Remarque. Le fait que la torsion gaussienne de V^2 est nulle est un cas spécial de la propriété suivant laquelle *la torsion gaussienne de chaque variété K_0 de codimension 2 est nulle.*

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BIBLIOGRAPHIE

1. Chen B. Y. — *Pseudo-umbilical Surfaces with Constant Gauss Curvature*, sous presse.
2. Chen B. Y. — *Minimal Surfaces with Constant Gauss Curvature*, Proc. Amer. Math. Soc., 34,2(1972).
3. Rosca R., Vanhecke L., Verstraelen L. — *Triade rectangulaire de variétés $\mathfrak{B}$ (de Bianchi) bidimensionnelles immergées dans un espace elliptique à quatre dimensions et généralisation dans un espace elliptique à 2 + n dimensions*, Glasnik Matematički, 7(27) (1972).
4. Vanhecke L. — *Pseudo-umbilical and minimal manifolds with constant riemannian curvature*, sous presse.
5. Kenmotsu K. — *Some remarks on minimal submanifolds*, Tôhoku Math. Journ., 22 (1970).

p-INDEXUL UNEI VARIETĂȚI

Rezumat

Fie $x: V^n \to P_e^{n+N}$ o imersie izometrică a unei varietăți V^n cu n dimensiuni într-un spațiu eliptic $(n+N)$-dimensional P_e^{n+N}. „p-indexul" lui V^n este numărul de tangente două cîte două ortogonale în fiecare punct din V^n astfel că, pentru fiecare din aceste tangente, există o varietate normală cînd punctul tangent variază pe varietate.

Fie acum K_{0c} o varietate V^n cu curbură riemanniană constantă avînd conexiune normală trivială și pentru care a doua formă fundamentală corespunzătoare punctului de curbură medie H posedă valori proprii constante în raport cu ds^2. În prezenta lucrare se studiază p-indexul unei asemenea varietăți K_{0c}. În particular, în cazul unui spațiu eliptic P_e^4, sîntem conduși la o configurație de suprafețe K_{0c} a cărei existență este demonstrată.

ON SOME SPECIAL CURVES ON THE SURFACE OF REFERENCE OF A RECTILINEAR CONGRUENCE. II

BY

H. K. N. TRIVEDI & S. C. RASTOGI

In the previous paper [9] of this series we have studied properties of a curve $C: x^i = x^i(s)$ $(i = 1, 2, 3)$, on the surface of reference $S: x^i = x^i(u^\alpha)$ $(\alpha = 1, 2)$ of a rectilinear congruence represented by the unit vector with contravariant components $\lambda^i = \lambda^i(u^\alpha)$ which is such that $d\lambda^i/ds$ is perpendicular to X^i (the contravariant components of the normal to S). The purpose of this paper is to study some more properties of this curve.

1. Preliminaries. At any point $P(x^i)$, of S, λ^i may be expressed as

$$\lambda^i = p^\alpha x^i{}_{,\alpha} + qX^i, \tag{1.1}$$

where p^α and q are the parameters.

Differentiating (1.1) with respect to s and using the equations of Gauss and Weingarten

$$x^i{}_{,\alpha\beta} = d_{\alpha\beta}\, X^i, \tag{1.2 a}$$

and

$$X^i_{,\beta} = -\, d_{\beta\gamma}\, g^{\gamma\delta}\, x^i{}_{,\delta}, \tag{1.2 b}$$

and the fact that $d\lambda^i/ds$ is perpendicular to X^i we get

$$d\lambda^i/ds = (p^\gamma_{,\alpha} - q d_{\alpha\sigma}\, g^{\sigma\gamma})\, x^i_{,\gamma}\, u'^\alpha \equiv \mu^\gamma_\alpha\, x^i_{,\gamma}\, u'^\alpha \tag{1.3}$$

and

$$(q_{,\alpha} + p^\beta d_{\alpha\beta})u'^\alpha = 0, \tag{1.4}$$

primes having been used for differentiation with respect to s.

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[1] the numbers in square brackets refer to the references in end.

Further differentiation of (1.3) and use of (1.2) in the resulting equation yields

(1.5) $$d^2\lambda^i/ds^2 = \overline{M}^{\gamma} x^i_{,\gamma} + \overline{N} X^i_{,}$$

where

(1.6) a $$M^{\gamma} \overset{\text{def}}{=} M^{\gamma}_{\alpha,\beta} u'^{\alpha} u'^{\beta} + \mu^{\gamma}_{\alpha} \rho^{\alpha}$$

and

(1.6) b $$\overline{N} \overset{\text{def}}{=} \mu^{\gamma}_{\alpha} d_{\gamma\beta} u'^{\alpha} u'^{\beta}$$

and we have put $u''^{\alpha} = \rho\alpha$.

The notations that we have used here are the same as used by Springer [4]. However for the vector product we shall use Cross instead of epsilon system.

2. Properties of the curve C. Since the vector with contravariant components $d\lambda^i/ds$ is perpendicular to X^i, therefore it lies in the tangent plane of S. In view of this property we may express the unit tangent to C, whose contravariant components are dx^i/ds, as a linear combination of $d\lambda^i/ds$ and $d\lambda^i/ds \times X^i$. Thus:

(2.1) $$dx^i/ds = a\,d\lambda^i/ds + b\,d\lambda^i/ds \times X^i,$$

where a and b are to be determined.

Multiplying (2.1) by λ^i and making use of $\lambda^i . d\lambda^i/ds = 0$ and (1.3) we get

(2.2) $$b = p_{\gamma} u'^{\gamma} / e_{\alpha\gamma}\, p^{\alpha} \mu^{\gamma}_{\beta}\, u'^{\beta}.$$

Multiplication of (2.1) by $d\lambda^i/ds$ yields

(2.3) $$a = \Phi/B^2,$$

where

(2.4) a $$\Phi \equiv \mu_{\alpha\beta} u'^{\alpha} u'^{\beta}$$

and

(2.4) b $$B \equiv | d\lambda^i/ds | .$$

Squaring (2.1) and making use of (2.2) and (2.3) we get

(2.5) $$\Phi^2/B^2 + (p_{\alpha} u'^{\alpha} B / e_{\alpha\gamma}\, p^{\alpha} \mu^{\gamma}_{\beta}\, u'^{\beta})^2 = 1.$$

Since λ^i is perpendicular to $d\lambda^i/ds$ therefore $\lambda^i . d\lambda^i/ds \times X^i$ $(= e_{\alpha\gamma} p^{\alpha} \mu^{\gamma}_{\beta} u'^{\beta})$ can vanish only if λ^i coincides with X^i, i. e., if the congruence is normal. Thus from (2.5), as $B \neq 0$, we observe that $p_{\alpha} u'^{\alpha} = 0$, i. e., C is the same curve which has been studied by Bhattacharya [1] and later on called hypernormal curve by other authors if and only if $\Phi^2 = B^2$ which with the help of (1.3) and (2.4) may be expressed as

(2.6) $$\mu_{\alpha\beta}\, u'^{\beta} u'^{\delta} (u^{\alpha}_{\delta} - \mu_{\delta\gamma}\, u'^{\alpha} u'^{\gamma}) = 0 .$$

Hence we have:

Theorem 2.1. *A necessary and sufficient condition for the curve C to be a hypernormal curve is that (2.6) be satisfied, provided that the congruence is not normal.*

Squaring (2.1) and putting the value of a from (2.3) we get

$$b = \sqrt{B^2 - \Phi^2}/B^2,$$

which along with (2.3) when used in (2.1) gives

(2.7) $$dx^i/ds = \Phi/B^2 . d\lambda^i/ds + \sqrt{B^2 - \Phi^2}/B^2\, d\lambda^i/ds \times X^i.$$

Since for the curve C to be a hypernormal curve $B^2 = \Phi^2$, therefore from (2.7) we have $dx^i/ds = \pm 1/B d\lambda^i/ds$, i. e., $d\lambda^i/ds$ coincides with the tangent vector to C with the possibility of their directions being same or opposite. From this we conclude:

Theorem 2.2. *A necessary and sufficient condition for the curve C to be a hypernormal curve is that $d\lambda^i/ds$ coincides with the tangent vector to C, the directions of $d\lambda^i/ds$ may be same or opposite to that of the tangent vector to C, provided the congruence under consideration is not normal.*

Differentiating (2.7) with respect to s we obtain

(2.8) $$d^2x^i/ds^2 = (\Phi/B^2)'\, d\lambda^i/ds + \Phi/B^2 d^2\lambda^i/ds^2 + (\sqrt{B^2 - \Phi^2}/B^2)' d\lambda^i/ds \times X^i + \\ + (\sqrt{B^2 - \Phi^2}/B^2)\, (d^2\lambda^i/ds^2 \times X^i + d\lambda^i/ds \times dX^i/ds).$$

Multiplying (2.8) by λ^i and using (1.2) b, (1.3) and (1.5), we get

(2.9) $$k_c = \Phi/B^2(\overline{M}^{\alpha} p_{\alpha} + q\overline{N}) + (\sqrt{B^2 - \Phi^2}/B^2)' e_{\alpha\beta}\, p^{\alpha} \mu^{\beta}_{\gamma} u'^{\gamma} + \\ + (\sqrt{B^2 - \Phi^2}/B^2) e_{\alpha\beta}\, (p^{\alpha} M^{\beta} - q d_{\gamma\delta}\, g^{\delta\beta}\, u'^{\gamma} \mu^{\alpha}_{\sigma} u'^{\sigma}) ,$$

where $k_c (= p^{\alpha} \rho_{\alpha} + q k_n)$ is the curvature of the congruence section of S for the direction of C as defined by Mishra [2]. Vanishing of k_c means C is a hyperasymptotic curve [2]. For brevity we shall use the word hyperasymptotic curvature for k_c. From (2.9) we have the following:

Theorem 2.3. *The necessary and sufficient condition for the curve C to be a hyperasymptotic curve is that the R. H. S. of (2.9) vanishes.*

By virtue of Theorem 2.1 and the equation (2.9) we observe:

Theorem 2.4. *The hyperasymptotic curvature of the curve C which is also a hypernormal curve is given by $k_c = \overline{M}^{\alpha} p_{\alpha} + q\overline{N}$.*

For the normal congruence and in view of (1.3), (1.6) b and (2.4), the relation (2.9) reduces to

$(\sqrt{d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\sigma\alpha}\, u'^{\beta} u'^{\delta}} - k_n^2)/(d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\sigma\alpha}\, u'^{\beta} u'^{\delta}).e_{\alpha\beta}\, d_{\gamma\delta}\, g^{\delta\beta}\, u'^{\gamma} d_{\sigma\rho}\, g^{\alpha\rho}\, u'^{\sigma} = 0,$

which implies either

(2.10) a
$$k_n = \sqrt{d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\sigma\alpha}\, u'^{\beta} u'^{\delta}}$$

or

(2.10) b
$$e_{\alpha\beta}\, d_{\gamma\delta}\, d_{\sigma\rho}\, g^{\delta\beta}\; g^{\alpha\rho}\, u'^{\gamma} u'^{\sigma} = 0.$$

From (2.10) we deduce the following:

Theorem 2.5. *For a normal congruence, the normal curvature of the curve C is given by (2.10)* a *provided* $e_{\alpha\beta}\, d_{\gamma\delta}\, d_{\sigma\rho}\, g^{\delta\beta}\, g^{\alpha\rho}\, u'^{\gamma}\, u'^{\sigma} \neq 0$.

Theorem 2.6. *For a normal congruence, the curve C satisfies the equation (2.10)* b *if and only if* $k_n \neq \sqrt{d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\sigma\alpha}\, u'^{\beta} u'^{\delta}}$.

We know that the union curvature k_n of C is given by [3]

$$q k_n = \lambda^i \cdot \frac{dx^i}{ds} \times \frac{d^2 x^i}{ds^2},$$

which on solving with the help of (2.7) and (2.8) becomes

(2.11)
$$q k_n = \frac{1}{B^4} e_{\alpha\beta}\, \mu^{\alpha}_{\gamma} u'^{\gamma} (B^2 \overline{M}^{\beta} - \Phi^2 p^{\beta} \overline{N}) + \Phi / B^2 \sqrt{B^2 - \Phi^2}\, d_{\alpha\beta}\, p^{\alpha} u'^{\beta}$$
$$+ \frac{q}{B\sqrt{B^2 - \Phi^2}} \{ B\Phi' - B'\Phi \}.$$

From this equation we obtain:

Theorem 2.7. *A curve C is a union curve if and only if*

$$\frac{1}{B^2 q} \mu^{\alpha}_{\gamma} u'^{\gamma} (B^2 \overline{M}^{\beta} - \Phi^2 p^{\beta} \overline{N}) + \frac{\Phi}{q} \sqrt{B^2 - \Phi^2}\, d_{\alpha\beta}\, p^{\alpha} u'^{\beta} +$$
$$+ B(B\Phi' - B'\Phi) / \sqrt{B^2 - \Phi^2} = 0.$$

Solving the scalar product $dx^i/ds \times d^2x^i/ds^2 . d\lambda^i/ds$ with the help of (1.3), (2.7) and (2.8) we obtain

(2.12)
$$dx^i/ds \times d^2x^i/ds^2 . d\lambda^i/ds = k_n \sqrt{B^2 - \Phi^2}.$$

If we solve the left hand member of (2.12) with the use of $dx^i/ds = x^i_{,\alpha} u'^{\alpha}$,

(2.13)
$$d^2x^i/ds^2 = \rho^{\beta} x^i_{,\beta} + k_n X^i,$$

and (1.3), then we obtain

(2.14)
$$k_n (e_{\alpha\beta}\, \mu^{\alpha}_{\gamma} u'^{\gamma} u'^{\beta} - \sqrt{B^2 - \Phi^2}) = 0.$$

Since vanishing of the scalar product $dx^i/ds \times d^2x^i/ds^2 . d\lambda^i/ds$ means the curve C is a 0^*-curve [5] and (2.14) reduces to an identity for $k_n = 0$, therefore we consider the curevs for which $k_n \neq 0$. Thus in view of Theorem (2.1), we have:

Theorem 2.8. *A necessary and sufficient condition for the curve C to be an 0^*-curve is that it be a hypernormal curve, provided (i) the congruence is not normal and (ii) C is not an asymptotic line.*

For a normal congruence, (2.14) reduces to

$$k_n [e_{\alpha\beta}\, d_{\delta\sigma}\, g^{\alpha\sigma}\, u'^{\beta} u'^{\delta} + \sqrt{d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\alpha\sigma}\, u'^{\beta} u'^{\delta} - k_n^2}] = 0.$$

Hence:

Theorem 2.9. *For a normal congruence, the curve C is an 0^*-curve if and only if the normal curvature $k_n (= 0)$ is given by*

$$k_n^2 = d_{\alpha\beta}\, d_{\delta\sigma}\, g^{\alpha\sigma}\, u'^{\beta} u'^{\delta}.$$

If the congruence is formed of one parameter family of curves in S then (2.14) becomes

$$k_n (e_{\alpha\beta}\, p^{\alpha}_{,\gamma}\, u'^{\gamma} u'^{\beta} - \sqrt{B^2 - \Phi^2}) = 0.$$

where $T (= p_{\alpha,\beta} u'^{\alpha} u'^{\beta})$ is the tendency of the congruence in the direction of the curve C. From this equation we obtain the following:

Theorem 2.10. *For the congruence formed of tangents to a one parameter family of curves in S, the curve C is an 0^*-curve if and only if the tendency of the congruence in the direction of the curve C is equal to the magnitude of $d\lambda^i/ds$, provided C is not an asymptotic line.*

We know that if the curve C is a N^*-curve [6], $dx^i/ds . d\lambda^i/ds = \Phi = 0$. Hence we have:

Theorem 2.11. *The curve C is a N^*-curve if and only if the Kummer's second fundamental form be a null form.*

In the previous paper [9] of this series we have obtained that

(2.15)
$$d\lambda^i/ds = \Phi dx^i/ds + \sqrt{B^2 - \Phi^2} (dx^i/ds \times X^i).$$

With the help of (2.13) and (2.15) we get

$$d^2x^i/ds^2 . d\lambda^i/ds = kg \sqrt{B^2 - \Phi^2},$$

which when solved with the help of (1.3) and (2.13), becomes

$$\mu_{\alpha\beta}\, \rho^{\alpha} u'^{\beta} = kg \sqrt{B^2 - \Phi^2},$$

which reduces to an identity if C is a geodesic. Since $d^2x^i/ds^2 . d\lambda^i/ds = 0$ for a R^*-curve [7], therefore in view of Theorem (2.1) we have:

Theorem 2.12. *The necessary and sufficient condition for a curve C to be a R^*-curve is that it be a hypernormal curve provided (i) the curve C is not a geodesic and (ii) the congruence is not normal.*

Next let us solve the product $\lambda^i \times d\lambda^i/ds.\ dx^i/ds$ (vanishing of which implies that C is a α-curve [8]) with the help of (1.1), (1.3) and (2.15), then we have

$$q(e_{\alpha\beta}\ \mu^{\alpha}_{\gamma} u'^{\gamma}\ u'^{\beta} - \sqrt{B^2 - \Phi^2}) = 0.$$

which reduces to an identity for a congruence formed of tangents to a one parameter family of curves in S. From this in view of Theorem (2.1), we observe:

Theorem 2.13. *The necessary and sufficient condition for a curve C to be an α-curve is that it be a hypernormal curve provided, neither the congruence be normal nor formed of tangents to a one parameter family of curves in S.*

If we solve the product $\lambda^i \times d\lambda^i/ds.d^2x^i/ds^2$ (vanishing of which means that C is a β-curve [8]) with the use of the sets of equations {(1.1), (1.3), (2.13)} and {(1.1), (2.13), (2.15)} we get

$$(2.16) \qquad e_{\alpha\beta}\mu^{\beta}_{\gamma}u'^{\gamma}(k_n p^{\alpha} - q\rho^{\alpha}) = \Phi e_{\alpha\beta}\ u'^{\alpha}(\ q\rho^{\beta} - k_n p^{\beta}) - k_n p_{\alpha} u'^{\alpha} \sqrt{B^2 - \Phi^2} = \\ = \Phi q k_u - k_n p_{\alpha} u'^{\alpha} \sqrt{B^2 - \Phi^2},$$

where k_u is the union curvature [3].

From 2.16 we have:

Theorem 2.14. *The necessary and sufficient condition for the curve C to be a β-curve is that the ratio of union curvature and normal curvature is given by*

$$k_u/k_n = \{\ p_{\alpha} u'^{\alpha} \sqrt{B^2 - \Phi^2}\ \}/\Phi q.$$

For a normal congruence (2.16) reduces to

$$e_{\alpha\beta} d_{\gamma\sigma}\ g^{\beta\sigma}\ u'^{\gamma}\rho^{\alpha} = k_n\ k_g,$$

which reduces to an identity if C be geodesic. Hence:

Theorem 2.15. *For a normal congruence, the curve C is a β-curve if and only if it is an asymptotic curve.*

If the congruence is formed of tangents to a one parameter family of curves in S, (2.16) reduces to

$$k_n(e_{\alpha\beta}\ p^{\beta}_{,\gamma}\ u'^{\gamma} p^{\alpha} - p_{\alpha} u'^{\alpha} \sqrt{B^2 - T^2}) = 0,$$

which reduces to an identity if $k_n = 0$ i. e., C is an asymptotic line. Thus:

Theorem 2.16. *If the congruence is formed of tangents to a one parameter family of curves in S, the curve C is a β-curve if and only if either it is a hypernormal curve or the magnitude of $d\lambda^i/ds$ is equal to the tendency of the congruence along the direction of the curve.*

Solving the product $(dx^i/ds \times d^2x^i/ds^2).(\lambda^i \times d\lambda^i/ds)$ (vanishing of which means that C is a γ-curve) with the application of the two sets of equations {(1.1), (1.3), (2.13)} and {(1.1), (2.13), (2.15)} we obtain

$$(2.17) \qquad \mu_{\alpha\beta}\ u'^{\beta}(\rho^{\alpha} p_{\gamma} u'^{\gamma} - k_c u'^{\alpha}) = p_{\alpha} u'^{\alpha} k_g \sqrt{B^2 - \Phi^2} - k_c \Phi.$$

From this equation we have:

Theorem 2.17. *The necessary and sufficient condition for a curve C to be a γ-curve is that its hyperasymptotic curvature be given by*

$$k_c = \frac{1}{\Phi}\, p_{\alpha}\ u'^{\alpha}\, k_g \sqrt{B^2 - \Phi^2}.$$

Cancelling the identical terms on the two sides of (2.17) we get

$$\mu_{\alpha\beta}\rho^{\alpha} u'^{\beta} = k_g \sqrt{B^2 - \Phi^2}\ ,$$

which gives

$$k_g = \mu_{\alpha\beta}\ \rho^{\alpha} u'^{\beta} / \sqrt{B^2 - \Phi^2}\ ,$$

i. e., an expression of geodesic curvature of C.

For a normal congruence (2.17) reduces to an identity and for the congruence formed of tangents to a one parameter family of curves in S, (2.17) becomes

$$p_{\alpha,\beta,}\rho^{\alpha} u'^{\beta}(p_{\gamma} u'^{\gamma}) = p_{\alpha} u'^{\alpha}\, k_g \sqrt{B^2 - T^2}\ .$$

If $p_{\gamma} u'^{\gamma} \neq 0$, the last equation gives

$$T = \sqrt{B^2 - \left(\frac{p_{\alpha,\beta}\rho^{\alpha} u'^{\beta}}{k_g}\right)^2},$$

which is an expression for the tendency of the congruence in the direction of the curve provided the congreunce under consideration is formed of tangents to a one parameter family of curves.

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REFERENCES

1. Bhattacharya P. B. — *Curves on the surface of reference whose normal plane at any point contains the ray of the congruence through that point*, Ganita, Vol. 1, 1950, pp. 23—29.
2. Mishra R. S. — *Union curves and hyperasymptotic curves on the surface of reference of a rectilinear congruence*, Bull. Cal. Math. Soc., Vol. 42, 1950, pp. 213—216.

3. Springer C. E. — *Union curves and union curvature*, Bull. Amer. Math. Soc., Vol. 51, 1945, pp. 686—691.
4. Springer C. E. — *Rectilinear congruences whose developables intersect a surface in its lines of curvature*, Bull. Amer. Math. Soc., Vol. 51, 1945, pp. 990—996.
5. Trivedi H. K. N. — *A problem on some special curves I*, (accepted for publication in Ganita).
6. Trivedi H. K. N. — *A problem on some special curves* II. Communicated for publication.
7. Trivedi H. K. N. — *A problem on some special curves* III. Communicated for publication.
8. Trivedi H. K. N. and Rastogi, S. C. — *On some curves in an Euclidean space of three dimensions* (Under publication).
9. Trivedi H. K. N. and Rastogi, S. C. — *On some special curves on the surface of reference of a rectilinear congruence* (Under publication).

ASUPRA UNOR CURBE SPECIALE PE SUPRAFAȚA DE REFERINȚĂ A UNEI CONGRUENȚE DE DREPTE. II

Rezumat

În articolul precedent din această serie [9] am studiat proprietățile unei curbe $C : x^i = x^i(s)$, $(i = 1, 2, 3)$, pe suprafața de referință $S : x^i = x^i(u^\alpha)$, $(\alpha = 1, 2)$ a unei congruențe de drepte reprezentată prin vectorul unitar cu componente contravariante $\lambda^i = \lambda^i(u^\alpha)$ care este astfel că $d\lambda^i/ds$ este perpendicular pe X^i (componentele contravariante ale normalei la S. Scopul articolului de față este de a studia unele proprietăți ale acestei curbe.

ON THE GENERAL PROBLEM OF ORTHOTROPIC BEAMS LOADED ON THE LATERAL SURFACE AND ON THE ENDS

BY

C. I. BORȘ

To the memory of professor Mendel Haimovici

1. Introduction. The problem of loaded beams on the ends and for which the lateral surface is free from tractions is known as Saint-Venant problem. The problem of loaded beams on the lateral surface when the tractions do not varie with the axial coordinate is known as Almansi-Michell problem and when the lateral surface tractions have polinomial forms with respect to axial coordinate the problem is known as Almansi problem.

The Saint-Venant problem is well-known. Concerning the Almansi-Michell and Almansi problem we know some papers, some of them for the isotropic case [1], [5], [7], [8] and others for the anisotropic case [2], [3], [6], [7].

Generally, there are finded solutions which satisfy the conditions on the lateral surface; the end conditions are passed into account of Saint-Venant problem

In this paper, we shall consider an elastic beam limited by two planes $x_3 = 0$, $x_3 = h (h > 0)$ and by a cylindrical surface $\mathcal{F}$.

We shall denote by $\mathcal{V}$ the domain occupied by the beam and by S the domain and also the area of a cross-section of the beam.

We shall take the axes of x_1, x_2, x_3 such that

$$\iint_S x_1\, d\sigma = 0, \quad \iint_S x_2\, d\sigma = 0, \quad \iint_S x_1 x_2\, d\sigma = 0.$$

All this means that x_3-axis is the central-line of the beam and the axes x_1 and x_2 are the principal axes of inertia of the end $x_3 = 0$.

Here, we shall improve the results of the paper [3], so that the finded solution will satisfy in addition to the lateral surface conditions,