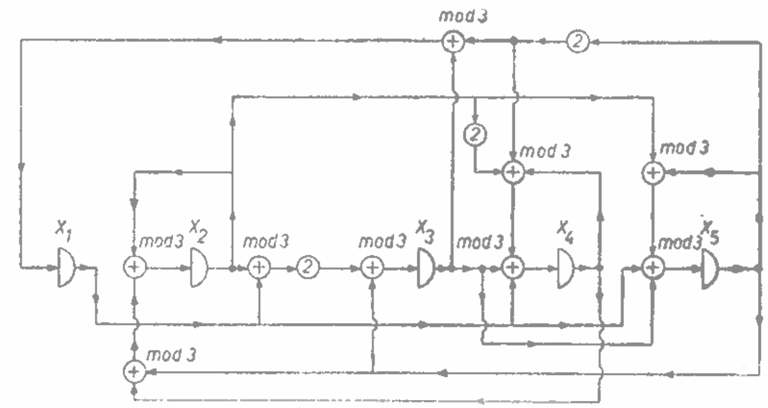

Fig. 3

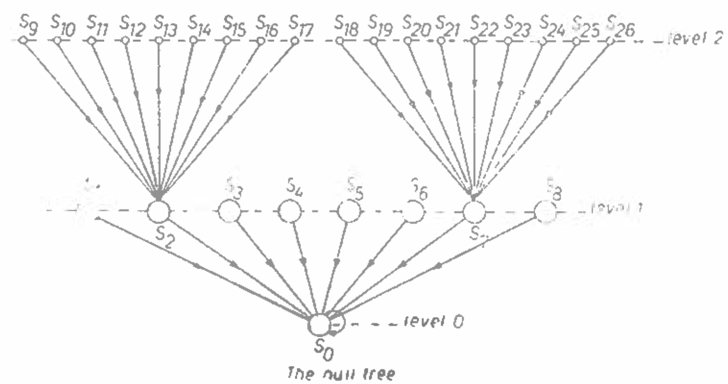

Fig. 4

The null tree is described by the specification [1, 8, 72] i. e. one state at level 0,8 states at level 1.72 states at level 2. This network has no cycle. The state graph (the null tree) of the network is indicated in Fig. 2. For the network of Fig. 3. we have: $\varphi(x) = x^3(x^2 + 1)$, $i_1(x) = x^2(x^2 + 1)$, $i_2(x) = x$, $i_3(x) = i_4(x) = i_5(x) = 1$, $\lambda_1 = 2$, $\lambda_2 = 1$, $k = 2$, $p = 3$, $n = 5$, $d_0 = 1$, $d_1 = 8$, $d_2 = 18$.

The null tree is thus described [1, 8, 18] and it is indicated in Fig. 4.

REFERENCES

1. Elspas B. — *The theory of autonomous linear sequential network*, IRE Trans. Circuit Theory, CT—6 (1959) 45—60.
2. Gill A. — *State graphs of Autonomous Linear Automata*, in *Automata Theory*, Academic Press, New York, 1966, p. 176—180.

ASUPRA TEORIEI REȚELELOR LINIARE SECVENȚIALE AUTONOME

Rezumat

Se studiază comportarea rețelelor liniare secvențiale autonome (automate liniare autonome) cu ajutorul descompunerii spațiului $V_n(p)$ al vectorilor din cîmpul finit $GF(p)$ în subspații ciclice. Se studiază structura arborelui nul prin numărul de stări de pe diferite niveluri. Se dă o altă demonstrație a propoziției 5 din [2]. Se dau 2 exemple de rețele liniare autonome, reprezentîndu-se arborii nuli.

ON THE DEFORMATION OF THE ANISOTROPIC SLIGHTLY CURVED BEAMS

BY

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Let us consider an anisotropic beam limited by two planes $x_3 = 0$, $x_3 = h(h > 0)$ and by two surfaces $\mathcal{S}_0$ and $\mathcal{S}_1$, without common points, given by the equations

$$f_i(x_1 + \frac{1}{n}\varepsilon x_3^n,\ x_2) = 0 \qquad (i = 0,1), \tag{1}$$

$\mathcal{S}_0$ being an external surface and $\mathcal{S}_1$ an internal one.

In the above equations ε is a small parameter so that the square and higher powers of ε can be neglected and n is a natural number.

Let the cavity bounded by the surface $\mathcal{S}_1$ be filled by another anisotropic material. We obtain in this way an anisotropic composite beam. We shall call such beams *slightly curved composite beams* of the n-th degree.

We shall study the deformation for such a beam when the tractions applied on the lateral surface $\mathcal{S}_1$ are polinomials of x_3.

Denote by $\mathcal{V}_0$ the domain of the beam between $\mathcal{S}_0$ and $\mathcal{S}_1$, by $\mathcal{V}_1$ the domain bounded by $\mathcal{S}_1$ and $\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1$.

Suppose there are no body forces.

1. Under these assumptions the stress components σ_{ij} must satisfy the equilibrium equations

$$\sigma_{ij,j} = 0 \quad \text{in} \quad \mathcal{V}. \tag{2}$$

We take into account the case when the tractions applied on the lateral surface are such that