deră fibrările de jeturi asociate unui fibrat vectorial foliat, obținîndu-se diagramele importante ($\Delta^k E$) și se introduc conexiunile adaptate și foliate de un ordin oarecare pe un fibrat vectorial foliat. Cel de-al doilea paragraf „Operatori diferențiali pe fibrări vectoriale foliate" se ocupă de unele proprietăți generale ale operatorilor diferențiali liniari „foliați" între două fibrări vectoriale foliate cu aceeași bază, adică operatori a căror parte transversă" depinde numai de foi. Tot aici sînt date exemple de asemenea operatori și se cercetează legătura între operatori oarecare și conexiuni. În fine, în ultimul paragraf „Probleme de existență" se calculează obstrucția coomologică la existența unui operator diferențial foliat de simbol dat și se demonstrează existența unor asemenea operatori atunci cînd există conexiuni foliate corespunzătoare. Menționăm că tot aici se indică un exemplu de foliație (construită de R. Roussarie), care nu admite o conexiune transversală foliată. Lucrarea are la bază teoriile expuse în [1], [6], [9] și [13].

Analele științifice ale Universității „Al. I. Cuza" din Iași
Tomul XX, s. I a, fasc. 2, 1974

CONFORMAL STRUCTURES ON BANACH VECTOR BUNDLES

BY

MIHAI ANASTASIE

In this paper, conformal structures, in particular Weyl structures on Banach vector bundles are defined. We prove that there is a one-to-one correspondence between the set of conformal structures on a Banach vector bundle and the set of reductions of its structural group to the conformal group.

The existence and the uniqueness of a connection without torsion, compatible with a Weyl structure on a Banach manifold are proved.

1. Linear conformal space. Conformal group. Let **E** be a real, infinit-dimensional linear space.

Definition 1.1. *A conformal structure on* **E** *is a set* $C(\mathbf{E})$ *of scalar products on* **E**, *denoted by* $(\,,\,)_a$, $a \in \mathcal{J}$ *which satisfy*

$$(1.1) \qquad (\,,\,)_a = \lambda_{ab}(\,,\,)_b \qquad a, b \in \mathcal{J},$$

where λ_{ab} *is a positive real number, and* $\mathcal{J}$ *a set of indices.*

Definition 1.2. *The linear space* **E** *with conformal structure* $C(\mathbf{E})$ *is called a conformal space.*

Remark 1.1. In the conformal space **E** the angle between two vectors can be defined by

$$(1.2) \qquad \cos(u, v) = \frac{(u, v)_a}{\| u \|_a \, \| v \|_a} \qquad \forall u, v \in \mathbf{E},\ a \in \mathcal{J},$$

where $\| u \|_a = \sqrt{(u, u)_a}$, the ratio of their lengths is well defined but their absolute lengths are not defined.

If $(\,,\,)$ is a fixed element of $C(\mathbf{E})$, (1.1) can be replaced by

$$(1.3) \qquad (\,,\,)_a = \lambda_a(\,,\,)_a \qquad \forall a \in \mathcal{J}.$$

This implies that all norms $u \to \| u \|_a$ are equivalent to the fixed norm $u \to \| u \| = \sqrt{(u, u)}$. In the following, the space **E** with the norm $\| . \|$ will be supposed to be complete.

Remark 1.2. Let $Ri\,(\mathbf{E})$ be the set of all scalar products on $\mathbf{E}$. We have

$$C\,(\mathbf{E}) \subseteq Ri\,(\mathbf{E}) \subseteq L_s^2\,(\mathbf{E}); \tag{1.4}$$

(Here $L_s^2\,(\mathbf{E})$ is the linear space of bilinear and symmetric maps $s : \mathbf{E} \times \mathbf{E} \to \mathbf{E}$).
Let $L\,(\mathbf{E})$ be the linear space of linear bounded operators on $\mathbf{E}$.

Definition 1.3. *We say that $A \in L\,(\mathbf{E})$ preserves the conformal structure of* $\mathbf{E}$ *if there is a unique $a \in \mathcal{I}$ such that*

$$(Au,\ Av) = (u,\ v)_a \qquad u,\ v \in \mathbf{E}.$$

If A^* denotes the adjoint operator of A with respect to the scalar product $(\ ,\ )$, then we have

Theorem 1.1. *Let* $\mathbf{E}$ *be a conformal space and $A \in L\,(\mathbf{E})$. The following conditions are equivalent*

1) *A preserves the conformal structure of* $\mathbf{E}$;

2) *There is a unique real number $k_A > 0$ such that*

$$A^*\,A = k_A\,I\ (I \text{ identity operator});$$

3) *A preserves the angle between vectors of* $\mathbf{E}$.

Proof. Obvious.

Definition 1.4. *An operator $A \in L\,(\mathbf{E})$ which satisfies one of the conditions of Theorem 1.1 will be called a conformal operator.*

Let $CO\,(\mathbf{E})$ be the set of invertible conformal operators and $O\,(\mathbf{E})$ the set of invertible operators which satisfy $A^*A = I$. We immediately obtain the following

Theorem 1.2. *The sets $CO\,(\mathbf{E})$ and $O\,(\mathbf{E})$ are subgroups of the group $GL\,(\mathbf{E})$ of all invertibles operators.*

We call $CO\,(\mathbf{E})$ and $O\,(\mathbf{E})$ the conformal group and the orthogonal group of $\mathbf{E}$, respectively.

Theorem 1.3. *There is an isomorphism*

$$\alpha : CO\,(\mathbf{E}) \to O\,(\mathbf{E}) \times R_+^*,$$

where R_+^* is the positive real multiplicative group.

Proof. For $A \in CO\,(\mathbf{E})$ and $A^*A = k_A\,I$, we put

$$\alpha\,(A) = \left(\frac{1}{\sqrt{k_A}}\,A,\ k_A\right) \text{and for } (B,\ \lambda) \in O\,(\mathbf{E}) \times R_+^*,\ \alpha^{-1}\,(B,\ \lambda) = \sqrt{\lambda}\,B.$$

Let $GL\,(\mathbf{E})$ be endowed with the topology induced by the norm topology of $L\,(\mathbf{E})$. We identify the group R_+^* with the homotheties group of $\mathbf{E}$. The following result is obvious.

Theorem 1.4. *The subgroups $CO\,(\mathbf{E})$, $O\,(\mathbf{E})$ and R_+^* are closed, and the map α from Theorem 1.3 is a topological isomorphism.*

2. Conformal structures on a Banach vector bundle. All vector bundles, manifolds and maps considered in the following sections will be assumed of class C^∞.

Let E and M be manifolds, modelled on Banach spaces, and suppose M connected. Let $\pi : E \to M$ be a vector bundle with fibre the conformal space $\mathbf{E}$. We denote by $Ri\,(\pi)$ the set of Riemannian metrics on π, and we define the following equivalence relation:

$$\Lambda : g \sim g' \Leftrightarrow g' = e^{\lambda} \cdot g \qquad g,\ g' \in Ri\,(\pi),$$

where λ is a smooth function on M.
(The use of exponential function is a handy way of ensuring positivity).

Definition 2.1. *A conformal structure on π is an equivalence class C with respect to Λ of Riemannian metrics on π.*

Remark 2.1. If the equivalence class C contains only one element, we obtain a Riemannian structure on π.

The proofs of the two following theorems are standard. (See [3, Ch. 7] for the particular case of the Riemannian structure).

Theorem 2.1. *Let $\pi : E \to M$ be a vector bundle with fibre* $\mathbf{E}$ *and suppose M admits a partition of unity. Then the vector bundle π admits a conformal structure.*

Theorem 2.2. *Let $\pi : E \to M$ and $\pi' : E' \to M'$ be vector bundles and let $f : E' \to E$ be a bundle morphism such that the map $f_{p'} : E'_{p'} \to E_{f(p')}$ where $E'_{p'} = \pi'^{-1}\,(p')$ and $E_{f(p')} = \pi^{-1}\,(f\,(p'))$, is injective and such that $f_{p'}\,(E'_{p'})$ has a complementary closed subspace in $E_{f(p')}$. Then a conformal structure on π canonically induces a conformal structure on π'.*

Definition 2.2. *The vector bundle $\pi : E \to M$ with fibre* $\mathbf{E}$ *admits a reduction of its structural group to $CO\,(\mathbf{E})$, if and only if there exists a bundle atlas $(U_i,\ \tau_i)_{i \in I}$, such that the maps $(\tau_j \circ \tau_i^{-1})_p : \mathbf{E} \to \mathbf{E}$ for each p of $U_i \cap U_j$ belong to the group $CO\,(\mathbf{E})$.*

Theorem 2.3. *Let $\pi : E \to M$ be a vector bundle with fibre* $\mathbf{E}$. *There exists a one-to-one correspondence between the set of reductions of the structural group to the conformal group and the set of conformal structures on π.*

Proof. Every reduction of π to the conformal group $CO\,(\mathbf{E})$ determines the conformal structure of π. Indeed, we define

$$g_{a,p}\,(v,\ w) = (\tau_{i,p}\,v,\ \tau_{i,p}\,w)_a,\ \forall v,\ w \in \mathbf{E} \text{ and } a \in \mathcal{I}.$$

The maps $g_a : p \to g_{a,p}$ define the sections of vector bundle $L_s^2\,(\pi)$ (See [3, Ch. 3] for the definition of this vector bundle) and the set $\{g_a\}$ is a conformal structure on π.

Conversely, let $\{g_j\}_{j \in J}$ be a conformal structure on π and let $(U_i,\ \tau_i)_{i \in I}$ be a bundle atlas for π. We consider an arbitrary map $\varepsilon : I \to J$ and let $g_i^{\varepsilon(i)}$ be the induced metric by g_j with $j = \varepsilon\,(i)$, on $U_i \times \mathbf{E}$ by the isomorphism τ_i. There exists a positive definite symmetric operator $A_{i,p}^{\varepsilon(i)}$ such that

$$g_{i,p}^{\varepsilon(i)}\,(v,\ w) = (A_{i,p}^{\varepsilon(i)}\,v,\ w),\ \forall\,p \in U_i,\ v,\ w \in \mathbf{E}.$$

We denote $B_{i,p} = \sqrt{A_{i,p}^{\varepsilon(i)}}$ and we put $\sigma_i = B_{i,p} \circ \tau_{i,p}$. Then (U_i, σ_i) is the bundle atlas we looked for. It is sufficient to prove that $B_i : U_i \times \mathbf{E} \to U_i \times \mathbf{E}$ which is defined on fibres by $B_{i,p}$ map $g_i^{\varepsilon(i)}$ on the scalar product $(\ ,\ )$ of $\mathbf{E}$. But we have

$$(B_{i,p}\, v, B_{i,p}\, w) = (A_{i,p}^{\varepsilon(i)}\, v, w) = g_i^{\varepsilon(i)}(v, w).$$

since $B_{i,p}$ is symmetric.

Definition 2.3. *Let $\pi : E \to M$ be a vector bundle with a conformal structure $\mathbf{C}$; $\mathbf{C}$ is called a Weyl structure if and only if there exists a map $W : \mathbf{C} \to C^\infty(T^* M)$ which satisfies*

$$W(e^\lambda \cdot g) = W(g) - d\lambda,$$

where $C^\infty(T^* M)$ denotes the set of sections in cotangent bundle of M.

Remark 2.1. A Riemannian metric g and a 1-form η on M determines a Weyl structure, namely $W : \mathbf{C} \to C^\infty(T^* M)$, where $\mathbf{C}$ is the equivalence class of g and $W(e^\lambda g) = \eta - d\lambda$.

Theorem 2.4. *Let $\pi : E \to M$ and $\pi' : E' \to M'$ be vector bundles with conformal structures and $f : E' \to E$ a bundle morphism compatible with this conformal structure (in the sense of Theorem 2.2). Every Weyl structure on π canonically induces a Weyl structure on π'.*

Proof. If (g, η) defines Weyl structure on π, then $(f^* g, f^* \eta)$ defines a Weyl structure on π'.

Theorem 2.5. *Let $\pi : E \to M$ be a vector bundle with a conformal structure, where M admits a partition of unity. Then π admits a Weyl structure.*

Proof. It is sufficient to prove that there is a global 1-form on M. But this follows from [1, lemma 1.3].

3. Connections compatibles with conformal structures. We shall give below some results from the connections theory on Banach vector bundles which we shall use later.

Theorem 3.1. *Let $\pi : E \to M$ be a vector bundle and M admits a partition of unity; then*

i) there exists a connection map K for π,

ii) there exists a canonic bijective map from the set of connection maps on π to the set of covariant derivatives on π given by

$$K \circ T\xi = \nabla\xi, \quad \forall \xi \in \mathscr{X}_E(M). \tag{3.1}$$

where $\mathscr{X}_E(M)$ is the set of the sections in π.

The proof is given in [1, Theorem 2.2].

Remark 3.1. a) $\nabla\xi$ is considered as a section in $L(\tau, \pi) : L(TM, E) \to M$ where $\tau : TM \to M$ is tangent bundle.

b) The implication $K \to \nabla$ is given by (3.1) and without the hypothesis of existence of the partition of unity on M.

Let $c : [0,1] \to M$ be a piecewise differentiable curve on M. We denote by $P_c|_{[t,t_0]}$, where $t, t_0 \in (0,1)$, the parallel displacement from $E_{c(t)}$ to $E_{c(t_0)}$ defined by the connection ∇, K. The map $\tilde{Q}_c : c^* E \to (0,1) \times E_{c(t_0)}$ defined by

$$\tilde{Q}_c(t, v) = (t, P_c|_{[t,t_0]} v) \qquad (t, v) \in c^* E \tag{3.2}$$

is a vector bundles isomorphism. See [1, Theorem 3.5].

Let $\mathscr{X}_E(c)$ be the vector space of section in π along the curve c and let $C^\infty((0,1), E_{c(t_0)})$ be the vector space of maps of class C^∞ from $(0,1)$, to $E_{c(t_0)}$. We consider the map $Q_c : \mathscr{X}_E(c) \to C^\infty((0,1), E_{c(t_0)})$ defined by

$$Y \to Q_c Y = pr_2 \circ \tilde{Q}_c(t, Y(t)) \;\; \forall Y \in \mathscr{X}_E(c),\; t \in (0,1). \tag{3.3}$$

Theorem 3.2. a) *Q_c is a vector space isomorphism;*

$$\text{b)}\ \frac{d}{dt}(Q_c Y) = Q_c(\nabla_c Y)$$

where $\nabla_c Y$ is covariant differentiation of section Y along curve c.

For proof see [1, Theorem 3.6].

In [4], the holonomy group of connection ∇, K with reference point p, denoted by $\Phi(p)$, is defined. For each p of M, the group $\Phi(p)$ can be realized as a subgroup of the structural group of π. The Theorem 2.11 of [4] suggests the following

Definition 3.1. *Let $\pi : E \to M$ be a vector bundle with a conformal structure $\mathbf{C}$. The connection ∇, K is compatible with the conformal structure $\mathbf{C}$ if and only if*

$$\Phi(p) \subseteq CO(\mathbf{E}), \qquad \forall p \in M. \tag{3.4}$$

where $\mathbf{E}$ is the fibre of π.

In the interesting case when $\mathbf{C}$ is in the same time a Weyl structure, we will use the following

Definition 3.2. (See [2]). *Let $\pi : E \to M$ be a vector bundle with a Weyl structure (g, η). The connection ∇, K is compatible with the Weyl structure (g, η) if and only if along every curve $c : [0,1] \to M$ and for at least one g from $\mathbf{C}$,*

$$g_p(Q_c^t Y, Q_c^t Z) = \exp\left[\int_0^t c^* \eta\right] g_{c(t)}(Y_t, Z_t); \tag{3.5}$$

where $Q_c^t = P_c|_{[t,t_0]}$, $p = c(0)$ and $Y_t, Z_t \in E_{c(t)}$.

Remark 3.2. If condition (3.5) is satisfied by one $g \in \mathbf{C}$, it will be satisfied by each $g' = e^\lambda \cdot g$ from $\mathbf{C}$.

Theorem 3.3. *Let $\pi : E \to M$ be a vector bundle with the Weyl structure (g, η) and let ∇, K be a connection on π.*

The following assertions are equivalent :

1) *The connection ∇, K is compatible with the Weyl structure (g, η) :*

$$2) \qquad \frac{d}{dt} g_c (Y, Z) = g_c (\nabla_c Y, Z) + g_c (Y, \nabla_c Z) - c^* \eta \cdot g_c (Y, Z)$$

for $\forall c : [0,1] \to M$ *and* $Y, Z \in \mathcal{X}_E (M)$.

3) $Xg (Y, Z) = g (\nabla_X Y, Z) + g (Y, \nabla_X Z) - \eta (X) g (Y, Z)$ *for* $\forall X \in \mathcal{X}_{TM} (M)$ *and* $Y, Z \in \mathcal{X}_E (M)$:

Proof. 1) $\to$ 2). For each curve c with $c (0) = p$, we shall denote $Y_{c(t)} = Y_t$, $Z_{c(t)} = Z_t$, $Y_p = Y$, $Z_p = Z$ for $Y, Z \in \mathcal{X}_E (M)$. It follows from (3.4) and b) of Theorem 3.2 that

$$\frac{d}{dt} g_c (Y, Z) = \lim_{t \to 0} \frac{1}{t} (g_{c(t)} (Y_t, Z_t) - g_p (Y, Z)) =$$

$$= \lim_{t \to 0} \frac{1}{t} \left(\exp \left[- \int_0^t c^* \eta \right] g_p (Q_c^t Y_t, Q_c^t Z_t) - g_p (Y, Z) \right) =$$

$$= \lim_{t \to 0} \frac{1}{t} \exp \left[- \int_0^t c^* \eta \right] (g_p (Q_c^t Y_t, Q_c^t Z_t) - g_p (Y, Z)) +$$

$$+ g_p (Y, Z) \lim_{t \to 0} \frac{1}{t} \left((\exp \left[- \int_0^t c^* \eta \right] - 1 \right) = g_p (\lim_{t \to 0} \frac{1}{t} (Q_c Y_t - Y), Z) +$$

$$+ g_p (Y, \lim_{t \to 0} (Q_c^t Z_t - Z)) + g_p (Y, Z) \frac{d}{dt} \exp \left[- \int_0^t c^* \eta \right] =$$

$$= g_p (\nabla_c Y, Z) + g_p (Y, \nabla_c Z) - c^* \eta \cdot g_p (Y, Z) \quad \text{i.e. 2)}.$$

2) $\to$ 1). Let Y, Z be parallel sections in π along c, i.e $Q_c^t Y_t = Y_p$ and $Q_c^t Z_t = Z_p$. Assertion 2) of Theorem becomes

$$\frac{d}{dt} g_{c(t)} (Y_t, Z_t) = - c^* \eta \cdot g_{c(t)} (Y_t, Z_t), \tag{3.6}$$

and we get (3.5) by integration.

The proof of 2) $\to$ 3) can be obtained in the same way as in the Riemannian case. See [1, Theorem 3.8]

Definition 3.3. *We shall say that a manifold M modeled by the conformal space $\mathbf{M}$ is endowed with a conformal structure if there is a collection of charts $(U_i; \varphi_i)$, covering M and satisfying*

$$D (\varphi_j \circ \varphi_i^{-1})_{\varphi_i(p)} \in CO (\mathbf{M}) \text{ for all } i, j \text{ and } p \in U_i \cap U_j, \tag{3.7}$$

where D denotes the differentiation operator.

Theorem 3.4. *A manifold M modelled by a conformal space $\mathbf{M}$ admits a conformal structure if and only if the tangent bundle TM admits a conformal structure.*

Proof. Let $(U_i; \varphi_i)$ be the collection of charts which defines the conformal structure on M. The transition maps of TM are $D(\varphi_j \circ \varphi_i^{-1})_{\varphi_i(p)}$ and belong to $CO (\mathbf{M})$ i.e. TM admits a reduction to conformal group, therefore a conformal structure by Theorem 2.3.

Conversely, a conformal structure on TM induces a reduction of this vector bundle to the conformal group $CO (\mathbf{M})$ i.e. the maps $D (\varphi_j \circ \varphi_i^{-1})_{\varphi_i(p)}$ belong to $CO (\mathbf{M})$.

Definition 3.4. *A conformal manifold M is called Weyl manifold if and only if the conformal structure of TM is a Weyl structure.*

Theorem 3.5. *Let M be a Weyl manifold, modelled by the conformal space $\mathbf{M}$. There exists a unique connection ∇, K, such that*

$$\text{i)} \quad Xg (Y, Z) = g (\nabla_X Y, Z) + g (Y, \nabla_X Z) - \eta (X) g (Y, Z)$$
$$\text{for } X, Y, Z \in \mathcal{X}_{TM} (M)$$

$$\text{ii)} \quad T (X, Y) \overset{\text{def}}{=} \nabla_X Y - \nabla_Y X - [X, Y] = 0, \quad \forall X, Y \in \mathcal{X}_{TM} (M)$$

where (g, η) is the Weyl structure of TM.

Proof. Existence. Let (U, φ) be a chart for M at p. We consider the following equation with Fréchet derivatives

$$\begin{aligned} 2g_\varphi (\Gamma_{\varphi(p)} ((u, v), w) &= Dg_\varphi |_{\varphi(p)} (u) (v, w) + \\ + Dg_\varphi |_{\varphi(p)} (v) (u, w) &- Dg_\varphi |_{\varphi(p)} (w) (u, v) + \eta_\varphi (u) g_\varphi (v, w) + \\ + \eta_\varphi (v) g_\varphi (u, w) &- \eta_\varphi (w) g_\varphi (u, v), \qquad \forall u, v, w \in \mathbf{M}, \end{aligned} \tag{3.8}$$

where g_φ and η_φ are local representatives of g and η, respectively. This equation defines a map $\Gamma_{\varphi(p)} \in L_s^2 (\mathbf{M}, \mathbf{M})$, such that $\varphi (p) \to \Gamma_{\varphi(p)}$ is of class C^∞. As the $\Gamma_{\varphi(p)}$ satisfies the usual transformation formula of a local connector, under change of trivialization, it defines a connection on M. The connection such obtained satisfies i) and ii) of Theorem. Indeed, ii) has the following local expression

$$T (X, Y)_{\varphi(p)} = \Gamma_{\varphi(p)} (X_{\varphi(p)}, Y_{\varphi(p)}) - \Gamma_{\varphi(p)} (Y_{\varphi(p)}, X_{\varphi(p)}) = 0.$$

This equality is satisfied because $\Gamma_{\varphi(p)}$ is a bilinear symmetric map. The local expression of condition i) is

$$Dg_\varphi |_{\varphi(p)} (X_{\varphi(p)}) (Y_{\varphi(p)}, Z_{\varphi(p)}) = g_{\varphi(p)} (\Gamma_{\varphi(p)} (X_{\varphi(p)}), Z_{\varphi(p)}) +$$

$$+ g_{\varphi(p)} (Y_{\varphi(p)}, \Gamma_{\varphi(p)} (X_{\varphi(p)}, Z_{\varphi(p)}) - \eta_{\varphi(p)} (X_{\varphi(p)}) g_{\varphi(p)} (Y_{\varphi(p)}, Z_{\varphi(p)}).$$

This equality can be easily verified using (3.8).

Uniqueness. Let $\Gamma'_{\varphi(p)}$ be another local connector which verifies i) and ii). It follows that $\Gamma'_{\varphi(p)}$ must satisfy equation (3.8) i.e. $\Gamma'_{\varphi(p)} = \Gamma_{\varphi(p)}$.

BIBLIOGRAPHY

1. Flaschel, P., Klingenberg W. — *Riemannsche Hilbertmanningfaltigkeiten.* Periodische Geodätische. Lecture Notes in Mathematics, Vol. 282, Springer-Verlag, 1972.
2. Folland, G. B. — *Weyl manifolds* J. Differential geometry 4 (1970) p. 145—153.
3. Lang, S. — *Introduction to differentiable manifolds*: Interscience, New York, 1962.
4. Maxim, L. — *Connections compatibles with Fredholm structures on Banach manifolds.* An. şt. Univ. Iaşi, Matematică, T. XVIII, fasc. 2 (1972), p. 389—399.
5. Miron, R. — *Espaces à structure conforme presque simplectique.* Colloquium Mathematicum, T. XXVI (1972), p. 207—215.

Received 30.XI.1973

STRUCTURI CONFORME PE FIBRATE VECTORIALE BANACH

Rezumat

În această Notă se defineşte structura conformă pe fibrate vectoriale Banach şi în particular, structura Weyl.

Se arată că există o corespondenţă bijectivă între mulţimea structurilor conforme pe un fibrat vectorial Banach şi mulţimea reducerilor grupului structural al acestui fibrat vectorial Banach la grupul conform. Se demonstrează că există şi este unică, o conexiune liniară fără torsiune, compatibilă cu o structură Weyl pe o varietate diferenţiabilă Banach.

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XX, s. I a, fasc. 2, 1974

PERIODIC SOLUTIONS OF FIRST ORDER HYPERBOLIC SYSTEMS CONTAINING A SMALL PARAMETER

BY

DAN PETROVANU

§0. This paper handles the problem of periodic solutions of the hyperbolic systems

$$u_x = \varepsilon F(x, y, u, v, \varepsilon), \quad v_y = \varepsilon G(x, y, u, v, \varepsilon), \tag{0.1}$$

where ε is a (small) parameter, u and v are the unknown vector functions (in general of different dimensions). By this same it is a continuation to author's paper [15]. It was already clear in [15] that there is a close analogy between the investigation of periodic solutions of Tricomi's problem (characteristic problem) for the system $u_x = F(x, y, u, v)$, $v_y = G(x, y, u, v)$ and the investigation of periodic solutions of the characteristic problem for the equation $u_{xy} = f(x, y, u, u_x, u_y)$. (See Cesari's papers [2], [3], [4], [5], see also Hale's paper [7]).

As in [15], we use the method of Cesari and Hale. The theoretic ground of this method should be found in [1] and also in [8], [9]. For further details and for different references, we send to [6], [9], [10]. The papers [6], [10] give an excellent survey on the use of this method in all the fields.

The reader should compare the results of this paper with the ones of [2], [3], [4], [5], [6], [7], [10], [11], [12], [13], [19], [22], [23], [26]. The papers [11], [12], [13], [14], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26] give a partial, but good indication of the results obtained in the field of periodic solutions for hyperbolic systems.

In handling the problem of periodic solutions for system (0.1) we could follow closer the method of Cesari-Hale as described in [2], [3], [4], that is, in our case to proceed as in [4] (as we did in [15], where we tried to keep close to the general lines of [2], [3]). For the sake of variety we choosed to keep closer to the general line of [7]. Furthermore, it was in the intention of the author to follow this line as close as possible, in order to make easy the comparison between the results of the present paper and the ones of [7] (and by this same the comparison with the results of [4] or those of [23] also).