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ASUPRA TEORIEI REȚELELOR LINIARE SECVENȚIALE AUTONOME

Rezumat

Se studiază comportarea rețelelor liniare secvențiale autonome (automate liniare autonome) cu ajutorul descompunerii spațiului $V_n(p)$ al vectorilor din cîmpul finit $GF(p)$ în subspații ciclice. Se studiază structura arborelui nul prin numărul de stări de pe diferite niveluri. Se dă o altă demonstrație a propoziției 5 din [2]. Se dau 2 exemple de rețele liniare autonome, reprezentîndu-se arborii nuli.

ON THE DEFORMATION OF THE ANISOTROPIC SLIGHTLY CURVED BEAMS

BY

G. M. KHATIASHVILI and C. I. BORŞ

Let us consider an anisotropic beam limited by two planes $x_3 = 0$, $x_3 = h (h > 0)$ and by two surfaces $\mathcal{S}_0$ and $\mathcal{S}_1$, without common points, given by the equations

(1) $$f_i\left(x_1 + \frac{1}{n}\varepsilon x_3^n,\ x_2\right) = 0 \qquad (i = 0,1),$$

$\mathcal{S}_0$ being an external surface and $\mathcal{S}_1$ an internal one.

In the above equations ε is a small parameter so that the square and higher powers of ε can be neglected and n is a natural number.

Let the cavity bounded by the surface $\mathcal{S}_1$ be filled by another anisotropic material. We obtain in this way an anisotropic composite beam. We shall call such beams *slightly curved composite beams* of the n-th degree.

We shall study the deformation for such a beam when the tractions applied on the lateral surface $\mathcal{S}_1$ are polinomials of x_3.

Denote by $\mathcal{V}_0$ the domain of the beam between $\mathcal{S}_0$ and $\mathcal{S}_1$, by $\mathcal{V}_1$ the domain bounded by $\mathcal{S}_1$ and $\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1$.

Suppose there are no body forces.

1. Under these assumptions the stress components σ_{ij} must satisfy the equilibrium equations

(2) $$\sigma_{ij,j} = 0 \quad \text{in} \quad \mathcal{V}.$$

We take into account the case when the tractions applied on the lateral surface are such that

(3) $$\sigma_{ij} n_j = \tau_i(x_1, x_2) x_3^k \text{ on } \mathcal{S}_0,$$

where n_i are the direction cosines of the exterior normal to the surface $\mathcal{S}_0$ and $\tau_i(x_1, x_2)$ are given functions of x_1 and x_2 having continuous first order derivatives and k is a positive integer.

On the surface of separation we must have

(4) $$\begin{cases} \{\sigma_{ij} n_j\}_1 = \{\sigma_{ij} n_j\}_0, \\ \{u_i\}_1 = \{u_i\}_0 \quad \text{on} \quad \mathcal{S}_1, \end{cases}$$

where u_i are the components of displacement and $\{...\}_0$, $\{...\}_1$ mean the values of the bracketed functions for the region $\mathcal{V}_0$ and $\mathcal{V}_1$ respectively.

At the ends we shall apply tractions such as to equilibrate the loads (3).

In the above formulae we used the summation convention over repeated indices. The index j after comma indicates partial differentiation with respect to x_j.

We shall find here a solution of the stated problem which satisfies the equations (2) and the boundary conditions (3) and (4). After that we have still to find a solution in order to satisfy the end conditions but this is another problem and we know how to solve it.

2. The materials of the beam are supposed to be anisotropic such that the planes parallel with the plane $x_3 = 0$ are planes of elastic symmetry. In this case the relations between the stress components σ_{ij} and the strain components γ_{ij} are given by [1, ch. 1; 4, ch. 1]

(5) $$\begin{aligned} \sigma_{11} &= s_{11}\gamma_{11} + s_{12}\gamma_{22} + s_{13}\gamma_{33} + s_{16}\gamma_{12}, \\ \sigma_{22} &= s_{12}\gamma_{11} + s_{22}\gamma_{22} + s_{23}\gamma_{33} + s_{26}\gamma_{12}, \\ \sigma_{33} &= s_{13}\gamma_{11} + s_{23}\gamma_{22} + s_{33}\gamma_{33} + s_{36}\gamma_{12}, \\ \sigma_{23} &= s_{44}\gamma_{23} + s_{45}\gamma_{31}, \quad \sigma_{31} = s_{45}\gamma_{23} + s_{55}\gamma_{31}, \\ \sigma_{12} &= s_{16}\gamma_{11} + s_{26}\gamma_{22} + s_{36}\gamma_{33} + s_{66}\gamma_{12}, \end{aligned}$$

where s_{kl} are moduli of elasticity and they are different in different domains $\mathcal{V}_i$.

The components of the strain γ_{ij} are given in terms of displacements u_i by

$$\gamma_{ii} = u_{i,i} \text{ (not summed)}, \quad \gamma_{ij} = u_{i,j} + u_{j,i} \quad (i \neq j).$$

The components γ_{ij} must satisfy the conditions of compatibility of Saint-Venant

$$\gamma_{11,22} + \gamma_{22,11} = \gamma_{12,12}, \ldots$$
$$(-\gamma_{23,1} + \gamma_{31,2} + \gamma_{12,3})_{,1} = 2\gamma_{11,23}, \ldots$$

3. If we make the transformation

(6 a) $$\xi_1 = x_1 + \frac{1}{n}\varepsilon x_3^n, \quad \xi_2 = x_2, \quad \xi_3 = x_3,$$

(6 b) $$x_1 = \xi_1 - \frac{1}{n}\varepsilon \xi_3^n, \quad x_2 = \xi_2, \quad x_3 = \xi_3,$$

then the surfaces (1) become

(7) $$f_i(\xi_1, \xi_2) = 0,$$

which are cylindrical surfaces $\mathcal{F}^*_i$ in the space ξ_1, ξ_2, ξ_3.

With an accuracy up to ε^2 it is easy to prove the following formulae [1, ch. X; 2].

(8) $$\frac{\partial}{\partial x_1} = \frac{\partial}{\partial \xi_1}, \quad \frac{\partial}{\partial x_2} = \frac{\partial}{\partial \xi_2}, \quad \frac{\partial}{\partial x_3} = \frac{\partial}{\partial \xi_3} + \varepsilon \xi_3^{n-1}\frac{\partial}{\partial \xi_1},$$

(9) $$n_1 = n_1^*, \quad n_2 = n_2^*, \quad n_3 = \varepsilon \xi_3^{n-1} n_1^*,$$

where n_i^* are the direction cosines of the exterior normal to the cylindrical surface (7).

With the same accuracy we have

(10) $$\tau_i(x_1, x_2) = \tau_i^*(\xi_1, \xi_2) - \frac{1}{n}\varepsilon \xi_3^n \frac{\partial \tau_i^*}{\partial \xi_1},$$

where τ_i^* have on obvious meaning.

4. Now let us come back to our problem defined by (2), (3) and (4). We shall try to find the solution in the form

(11) $$u_i = u_i^*(\xi_1, \xi_2) + \varepsilon u_i^0,$$

where u_i^0 are additional unknown displacements and u_i^* are the displacements of the Almansi problem, for the cylindrical composite beam (7), defined by

(12 a) $$\begin{cases} \sigma^*_{ij,j} = 0 \text{ in } \mathcal{V}^*, \\ \sigma^*_{ij} n^*_j = \tau^*_i(\xi_1, \xi_2)\xi_3^k \text{ on } \mathcal{S}^*_0; \end{cases}$$

(12 b) $$\begin{cases} \{\sigma^*_{ij} n^*_j\}_1 = \{\sigma^*_{ij} n^*_j\}_0, \\ \{u_i\}_1 = \{u_i\}_0 \text{ on } \mathcal{S}^*_1. \end{cases}$$

This problem was solved by K h a t i a s h v i l i in [3] and we suppose here that its solution is known. In [3] it is shown that the solution of the above Almansi problem is of the form

$$\sigma^*_{ij} = \sum_{p=0}^{k+2} \mathfrak{L}^{(p)}_{ij}(\xi_1,\ \xi_2)\,\xi_3^p, \qquad u_i = \sum_{p=0}^{k+4} \mathfrak{L}^{(p)}_{i}(\xi_1,\ \xi_2)\,\xi_3^p,$$

where $\mathfrak{L}^{(p)}_{ij}(\xi_1,\ \xi_2)$ and $\mathfrak{L}^{(p)}_{i}(\xi_1,\ \xi_2)$ are known functions.

The stresses σ_{ij} corresponding to the displacements (11) are given by

$$\sigma_{ii} = \sigma^*_{ii} + \varepsilon s_{i3}\,\xi_3^{n-1}\,\frac{\partial u^*_3}{\partial \xi_1} + \varepsilon\tau_{ii} \qquad (i = 1,\ 2,\ 3:\ \text{not summed}),$$

(13)

$$\sigma_{23} = \sigma^*_{23} + \varepsilon\,\xi_3^{n-1}\left(s_{44}\frac{\partial u^*_2}{\partial \xi_1} + s_{45}\frac{\partial u^*_1}{\partial \xi_1}\right) + \varepsilon\tau_{23},$$

$$\sigma_{31} = \sigma^*_{31} + \varepsilon\,\xi_3^{n-1}\left(s_{45}\frac{\partial u^*_2}{\partial \xi_1} + s_{55}\frac{\partial u^*_1}{\partial \xi_1}\right) + \varepsilon\tau_{31},$$

$$\sigma_{12} = \sigma^*_{12} + \varepsilon s_{36}\,\xi_3^{n-1}\,\frac{\partial u^*_3}{\partial \xi_1} + \varepsilon\tau_{12},$$

where σ^*_{ij} are the stresses of the mentioned Almansi problem and τ^*_{ij} are the stresses corresponding to the unknown displacements u^0_i.

It is easy to prove that the stresses σ_{ij} will satisfy the compatibility conditions if the stresses τ_{ij} verify them in the space $\xi_1,\ \xi_2,\ \xi_3$.

Taking into account (2) and (13) we find that the stresses τ_{ij} must satisfy the equations

(14) $$\tau_{ij,j} + X_i = 0 \ \text{ in } \ \mathfrak{P}^{*}\,{}^{1)},$$

where

$$X_1 = (n-1)\left(s_{45}\frac{\partial u^*_2}{\partial \xi_1} + s_{55}\frac{\partial u^*_1}{\partial \xi_1}\right)\xi_3^{n-2} + \left(s_{45}\frac{\partial^2 u^*_2}{\partial \xi_1\,\partial \xi_3} + s_{55}\frac{\partial^2 u^*_1}{\partial \xi_1\,\partial \xi_3} + \right.$$

$$\left. + s_{13}\frac{\partial^2 u^*_3}{\partial \xi_1^2} + s_{36}\frac{\partial^2 u^*_3}{\partial \xi_1\,\partial \xi_2} + \frac{\partial \sigma^*_{31}}{\partial \xi_1}\right)\xi_3^{n-1},$$

$$X_2 = (n-1)\left(s_{44}\frac{\partial u^*_2}{\partial \xi_1} + s_{45}\frac{\partial u^*_1}{\partial \xi_1}\right)\xi_3^{n-2} + \left(s_{44}\frac{\partial^2 u^*_2}{\partial \xi_1\,\partial \xi_3} + s_{45}\frac{\partial^2 u^*_1}{\partial \xi_1\,\partial \xi_3} + \right.$$

$$\left. + s_{23}\frac{\partial^2 u^*_3}{\partial \xi_1\,\partial \xi_2} + s_{36}\frac{\partial^2 u^*_3}{\partial \xi_1^2} + \frac{\partial \sigma^*_{23}}{\partial \xi_2}\right)\xi_3^{n-1},$$

$$X_3 = (n-1)s_{33}\frac{\partial u^*_3}{\partial \xi_1}\,\xi_3^{n-2} + \left(s_{45}\frac{\partial^2 u^*_2}{\partial \xi_1^2} + s_{55}\frac{\partial^2 u^*_1}{\partial \xi_1^2} + s_{44}\frac{\partial^2 u^*_2}{\partial \xi_1\,\partial \xi_2} + s_{45}\frac{\partial^2 u^*_1}{\partial \xi_1\,\partial \xi_2} + \right.$$

$$\left. + s_{33}\frac{\partial^2 u^*_3}{\partial \xi_1\,\partial \xi_3} + \frac{\partial \sigma^*_{33}}{\partial \xi_1}\right)\xi_3^{n-1}.$$

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1) Here and in (12) the indices 1, 2, 3 after comma indicate partial differentiation with respect to $\xi_1,\ \xi_2,\ \xi_3$.

Making use of the equations (18), (12), (11), (10), from the boundary conditions (3), (4) we obtain for the determination of the stresses τ_{ij} and displacements u^0_i the following boundary conditions:

(15)
$$\begin{cases} \tau_{ij}n^* = \Lambda_i \ \text{ on } \ \mathfrak{S}^*_0; \\ \{\tau_{ij}n^*_j\}_1 - \{\tau_{ij}n^*_j\}_0 = \{\Lambda^*_i\}_0 - \{\Lambda^*_i\}_1 \\ \{u^0_i\}_0 = \{u^0_i\}_1 \ \text{ on } \ \mathfrak{S}^*_1, \end{cases}$$

where

$$\Lambda_i = \Lambda^*_i - \frac{1}{n}\frac{\partial \tau_i}{\partial \xi_1}\,\xi_3^{n+k},$$

$$\Lambda^*_1 = -\,\xi_3^{n-1}\left(s_{13}\frac{\partial u^*_3}{\partial \xi_1}\,n^*_1 + s_{36}\frac{\partial u^*_3}{\partial \xi_1}\,n^*_2 + \sigma^*_{31}n^*_1\right),$$

$$\Lambda^*_2 = -\,\xi_3^{n-1}\left(s_{23}\frac{\partial u^*_3}{\partial \xi_1}\,n^*_2 + s_{36}\frac{\partial u^*_3}{\partial \xi_1}\,n^*_1 + \sigma^*_{23}n^*_1\right),$$

$$\Lambda^*_3 = -\,\xi_3^{n-1}\left[s_{44}\frac{\partial u^*_2}{\partial \xi_1}\,n^*_2 + s_{45}\left(\frac{\partial u^*_2}{\partial \xi_1}\,n^*_1 + \frac{\partial u^*_1}{\partial \xi_1}\,n^*_2\right) + s_{55}\frac{\partial u^*_1}{\partial \xi_1}\,n^*_1 + \sigma^*_{33}n^*_1\right].$$

The problem defined by (14) and (15) which is just found for the determination of the stresses τ_{ij} and displacements u^0_i, is an Almansi problem of the cylindrical composite beam (7) with given jumps for the body forces X_i and for $\tau_{ij}n^*_j$ on the surface $\mathfrak{S}^*_1$.

This problem was solved in the paper [3] too.

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DEFORMAREA BARELOR ANIZOTROPE UŞOR CURBATE

Rezumat

Se consideră o bară mărginită de către planele $x_3 = 0$, $x_3 = h$ şi de suprafeţele (1). Se presupune că ε este un parametru mic astfel încît termenii în ε^2 pot fi neglijaţi. Domeniile $\mathscr{V}_0$ şi $\mathscr{V}_1$ mărginite de cele două plane şi de suprafeţele $\mathscr{S}_0$, $\mathscr{S}_1$ respectiv $\mathscr{S}_1$ sînt ocupate de materiale anizotrope diferite. Se studiază deformarea unei astfel de bare în cazul cînd încărcările de pe suprafaţa $\mathscr{S}_0$ sînt de forma (3). În cele din urmă problema este redusă la o problemă Almansi definită de (14) şi (15) pe care ştim să o rezolvăm [3].

ON THE QUASISTATIC TORSION OF VISCOELASTIC BEAMS

BY

GH. CIOBANU

1. Let be an isotropic homogeneous and linear viscoelastic material with the constitutive equations of the form [1] ch. 1, [6]

$$\tau_{ij}(x, t) = \lambda(t)\theta(x, t)\delta_{ij} + 2\mu(t)\varepsilon_{ij}(x, t) + \tag{1.1}$$
$$+ \int_0^t \Big[l(t, \tau)\theta(x, \tau)\delta_{ij} + 2m(t, \tau)\,\varepsilon_{ij}(x, \tau) \Big] d\tau$$

with respect to a rectangular coordinate system $Ox_1x_2x_3$. Here the functions $\tau_{ij}(x, t)$ and $\varepsilon_{ij}(x, t)$ are the stress tensor and the strain tensor at the point $x(x_1, x_2, x_3)$, at the moment t and $\theta(x, t) = \varepsilon_{ii}(x, t)$.

Latin subscripts have the range 1, 2, 3 and the usual convention of summation over repeated subscripts is adopted.

If $u_i(x, t)$ are the components of the displacement vector at the point x at the moment t, then

$$2\varepsilon_{ij}(x, t) = u_{i,j}(x, t) + u_{j,i}(x, t). \tag{1.2}$$

The index i after comma indicates partial differentiation with respect to x_i.

The constitutive equations (1.1) show that the viscoelastic material is assumed initially undisturbed.

If $\lambda(t)$ and $\mu(t)$ are constants then (1.1) are the constitutive equations suggested by V o l t e r r a [8]. Moreover, if $l(t, \tau) = g(t - \tau)$, $m(t, \tau) = h(t - \tau)$, (1.1) is identical in form to the accepted general linear viscoelastic constitutive equations [5], [3], [7].

We assume the elastic moduli $\lambda(t)$ and $\mu(t)$ are continuous and bounded functions on $[0, \infty)$. We suppose also there exists positive constants λ_0,