Der von der Knotenpunktmenge $\{x, x_1^1, x_2^1, x_1^2, x_2^2, x_1^3, x_2^3\}$ in $\overline{G}$ aufgespannte Teilgraph ist das gesuchte Loch (Abb. 1 und 1).

Der Satz 2 ist damit bewiesen.

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ASUPRA GRAFURILOR PERFECTE ȘI CRITIC IMPERFECTE

Rezumat

Într-o lucrare publicată anterior [2] autorul introduce noțiunea de graf critic α-imperfect, noțiune ce s-a dovedit a fi foarte utilă în studiul grafurilor perfecte. Studiul proprietăților grafurilor critic α-imperfecte a permis demonstrarea în mod unitar și mai simplu a multor teoreme cunoscute și în același timp obținerea unor noi rezultate interesante relative la grafurile perfecte. Astfel, cunoscuta ipoteză a lui Berge și Gilmore asupra grafurilor perfecte este echivalentă cu următoarea: un graf G este atunci și numai atunci critic α-imperfect, dacă G sau complementul său $\overline{G}$ sînt cicluri elementare L_{2k+1} de lungime $2k+1$ ($k \geqslant 2$). Lucrarea de față este strîns legată de cea menționată, demonstrîndu-se alte proprietăți ale grafurilor critic α-imperfecte. O proprietate a anumitor mulțimi de articulație ale unui graf critic α-imperfect, demonstrată în lucrare (teorema 1), are, printre consecințele sale, afirmații cu ajutorul cărora se pot demonstra ușor două teoreme cunoscute (teorema lui Hajnal-Surányi și teorema lui Dilworth) asupra grafurilor α-perfecte.

Lucrarea se încheie cu o metodă de demonstrare a ipotezei Berge-Gilmore pentru grafuri cu numărul de stabilitate doi, metodă care probabil poate fi generalizată pentru grafuri cu orice număr de stabilitate.

ON LINEAR SEQUENTIAL MACHINES WITH VARIABLE STRUCTURE

BY

V. FELEA

Let M be a linear sequential machine with variable structure, with l inputs and m outputs. We denote by $X(t)$, $Y(t)$ the input and output vectors, respectively, at the time t.

For a natural number h, we consider M_1 a linear sequential machine with variable structure, with lh inputs and mh outputs, such that if denote by $X_1(t)$, $Y_1(t)$ the input and output vectors, respectively, at the time t, then we have

$$X_1(r) = \begin{bmatrix} X(rh) \\ X(rh+1) \\ \vdots \\ X(rh+h-1) \end{bmatrix}, \quad Y_1(r) = \begin{bmatrix} Y(rh) \\ Y(rh+1). \\ \vdots \\ Y(rh+h-1) \end{bmatrix}.$$

If the structure of M is known, then the characterizing matrices of M_1 can be constructed.

One obtains in a particular case the series-to-parallel transformations of linear sequential circuits studied by Gill [1].

Definition. *A sequential machine with variable structure is* $M = (S, X, Y, f)$, *where*:

1. S *is a nonempty set of states*;
2. X *is a nonempty set of input symbols*;
3. Y *is a nonempty set of output symbols*;
4. $f : N \to S^{S \times X} \times Y^{S \times X}$.

Thus $f(t) = (\delta_t, \lambda_t)$ with $\delta_t : S \times X \to S$ and $\lambda_t : S \times X \to Y$ for $t = 0, 1, 2, \ldots$. δ_t are called *transition* functions and λ_t *output* functions. Let F be a field and k a non-negative integer. Then F_k denotes the vector space of column vectors of dimension k over F. By definition $F_0 = \{O\}$. We shall say that a sequential machine with variable structure is finite, when, S, X, and Y are finite.

Definition. *A linear sequential machine with variable structure (VLSM for short) is a sequential machine with variable structure for which there exist a field F and non-negative integers n, l, m such that $S = F_n$, $X = F_l$, $Y = F_m$, there exist $n \times n$ matrices $A(t)$, $n \times l$ matrices $B(t)$, $m \times n$ matrices $C(t)$, $m \times l$ matrices $D(t)$ over F, where $t = 0, 1, 2, \ldots$ such that for each $(s, x) \in S \times X$*

(1) $$\delta_t(s, x) = A(t)s + B(t)x$$

(2) $$\lambda_t(s, x) = C(t)s + D(t)x.$$

Such an VLSM will be denoted by $(F, A(t), B(t), C(t), D(t), t = 0, 1, 2, \ldots)$.

Let $X(t)$, $s(t)$, $Y(t)$ be the input, state, output vectors respectively at the moment t, and $s(0)$ be the state at the moment zero.

Then the relations (1) and (2) become:

(3) $$s(t+1) = A(t)s(t) + B(t)X(t),$$

(4) $$Y(t) = C(t)s(t) + D(t)X(t).$$

We shall give a theorem about VLSM without proof, which is a immediate generalization of a well-known theorem about linear sequential machines (theorem 1.6, [2]).

Theorem 1. *Let $M = (F, A(t), B(t), C(t), D(t), t = 0, 1, 2, \ldots)$ be a* VSLM. *For each $s \in F_n$ and $X(0), X(1), \ldots, X(t) \in F_l$ and $t = 0, 1, 2, \ldots$ yields:*

(5) $$s(t+1) = \left(\prod_{k=t}^{0} A(k)\right)s + \sum_{v=0}^{t}\left(\prod_{k=1}^{t-v} A(t+1-k)\right)B(v)X(v)$$

(6) $$Y(t) = C(t)\left(\prod_{k=t-1}^{0} A(k)\right)s + \sum_{v=0}^{t-1} C(t)\left(\prod_{k=1}^{t-v-1} A(t-k)\right)B(v)X(v) + D(t)X(t)$$

where s is the initial state, i. e. $s = s(0)$.

We accept the conventions: $\prod_{k=1}^{0} F(k) = 1$ and $\prod_{k=-1}^{0} F(k) = 1$.

Definition. *Let $M = (F, A(t), B(t), B(t), C(t), D(t), t = 0, 1, 2, \ldots)$ be a* VSLM. *We define the translation T corresponding to M:*

$$T = T(M) = \bigcup_{t=0}^{\infty} (X(0)\,X(1) \ldots X(t),\ Y(0)\,Y(1) \ldots Y(t))$$

with $s = s(0)$ the initial state of M. Let us denote:

$$X_1(r) = \begin{bmatrix} X(rh) \\ X(rh+1) \\ \vdots \\ X(rh+h-1) \end{bmatrix} \qquad Y_1(r) = \begin{bmatrix} Y(rh) \\ Y(rh+1) \\ \vdots \\ Y(rh+h-1) \end{bmatrix}$$

where h is a natural number, and:

$$T_1^{(h)} = \bigcup_{r=0}^{\infty} (X_1(0)X_1(1) \ldots X_1(r),\ Y_1(0)Y_1(1) \ldots Y_1(r)).$$

If the VSLM which realizes T is known, then we shall determine another VSLM M_1, which realizes $T_1^{(h)}$.

Theorem 2. *If $M = (F, A(t), B(t), C(t), D(t), t =, 0, 1, 2, \ldots)$ is a* VSLM *with the initial state s, then $T_1^{(h)}$ is realized by $M_1 = (F, A_1, (t), B_1(t), C_1(t), D_1(t), t = 0, 1, 2, \ldots)$ with the initial state s, where:*

(7) $$A_1(r) = \prod_{i=h-1}^{0} A(rh+i).$$

(8) $$B_1(r) = \left|\left(\prod_{\beta=h-1}^{1} A(rh+\beta)\right)B(rh), \left(\prod_{\beta=h-2}^{1} A(rh+\beta)\,B(rh+1), \ldots, \ldots, A(rh+1)\,B(rh+h-2),\quad B(rh+h-1)\right)\right|,$$

(9) $$C_1(r) = \begin{bmatrix} C(rh) \\ C(rh+1)\,A(rh) \\ C(rh+2)\,A(rh+1)\,A(rh) \\ \vdots \\ C(rh+h-1)\,\prod_{h-2}^{0} A(rh+i) \end{bmatrix},$$

(10) $$D_1(r) = \begin{bmatrix} D(rh) & & & \\ C(rh+1)\,B(rh) & D(rh+1) & & \\ C(rh+2)\,A(rh+1)\,B(rh) & C(rh+2)\,B(rh+1) & D(rh+2) & \\ \cdots & \cdots & \cdots & \cdots \\ C(rh+h-1)\prod_{h-2}^{1} A(rh+i)\,B(rh) & C(rh+h-1) \times \left(\prod_{h-2}^{0} A(rh+i)\right)B(rh+1) & \ldots\ C(rh+h-1) \times B(rh+h-2) & D(rh+h-1) \end{bmatrix},$$

Proof. We may assume that $h \geqslant 2$. We have

$t = rh + j - 1$, where $1 \leqslant j \leqslant h$ and

$$\prod_{k=rh+j-2}^{0} A(k) = \prod_{i=j-2}^{0} A(rh+i) \prod_{\mu=r-1}^{0} \left(\prod_{i=h-1}^{0} A(\mu h + i) \right). \tag{11}$$

Using (6) we obtain:

$$Y(rh+j-1) = C(rh+j-1) \prod_{k=rh+j-2}^{0} A(k)s + \sum_{v=0}^{rh+j-2} C(rh+j-1).$$

$$\sum_{k=1}^{rh+j-1-v-1} A(rh+j-1-k)\Big) B(v)\, X(v) + D(rh+j-1)X(rh+j-1) = \tag{12}$$

$$= S_1 + S_2 + S_3,$$

$$S_1 = C(rh+j-1) \prod_{k=rh+j-2}^{0} A(k)s = C(rh+j-1) \prod_{i=j-2}^{0} A(rh+i) \tag{13}$$

$$\prod_{\mu=r-1}^{0} \left(\prod_{i=h-1}^{0} (A(\mu h + i) \right) s,$$

$$S_3 = D(rh+j-1)X(rh+j-1),$$

$$S_2 = \sum_{v=0}^{rh+j-2} C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-v-1} A(rh+j-1-k) \right) B(v)\, X(v) =$$

$$= \sum_{\mu=0}^{r-1} \left[\sum_{i=0}^{h-1} C(rh+j-2) \left(\prod_{k=1}^{rh+j-1-\mu h-i-1} A(rh+j-1-k) \right) \times \right.$$

$$\left. \times B(\mu h + i)\, X(\mu h + i) \right] + \sum_{i=0}^{j-2} C(rh+j-1)$$

$$\left(\prod_{k=1}^{rh+j-1-rh-i-1} A(rh+j-1-k) \right) B(rh+i)\, X(rh+i) =$$

$$= \sum_{\mu=0}^{r-1} C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-\mu h-1} A(rh+j-1-k) \right) \times$$

$$\times B(\mu h)\, X(\mu h) + \sum_{\mu=0}^{r-1} C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-\mu h-2} A(rh+j-1-k) \right) \times$$

$$\times B(\mu h + 1)\, X(\mu h + 1) + \ldots + \sum_{\mu=0}^{r-1} C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-\mu h-(h-1)} \times \right.$$

$$\left. \times A(rh+j-1-k) \right) B(\mu h + h - 2)\, X(\mu h + h - 2) +$$

$$+ \sum_{\mu=0}^{r-1} C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-\mu h-h} A(rh+j-1-k) \right) B(\mu h + h - 1) \times$$

$$\times X(\mu h + h - 1) + C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-rh-1} A(rh+j-1-k) \right) \times$$

$$\times B(rh)X(rh) + C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-rh-2} A(rh+j-1-k) \right) \times$$

$$\times B(rh+1)X(rh+1) + \ldots +$$

$$+ C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-rh-(j-2)} A(rh+j-1-k) \right) B(rh+j-3) \times$$

$$\times X(rh+j-3) + C(rh+j-1) \left(\prod_{k=1}^{rh+j-1-rh-(j-1)} A(rh+j-1-k) \right) \times \tag{14}$$

$$\times B(rh+j-2)\, X(rh+j-2).$$

To write (14), we have used:

$$\sum_{v=0}^{t-1} F(v) = \sum_{v=0}^{rh+j-2} F(v) = \sum_{\mu=0}^{r-1} \left(\sum_{i=0}^{h-1} F(\mu h + i) \right) + \sum_{i=0}^{j-2} F(rh+i). \tag{15}$$

Let us now express the products, which appear in S_2:

$$\prod_{k=1}^{rh+j-1-\mu h-1} A(rh+j-1-k) = \prod_{k=1}^{(r-\mu)h+j-2} A(rh+j-1-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k)$$

$$\prod_{k=j}^{(r-\mu)h+j-2} A(rh+j-1-k) = \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=1}^{(r-\mu)h-1} A(rh-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{\alpha=1}^{(r-\mu-1)} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right) \prod_{\beta=h-1}^{1} A(\mu h + \beta).$$

$$\prod_{k=1}^{(rh+j-1)-\mu h-2} A(rh+j-1-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=j}^{(r-\mu)h+j-3} A(rh+j-1-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=1}^{(r-\mu)h-2} A(rh-k) = \prod_{k=1}^{j-1} A(rh+j-1-k)$$

$$\prod_{\alpha=1}^{r-\mu-1} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right) \prod_{\beta=h-2}^{1} A(\mu h + \beta).$$

$$\prod_{k=1}^{rh+j-1-\mu h-(h-1)} A(rh+j-1-k) = \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=j}^{(r-\mu)h+j-h} A(rh+j-1-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=1}^{(r-\mu)h-(h-1)} A(rh-k) = \prod_{k=1}^{j-1} A(rh+j-1-k)$$

$$\prod_{\alpha=1}^{(r-\mu-1)} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)\, h+\beta) \right) \prod_{\beta=h-(h-1)}^{1} A(\mu h+\beta).$$

$$\prod_{k=1}^{rh+j-1-\mu h-h} A(rh+j-1-k) = \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=j}^{(r-\mu)h+j-h-1} A(rh+j-1-k) =$$

$$= \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{k=1}^{(r-\mu)h-h} A(rh-k) = \prod_{k=1}^{j-1} A(rh+j-1-k).$$

$$\prod_{\alpha=1}^{(r-\mu-1)} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right). \tag{16}$$

Substituting (16) in (14) and using (13), (12) becomes:

$$Y(rh+j-1) = C(rh+j-1) \prod_{i=j-2}^{0} A(rh+i) \prod_{\mu=r-1}^{0} \left(\prod_{i=h-1}^{0} A(\mu h+i) \right) s +$$

$$+ \sum_{\mu=0}^{r-1} C(rh+j-1) \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{\alpha=1}^{r-\mu-1} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta)\right) \times$$

$$\times \left(\prod_{\beta=h-1}^{1} A(\mu h+\beta) \right) B(\mu h) X(\mu h) + \sum_{\mu=0}^{r-1} C(rh+j-1).$$

$$\prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{\alpha=1}^{r-\mu-1} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right) \left(\prod_{\beta=h-2}^{1} A(\mu h+\beta) \right) B(\mu h+1).$$

$$X(\mu h+1) + \ldots + \sum_{\mu=0}^{r-1} C(rh+j-1) \prod_{k=1}^{j-1} A(rh+j-1-k).$$

$$\prod_{\alpha=1}^{r-\mu-1} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right) \left(\prod_{\beta=h-(h-1)}^{1} A(\mu h+\beta) \right) B(\mu h+h-2) X(\mu h+h-2) +$$

$$+ \sum_{\mu=0}^{r-1} C(rh+j-1) \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{\alpha=1}^{r-\mu-1} \left(\prod_{\beta=h-1}^{0} A((r-\alpha)h+\beta) \right) B(\mu h+h-1)$$

$$X(\mu h+h-1) + C(rh+j-1) \prod_{k=1}^{j-2} A(rh+j-1-k) B(rh) X(rh) +$$

$$+ C(rh+j-1) \prod_{k=1}^{j-3} A(rh+j-1-k)\, B(rh+1) X(rh+1) + \ldots +$$

$$+ C(rh+j-1) \prod_{k=1}^{1} A(rh+j-1-k) B(rh+j-3)\, X(rh+j-3) +$$

$$+ C(rh+j-1) B(rh+j-2) X(rh+j-2).$$

Using (7) we obtain:

$$Y(rh+j-1) = C(rh+j-1) \prod_{i=j-2}^{0} A(rh+i) \prod_{\mu=r-1}^{0} A_1(\mu) s +$$

$$+ \prod_{\mu=0}^{r-1} C(rh+j-1) \prod_{k=1}^{j-1} A(rh+j-1-k) \prod_{\alpha=1}^{r-\mu-1} A_1(r-\alpha) \left| \left(\prod_{\beta=h-1}^{1} A(\mu h+\beta)\right) \times \right.$$

$$B(\mu h) X(\mu h) + \left(\prod_{\beta=h-2}^{1} A(\mu h+\beta) \right) B(\mu h+1) X(\mu h+1) + \ldots +$$

$$\left. + \left(\prod_{\beta=1}^{1} A(\mu h+\beta) \right) B(\mu h+h-2) X(\mu h+h-2) + B(\mu h+h-1)\, X(\mu h+h-1) \right| +$$

$$+ C(rh+j-1) \prod_{k=1}^{j-2} A(rh+j-l-k) B(rh) X(rh) +$$

$$+ C(rh+j-1) \prod_{k=1}^{j-3} A(rh+j-1-k) B(rh+1) X(rh+1) + \ldots + \tag{17}$$

$$+ C(rh+j-1) \prod_{k=1}^{1} A(rh+j-1-k)\, B(rh+j-3) X(rh+j-3) +$$

$$+ C(rh+j-1) B(rh+j-2) X(rh+j-2) + D(rh+j-1) X(rh+j-1).$$

If the initial state of M_1 is s and the input vector is $X_1(r)$, then the output vector $Y_1'(r)$ is obtained by (6):

$$Y_1'(r) = C_1(r) \prod_{k=r-1}^{0} A_1(k) s + \sum_{\mu=0}^{r-1} C_1(r) \left(\prod_{\alpha=1}^{r-\mu-1} A_1(r-\alpha)) B_1(\mu) \right. \times$$

$$\times X_1(\mu) + D_1(r) X_1(r).$$

From (7), (8), (9), (10), (17) it follows that

$$Y_1'(r) = X_1(r);$$

that concludes the proof of the theorem.

Definition. *Let* $M = (F, A(t), B(t), C(t), D(t), t = 0, 1, 2, \ldots)$ *be a* VSLM. *If* $A(t)$, $B(t)$, $C(t)$, $D(t)$, *are independent of* t, *then one obtains a linear sequential machines* (LSM *for short*). In this case, and for $m = l = 1$, we find again theorem 1 proved by Gill in [1].

Corollary. *Let* $M = (F, A(t), B(t), C(t), D(t), t = 0, 1, 2, \ldots)$ *be a* VSLM. *If* $A(t)$, $B(t)$, $C(t)$, $D(t)$ *are almost periodical i. e. there exist natural numbers* $l_1, l_2, l_3, l_4, \pi_1, \pi_2, \pi_3, \pi_4$ *such that:*

$$A(t+\pi_1) = A(t) \text{ for } t \geqslant l_1, \quad B(t+\pi_2) = B(t) \text{ for } t \geqslant l_2,$$
$$C(t+\pi_3) = C(t) \text{ for } t \geqslant l_3 \text{ and } D(t+\pi_4) = D(t) \text{ for } t \geqslant l_4.$$

Let $s(0)$ *be the initial state of* M *and*

$$l = \max_{1 < i < 4} (l_i) \quad \pi = \text{l. c. m.}\ (\pi_1, \pi_2, \pi_3, \pi_4).$$

Then $T^{(\pi)}$ *with the initial state* $s = s(l)$ *is realized by a* LSM.

Example. Let us consider $F = \{0, 1, 2\} = GF(3)$ and

$$A(t) = \begin{bmatrix} 1 & t^2 & t+1 \\ t^2+t+1 & 0 & t^2+2 \\ 1 & 2 & t \end{bmatrix}, \qquad B(t) = \begin{bmatrix} t^2 & 1 \\ 0 & t \\ 2 & 2t \end{bmatrix},$$

$$C(t) = \begin{bmatrix} 1 & t & t^2 \\ 0 & 2 & t+1 \end{bmatrix}, \qquad D(t) = \begin{bmatrix} t^2+1 & 0 \\ 1 & t \end{bmatrix},$$

where the elements of the matrices $A(t)$, $B(t)$, $C(t)$, $D(t)$ are polynomials $p(t)$ over $GF(3)$ and for $t \in N$ $p(t) = p(\alpha)$ where $t \equiv \alpha \pmod 3$ and $\alpha \in F$. We consider $h = 2$. Then we obtain:

$$A_1(r) = \begin{bmatrix} r^2+1 & r^2+r+1 & 2r^2 \\ 2r^2+r & 2r & r \\ 2r^2+1 & r^2+r+2 & r+2 \end{bmatrix},$$

$$B_1(r) = \begin{bmatrix} r^2+r+1 & r^2+1 & r^2+r+1 & 1 \\ 2r & 2r^2+r & 0 & 2r+1 \\ r^2+r+2 & 2r^2+2r+1 & 2 & r+2 \end{bmatrix},$$

$$C_1(r) = \begin{bmatrix} 1 & 2r & r^2 \\ 0 & 2 & 2r+1 \\ r & 2r+2 & 0 \\ 2r^2+1 & r+1 & r+1 \end{bmatrix},$$

$$D_1(r) = \begin{bmatrix} r^2+1 & 0 & 0 & 0 \\ 1 & 2r & 0 & 0 \\ 2r+2 & 2r^2+r+1 & r^2+r+2 & 0 \\ r+1 & 2r^2 & 1 & 2r+1 \end{bmatrix},$$

If we consider:

$$s = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad X(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad X(3) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$X(4) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad X(5) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad X(6) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad X(7) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

then we have: $Y(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $s(1) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $Y(1) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $s(2) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$,

$$Y(2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad s(3) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad Y(3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad s(4) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad Y(4) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$s(5) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad Y(5) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad s(6) = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad Y(6) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$s(7) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad Y(7) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

$$X_1(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad X_1(1) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad X_1(2) = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad X_1(3) = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix},$$

$$Y_1(0) = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} Y(0) \\ Y(1) \end{bmatrix}, \quad s_1(1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad Y_1(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} Y(2) \\ Y(3) \end{bmatrix}$$

$$s_1(2) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \quad Y_1(2) = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} Y(4) \\ Y(5) \end{bmatrix} \quad s_1(3) = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

$$Y_1(3) = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} Y(6) \\ Y(7) \end{bmatrix}.$$

REFERENCES

1. Gill Arthur — *On the series-to-parallel transformation of linear sequential circuits*, IEEE Trans. Electron. Comput., 1966, 15, Nr. 1, 107—108.
2. Gallaire H., Harrison M. A. — *Decomposition of Linear Sequential Machines*, Math. Systems Theory, vol. 3, No. 3, 1969, 246—287.

ASUPRA MAŞINILOR LINIARE SECVENŢIALE CU STRUCTURĂ VARIABILĂ

Rezumat

Se studiază transformarea serie-paralelă a maşinilor liniare secvenţiale cu structură variabilă. Se obţine, în caz particular, transformarea serie-paralelă a circuitelor liniare secvenţiale tratată de Gill în [1].

RECENZII

HERMANN MINKOWSKI BRIEFE and DAVID HILBERT. Cu contribuţii şi sub redacţia lui L. Rüdenberg şi H. Zassenhaus. Springer-Verlag, Berlin-Heidelberg-New York, 1973, 166 p.

Din prefaţa cărţii scrisă în 1972 de D-na Lily Rüdenberg, fiica lui Hermann Minkowski, rezultă că scrisorile acestuia din urmă, către colegul şi prietenul său David Hilbert, au fost păstrate de către David Hilbert şi notate pe margini probabil pentru se putea referi la ele în răspunsul său. În 1930, deci după moartea lui Minkowski, D-na Käthe Hilbert, soţia lui David Hilbert a dat aceste scrisori D-nei Minkowski, care le-a păstrat cu pietate şi le-a cedat ulterior Institutului Matematic din Göttingen. D-na Minkowski, la rîndul ei, a dat scrisorile lui Hilbert către Minkowski, care şi acestea, fuseseră păstrate de destinatar, D-nei Hilbert. Din păcate aceste scrisori n-au mai putut fi găsite.

Este deosebit de interesant şi impresionant pentru cititorul de azi să urmărească în aceste scrisori mersul gîndirii acestor deschizători de drumuri, care au lăsat urme neşterse în istoria matematicii moderne. Se găsesc în ele, de multe ori cu ton glumeţ, uneori bucuriile, alteori îndoielile lor cu privire la problemele care îi frămîntau şi care constituie matematica epocii moderne. Este tot atît de impresionant să întîlneşti referiri, fie la viaţa personală, fie la lucrările colegilor lor Hurwitz, Hölder, Burckhardt etc.; nume care parcă au încetat a aparţine unor persoane, ci au devenit nume de teoreme, de noţiuni sau de capitole ale matematicii.

Pe lîngă prefaţa D-nei Lily Rüdenberg, culegerea de scrisori este precedată de două articole, scrise de Hans Zassenhaus, unul despre preistoria unei comunicări făcute de Hilbert şi Minkowski în 1893 la D.M.V. despre dezvoltarea teoriei numerelor, şi al doilea consacrat lui Friedrich Althoff, unul din înalţii funcţionari ai Ministerului Culturii în Prusia, care a jucat un rol important în dezvoltarea învăţămîntului superior în Germania.

Adolf Haimovici

CAMBRIDGE SUMMER SCHOOL IN MATHEMATICAL LOGIC. Held in Cambridge, England, August 1—21, 1971. Springer-Verlag, Berlin-Heidelberg-New York, 1973, X + 660 p.

Cartea conţine o mare parte a expunerilor prezentate în cadrul Cursurilor de vară de logică matematică ce au avut loc la Cambridge între 1—21 august 1971 şi au fost grupate în cinci părţi: teoria modelelor, intuiţionism matematic, metoda priorităţii, combinatorică infinită şi automate finite. În legătură cu teoria mode-