5. Browder F. E. — *Nonlinear elliptic boundary-value problems*, Bull. American Math. Soc., 69 (1963), pp. 862—869.
6. Smirnov V. I. — *A cours of higher Mathematics*, Oxford, Pergamon Press, 1964, vol. 5.

DESPRE EXISTENȚA ȘI UNICITATEA SOLUȚIEI UNEI PROBLEME CU VALORI PE FRONTIERĂ ÎN TEORIA FLUIDELOR MICROPOLARE INCOMPRESIBILE

Rezumat

Se cercetează existența și unicitatea curgerii staționare tridimensionale a unui fluid micropolar incompresibil în cazul condițiilor omogene pe frontieră și a teoriei liniare propusă de Eringen în [2].

ON THE IRROTATIONAL MOTIONS OF MICROPOLAR FLUIDS

BY

MARIAN ARON

1. Introduction. The field equations for incompressible micropolar fluids, as given by Eringen [1] are

$$\operatorname{div} \mathbf{v} = 0, \tag{1.1}$$

$$\begin{gathered}(\lambda + 2\mu + k)\operatorname{grad}\operatorname{div}\mathbf{v} - (\mu + k)\operatorname{curl}\operatorname{curl}\mathbf{v} + \\ k\operatorname{curl}\boldsymbol{\nu} - \operatorname{grad} p + \rho\mathbf{f} = \rho\left[\frac{\partial \mathbf{v}}{\partial t} - \mathbf{v}\times\operatorname{curl}\mathbf{v} + \frac{1}{2}\operatorname{grad}(\mathbf{v}^2)\right],\end{gathered} \tag{1.2}$$

and

$$(\alpha + \beta + \gamma)\operatorname{grad}\operatorname{div}\boldsymbol{\nu} - \gamma\operatorname{curl}\operatorname{curl}\boldsymbol{\nu} + k\operatorname{curl}\mathbf{v} - 2k\boldsymbol{\nu} + \rho\mathbf{l} = \rho j\dot{\boldsymbol{\nu}}, \tag{1.3}$$

in which $\mathbf{v}$, $\boldsymbol{\nu}$, $\mathbf{f}$ and $\mathbf{l}$ represent respectively the velocity, micro-rotation, body force and body couple vectors. The constants ρ and j are the density and gyration parameters while $\alpha, \beta, \gamma, \lambda, \mu, k$ are the viscosity coefficients of micropolar fluids. The material constants conform to the inequalities :

$$\begin{gathered} k \geqslant 0, \quad 2\mu + k \geqslant 0, \quad 3\lambda + 2\mu + k \geqslant 0, \\ 3\alpha + \beta + \gamma \geqslant 0, \quad \gamma + \beta \geqslant 0, \quad \gamma - \beta \geqslant 0. \end{gathered} \tag{1.4}$$

p is an unknown pressure and a dot denotes the material time derivative.

In the present Note, we consider steady – state isochoric motions with lamellar velocity fields and prove in section 2 that if $\dot{\boldsymbol{\nu}} = 0$ then, the field equations are satisfied if and only if the micror-otation velocity fields are also lamellar and the corresponding potential satisfies a Helmholtz equation. In section 3 we indicate a class of dynamically possible accelerationless motions of considered type.

2. The potential of the micro-rotation velocity vector. Let φ be the potential of the velocity vector, ψ be the potential of the body force and η be the potential of the body couple. Then, according to our hypoteses

$$\text{curl } \mathbf{v} = 0, \quad \text{div } \mathbf{v} = 0, \quad \dot{\nu} = 0, \tag{2.1}$$

we have to solve the following system

$$k \text{ curl } \nu = \text{grad } (p - \rho\psi + \frac{1}{2} v^2). \tag{2.2}$$

$$(\alpha + \beta + \gamma) \text{ grad div } \nu - \gamma \text{ curl curl } \nu - 2k\nu + \rho \text{ grad } \eta = 0. \tag{2.3}$$

By application of curl operator to equation (2.2) one obtains [1])

$$\text{curl curl } \nu = 0, \tag{2.4}$$

so that the equation (2.3) becomes

$$(\alpha + \beta + \gamma) \text{ grad div } \nu - 2k\nu + \rho \text{ grad } \eta = 0. \tag{2.5}$$

We apply again the curl operator to equation (2.5) and we get

$$\text{curl } \nu = 0. \tag{2.6}$$

From the equation (2.2), it follows now

$$p = h(t) - \frac{1}{2} v^2 + \rho\psi. \tag{2.7}$$

where $h(t)$ is an arbitrary function of t.

Let χ be the potential of the micro-rotation velocity vector. Then, from (2.5) we find that the function χ must satisfy the Helmholtz equation

$$(\alpha + \beta + \gamma)\Delta\chi - 2k\chi + \rho\eta = 0. \tag{2.8}$$

3. A class of flows. We consider an orthogonal coordinate cartesian system $Oxyz$ and look for a solution of the system given by (2.8) and

$$\text{curl } \mathbf{v} = 0, \quad \text{div } \mathbf{v} = 0, \tag{3.1}$$

$$(\text{grad grad } \chi). \mathbf{v} = 0. \tag{3.2}$$

A particular solution of the system (3.1) is given by:

[1]) When $k = 0$, the field equations are decoupled and the stress tensor is symmetric. The global motion of the fluid is then unaffected by the micro-rotation of the fluid particle. According to (1.4) we suppose $k > 0$.

$$v_1 = \frac{\partial\varphi}{\partial x}, \quad v_2 = \frac{\partial\varphi}{\partial y}, \quad \Delta\varphi = 0, \quad v_3 = 0. \tag{3.3}$$

Taking into account (1.4), in the absence of the body couple, without any loss of generality we can assume

$$\alpha + \beta + \gamma > 0. \tag{3.4}$$

We consider now the particular solution of the Helmholtz equation (2.8) given by

$$\chi = C_1 \exp(\varepsilon z) + C_2 \exp(-\varepsilon z), \tag{3.5}$$

where

$$\varepsilon^2 = \frac{2k}{\alpha + \beta + \gamma}, \tag{3.6}$$

and where C_1 and C_2 are arbitrary constants.

It is clear that (3.3) and (3.5) verify the conditions (3.2).

In this way we obtained the class of dynamically possible motions defined by (3.3) and

$$\nu_1 = 0, \quad \nu_2 = 0, \quad \nu_3 = \varepsilon[C_1 \exp(\varepsilon z) - C_2 \exp(-\varepsilon z)], \tag{3.7}$$

which satisfies to all the above mentioned requirements.

REFERENCES

1. Eringen A. C. — *Theory of Micropolar Fluids*. Journ. Math. Mech., 16 (1966), p. 1.

ASUPRA MIŞCĂRILOR IROTAŢIONALE ALE FLUIDELOR MICROPOLARE

Rezumat

În această Notă se consideră mişcările staţionare şi irotaţionale ale fluidelor micropolare incompresibile şi se demonstrează, în cazul în care derivata materială a vectorului micro-rotaţiei este nulă, că este necesar şi suficient ca vectorul micro-rotaţiei să fie lamelar şi ca potenţialul corespunzător să satisfacă o ecuaţie de tip Helmholtz pentru ca ecuaţiile de cîmp să fie satisfăcute. Se obţine de asemenea o clasă de curgeri cu proprietăţile menţionte mai sus.