SUR LA DIVISION DANS L'ANNEAU DES QUATERNIONS ENTIERS

Résumé

Dans cette Note nous étudions la possibilité de déterminer le quotient et le reste de la division à gauche ou à droite de deux quaternions entiers et on conclut que c'est possible dans le demigroupe des quaternions entiers dont les normes sont impaires, ce qui permet la définition et la construction du plus grand commun diviseur à gauche ou à droite de deux quaternions de normes impaires.

Enfin on donne une condition nécessaire et suffisante pour qu'un quaternion entier soit irréductible.

ON THE ISOMORPHISM OF DIGRAPHS

BY

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Theorem 1. *Let D_1, D_2 be two digraphs with the same number of nodes and A_1, A_2 be their adjacency matrices obtained for some arrangement of the nodes. The digraphs D_1 and D_2 are isomorphic if and only if there exists a permutation matrix P so that*

$$A_1P = PA_2.$$

The proof results from the definition of the isomorphism for two digraphs.

If D is a digraph with p nodes and A its adjacency matrix, let us consider the digraphs $D^j(1 \leqslant j \leqslant p)$ obtained from D by removing all the incident to v_j lines and let A^j be their corresponding matrices for the same degree of nodes. We shall denote by $a \oplus b = \max(a, b)$, a and b being real numbers.

Main Lemma. *If $p > 3$, then*

$$A = A \oplus A^2 \oplus A^3 \oplus \ldots \oplus A^p.$$

Proof. Let a_{ij} be the element of i-th row and j-th column of the matrix A and b_{ij} the element of i-th row and j-th column of the matrix $B = A^1 \oplus A^2 \oplus A^3 \oplus \ldots \oplus A^p$.

Case 1: If $a_{ij} = 1$, because $p > 3$, it follows that there exists $k \neq i, j$ so that $a_{ij}^k = 1$, where a_{ij}^k denotes the element of i-th row and j-th column of the matrix A^k, hence $b_{ij} = 1$.

Case 2: If $a_{ij} = 0$, it follows that for all integer k, $1 \leqslant k \leqslant p$, $a_{ij}^k = 0$, hence $b_{ij} = 0$ and therefore the lemma is proved.

Now, let us consider that we know the digraphs D_1 and D_2 with the same number of nodes and that for some order of nodes from D_1, there exists an order of nodes from D_2 so that D_1^j is isomorphic with D_2^j for all j, $1 \leqslant j \leqslant p$. Let A_1, A_2, A_1^j, A_2^j be the adjacency matrices of digraphs D_1, D_2, D_1^j, D_2^j respectively.

We now give a necessary and sufficient condition, so that the digraphs D_1 and D_2 are isomorphic.

Theorem 2. *The digraphs D_1 and D_2 are isomorphic if and only if there exists a permutation matrix P so that for all j, $1 \leqslant j \leqslant p$,*

$$A_1^jP = PA_2^j.$$

Proof. If D_1 and D_2 are isomorphic, then by arranging the nodes of digraph D_2 with respect to their isomorphism it follows that $A_1^j = A^j$ for all j, $1 \leqslant j \leqslant p$, and hence, we can consider the matrix

$$E = \begin{pmatrix} 1 & 0 \dots 0 \\ \cdot & \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ 0 \dots 0 & 1 \end{pmatrix}$$

as asked permutation matrix.

Conversely, if there exists a permutation matrix P so that $A_1^jP = PA_2^j$, it results that

$$A_1^1P \oplus A_1^2P \oplus \dots \oplus A_1^pP = PA_2^1 \oplus PA_2^2 \oplus \dots \oplus PA_2^p,$$

and hence

$$(A_1^1 \oplus A_1^2 \oplus \dots \oplus A_1^p)P = P(A_2^1 \oplus A_2^2 \oplus \dots \oplus A_2^p)$$

and according to main lemma $A_1P = PA_2$ and now, according to Theorem 1, D_1 and D_2 are isomorphic.

This theorem allows us to establish if two digraphs D_1 and D_2 are isomorphic when we know that the digraphs D_1^j and D_2^j are isomorphic; it follows that the system

$$A_1^jX = XA_2^j, \qquad j = 1, 2, \dots, p,$$

where A_1^j and A_2^j are adjacency matrices of the digraphs D_1^j and D_2^j respectively and X is a permutation matrix, is compatible if and only if the digraphs D_1 and D_2 are isomorphic.

ASUPRA IZOMORFISMULUI DIGRAFURILOR

Rezumat

În această Notă se arată că două digrafuri D_1 și D_2 sînt izomorfe dacă și numai dacă sistemul de ecuații matriciale $A_1^jX = XA_2^j$ $(1 \leqslant j \leqslant p)$ este compatibil, A_1^j și A_2^j fiind matricele de incidență ale grafurilor D_i^j— obținute din D_i prin înlăturarea muchiilor incidente cu vîrful v_j.

ON A PROBLEM OF FRANK HARARY

BY

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The well known Ulam's conjecture states that two graphs are isomorphic if and only if the subgraphs obtained by removing one after another a vertex are isomorphic. P. J. Kelly [2] proved that this conjecture is true for trees. Frank Harary and Ed. Palmer [3] showed that it is true for tournements which are not strong connexe.

In this paper we deal with the following Harary's problem, [1], for undirected graphs: let $\bar{G}^j = (\bar{V}^j, \bar{U}^j)$ be p graphs, with $|\bar{V}^j| = p$, $(j = 1, \dots, p)$, each of them having at least an isolated vertex; we wish to establish if there exists a graph $G = (V, U)$, such that for every $j = 1, \dots, p$, the graph $G^j = (V, U^j)$, in which U^j gets from U by removing all the edges linking the vertex $v_j \in V$ with the other vertices of G, is isomorphic to the given graph $\bar{G}^j$.

Let $G = (V, U)$ be a graph without loops, with $|V| = p$, $|U| = q$, and denote by $\rho(v)$ the degree of a vertex $v \in V$. Consider the square matrix $A = (a_{ij})$ given by:

$$(1) \qquad a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in U, \\ 0 & \text{otherwise,} \end{cases} \qquad (i, j = 1, \dots, p).$$

Obviously, we have $a_{ij} = a_{ji}$ and

$$\sum_i a_{ij} = \rho(v_j), \qquad (i, j = 1, \dots, p).$$

Let x_i be the number of vertices having the degree i in G, $(0 \leqslant i \leqslant p-1)$, and x_i^j be the number of vertices having the degree i in G^j. We can state the relationship between these numbers as