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THE ALGEBRA OF SPINORS OVER AN ARBITRARY FIELD

Summary

In this paper we define the algebra of spinors in a linear space, which is finite-dimensional, over a commutative field (of characteristic $\neq 2$), in which is given a bilinear symmetric form. Further the properties of this algebra are studied.

A FIXED POINT THEOREM IN UNIFORM SPACES

BY

N. GHEORGHIU and E. ROTARU

This Note proves a fixed point theorem in uniform spaces suggested by a theorem of Edelstein [1] stated for metric spaces. In comparison with the usual condition of contraction of Banach's theorem, in this of Edelstein a condition of local contraction is suggested. Other two fixed point theorems in uniform spaces are to be noted in [2] and [3].

An application is indicated for linear operators in locally convex spaces.

Theorem 1. *Let X be a uniform sequentially complete space and let $\{\rho_\alpha\}_{\alpha\in A}$ be a saturated and separated by semimetrics set defining the uniform topology of X. Let f be an application from X into itself which is assumed to satisfy the following conditions:*

A_1) *There are a mapping $\varphi : A \to A$, and numbers $\varepsilon_\alpha > 0$, $\lambda_\alpha \in [0, 1]$ for $\alpha \in A$ such that*

$$\rho_\alpha(x, y) \leqslant \varepsilon_\alpha \textit{ implies } \rho_\alpha(fx, fy) \leqslant \lambda_\alpha \rho_{\varphi(\alpha)}(x, y) \tag{1}$$

for any $\alpha \in A$;

A_2) *There is $x_0 \in X$ such that for each $\alpha \in A$ there exists a finite set of points $x_0 = u_0, u_1, \ldots, u_p = fx_0$ (eventually dependent of α) satisfying the inequalities*

$$\rho_{\varphi^n(\alpha)}(u_{i-1}, u_i) \leqslant \varepsilon_\alpha \tag{2}$$

for any $n \in N$, $i = 1, 2, 3, \ldots, p$;

A_3) *The series*

$$\sum_{n=0}^{\infty} \lambda_\alpha \lambda_{\varphi(\alpha)} \cdots \lambda_{\varphi^n(\alpha)} \tag{3}$$

are convergent.

Then the sequence

$$x_0,\ x_n = fx_{n-1},\ n \in N. \tag{4}$$

converges to a solution of the equation

$$fx = x. \tag{5}$$

Proof. In order to get the convergence of the sequence (4) it suffices to show that for each semimetric ρ_α, $\alpha \in A$, the sequence (4) is Cauchy. Let $\alpha \in A$ be fixed. Using the conditions that were put, we have successively

$$\begin{aligned} &\rho_\alpha(fu_{i-1}, u_i) \leqslant \lambda_\alpha \rho_{\varphi(\alpha)}(u_{i-1}, u_i) \leqslant \lambda_\alpha \varepsilon_\alpha, \\ &\rho_\alpha(f^2u_{i-1}, f^2u_i) \leqslant \lambda_\alpha \rho_{\varphi(\alpha)}(fu_{i-1}, fu_i) \leqslant \lambda_\alpha \lambda_{\varphi(\alpha)} \varepsilon_\alpha, \end{aligned} \tag{6}$$

and, by induction, it follows that

$$\rho_\alpha(f^nu_{i-1}, f^nu_i) \leqslant \lambda_\alpha \lambda_{\varphi(\alpha)} \cdots \lambda_{\varphi^{n-1}(\alpha)} \varepsilon_\alpha \tag{7}$$

for $n \in N$ and $i = 1, 2, 3, \ldots, p$.

By using the inequality (7), we obtain:

$$\begin{aligned} &\rho_\alpha(x_n, x_{n+1}) = \rho_\alpha(f^nx_0, f^nx_1) \leqslant \rho_\alpha(f^nu_0, f^nu_1) + \\ &+ \rho_\alpha(f^nu_1, f^nu_2) + \ldots + \rho_\alpha(f^nu_{p-1}, f^nu_p) \leqslant p\varepsilon_\alpha \lambda_\alpha \lambda_{\varphi(\alpha)} \cdots \lambda_{\varphi^{n-1}(\alpha)}. \end{aligned} \tag{8}$$

Finally, for two natural numbers $m > n$, it follows by (8) that

$$\rho_\alpha(x_n, x_m) \leqslant p\varepsilon_\alpha \sum_{j=n-1}^{m-2} \lambda_\alpha \lambda_{\varphi(\alpha)} \cdots \lambda_{\varphi^j(\alpha)}$$

which, due to A_3, shows that the sequence (4) satisfies Cauchy's condition and hence is convergent. We set $x' = \lim\limits_{n\to\infty} x_n$. By passing to limit in (4) we get $fx' = x'$, because f is continuous on X as can be seen from (1). So Theorem 1 is completely proved.

Remark 1. When X is a metric space, Theorem 1 becomes the theorem included in Edelstein's paper.

Remark 2. If there exists a natural number j so that $\varphi^j = \varphi$, then Theorem 1 has a much simpler statement.

Remark 3. Suppose that Assumption A_2 is replaced by:

A_2') *For every pair* $(x', x'') \in X \times X$ *and for every* $\alpha \in A$, *there are* $x' = u_0, u_1, u_2, \ldots, u_p = x''$ *satisfying* (2) *for* $n \in N$ and $i = 1, 2, 3 \ldots, p$. Then the solution of equation (5) is unique.

Indeed if x' and x'' are two solutions, for each $n \in N$, we have:

$$\rho_\alpha(x', x'') = \rho_\alpha(f^nx', f^nx'');$$

inserting the points u_i we find:

$$\rho_\alpha(x', x'') \leqslant \lambda_\alpha \lambda_{\varphi(\alpha)} \cdots \lambda_{\varphi^{n-1}(\alpha)} \varepsilon_\alpha.$$

This fact together with the convergence of the series (3) imply that

$$\rho_\alpha(x', x'') = 0,$$

for any $\alpha \in A$. Hence $x' = x''$.

Now let us illustrate Theorem 1 by solving the equation

$$x = y + Tx, \tag{9}$$

where T is a linear operator on a locally convex space.

Let X be a locally convex space sequentially complete, its topology being defined by a family which is saturated and separated by seminorms $\{p_\alpha\}_{\alpha\in A}$. It is known that X with this topology is a uniform space, whose uniforme structure gives the initial topology of a locally convex space whose family of semimetrics is

$$\rho_\alpha(x, y) = p_\alpha(x - y), \quad \alpha \in A.$$

If $T: X \to X$ is a continuous linear operator, then there are numbers $\lambda_\alpha \geqslant 0$, such that

$$p_\alpha(Tx) \leqslant \lambda_\alpha p_{\varphi(\alpha)}(x) \tag{10}$$

for any $x \in X$ and $\alpha \in A$, ([4], p. 49).

Then from Theorem 1 we obtain:

Theorem 2. *Let* X *be a locally convex and sequentially complete linear space. Let* T *be a linear continuous operator from* X *into itself. Suppose that the following assumptions are satisfied:*

B_1) *The constants* λ_α *from (10) satisfy the conditions* $\lambda_\alpha \leqslant 1$.

B_2) *For each* $y \in X$ *the set* $\{p_{\varphi^n(\alpha)}(y),\ n = 0, 1, 2, \ldots\}$ *is bounded for each* $\alpha \in A$.

B_3) *The series (3) is convergent.*

Then the sequence

$$x_0 = y, \quad x_{n+1} = y + Tx_n, \quad n = 0, 1, 2, \ldots \tag{11}$$

converges to a solution of equation (9).

Proof. It is enough to prove that the conditions of Theorem 2 imply the assumptions of Theorem 1 with $fx = y + Tx$. Certainly, A_1 is satisfied due to B_1 and (10) : A_3 is implied by B_3. The proof is achieved showing that B_2 implies A_2, α being fixed. To this end let M be a positive number, which can depend on α, so that

$$p_{\varphi^n(\alpha)}(y) \leqslant M, \quad n = 0, 1, 2, \ldots \tag{12}$$

Now fixing a natural number s, (12) gets the form :

$$p_{\varphi^n(\alpha)}\left(\frac{y}{s}\right) \leqslant \frac{M}{s}. \tag{13}$$

or

$$p_{\varphi^n(\alpha)}\left(\frac{i-1}{s}y, \frac{i}{s}y\right) = p_{\varphi^n(\alpha)}\left(\frac{y}{s}\right) \leqslant \frac{M}{s} = \varepsilon_\alpha \tag{14}$$

for any $n \in N$ and $i = 1, 2, \ldots, s$. which shows that the condition A_2 is carried out with $x_0 = 0$, $x_1 = y$, $u_i = y/s$ for $i = 0, 1, 2, 3, \ldots, s$.

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O TEOREMĂ DE PUNCT FIX IN SPAȚII UNIFORME

Rezumat

În această Notă se demonstrează o teoremă de punct fix în spații uniforme pentru aplicații local contractante ; în continuare se aplică acest rezultat pentru operatori liniari continui pe spații local convexe secvențial complete.

ESPACES *H*-BORNOLOGIQUES

PAR

TEODOR PRECUPANU

Nous avons défini dans cette Note les espaces *H*-bornologiques pour la catégorie des espaces linéaires topologiques *H*-localement convexes, par analogie avec les espaces bornologiques introduits pour la catégorie des espaces linéaire topologiquies localement convexes ([1], ch. 3). Rappelons qu'un espace linéaire topologique est dit *H*-localement convexe si la topologie peut être définie à l'aide d'une famille de seminormes hilbertiennes. Conformément à la théorie générale des espaces *H*-localement convexes, [6], [2]§ 4 D et [7], la place des ensembles convexes est prise par les ensembles *H*-convexes.

Definition 0. *On dit qu'une partie M d'un espace linéaire est H-convexe si pour chaque $\alpha \geqslant 0$, $\beta \geqslant 0$ et $x, y \in M$ il existe $\alpha_1 \geqslant 0$, $\beta_1 \geqslant 0$ avec les propriétés suivantes :*

H_1. $\alpha_1^2 + \beta_1^2 \leqslant 2(\alpha^2 + \beta^2)$,

H_2. $\alpha x + \beta y \in \alpha_1 M$,

H_3. $\alpha x - \beta y \in \beta_1 M$.

On peut observer immédiatement qu'une intersection de parties *H*-convexes n'est pas toujours une partie *H*-convexe et donc nous n'avons pas la possibilité d'introduire la notion d'enveloppe *H*-convexe pour une partie arbitraire.

A l'aide de la notion de partie *H*-convexe on a donné une caractérisation simple pour les espaces *H*-localement convexes ([6], th. B).

Théorème 0. *Un espace linéaire topologique est H-localement convexe si et seulement si la topologie peut être définie à l'aide d'un système fondamental de voisinages équilibrés et H-convexes de l'origine.*