Mulţimea măsurilor exterioare cardinal-secvenţiale se organizează ca o semilatice disjunctivă ceea ce permite introducerea şi studierea noţiunii de măsurabilitate a unui limbaj în raport cu un ideal H din această semilatice, o generalizare a noţiunii de regularitate. Se stabilesc caracterizări ale limbajelor H-măsurabile şi proprietăţi de închidere, regăsindu-se astfel proprietăţi de închidere ale clasei limbajelor regulate. Se arată că dacă o familie de măsuri exterioare cardinal-secvenţiale constituie un ideal complet, familia de algebre unare ataşate automatelor conexe ce induc aceste măsuri este o varietate.

În final, după prezentarea unor extensiuni globale ale măsurilor exterioare cardinal-secvenţiale se studiază unele proprietăţi ale funcţiilor măsurabile dintre spaţii cu măsuri exterioare cardinal-secvenţiale.

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XX, s. II a, fasc. 2, 1974

NUMERICAL SOLUTIONS OF TWO NEURAL MODELS AND THE CORRESPONDING *FORTRAN* PROGRAMMING*)

BY

K. V. LEUNG, D. MANGERON, M. N. OGUZTORELI, R. B. STEIN

In the very ample and important framework of BIONICS, whose main lines of research are to explore the principles of computation in living organisms, to establish the structural and functional organization of such „biological computers", and to utilize this knowledge in the design and construction of cognitive systems, conversant systems, inductive inference machines, etc., the authors investigated the solutions of the neural equations considered in [17]. After studying the stability of the steady state solutions to these equations in general, they discussed finite systems with more details, because their importance in laboratory work and computer simulations.

In what follows the numerical analysis of the solutions is given and FORTRAN programs for the methods of variable step size and variable order of approximations is exposed.

A partial list of relevant references is given at the end of the present work.

I. Introduction. The infinite system of ordinary differential equations

$$\left(\tau\frac{d}{dt}+1\right)\log\frac{x_i}{1-x_i}=\gamma_i+\sum_{j=1}^{\infty}\beta_{ij}\,x_j \tag{I.1}$$

and

$$\left(\tau\frac{d}{dt}+\alpha_i\right)x_i=\frac{1}{1+\exp\left\{-\left[\gamma_i+\sum_{j=1}^{\infty}\beta_{ij}\,x_j\right]\right\}}, \tag{I.2}$$

$i=1, 2, 3, \ldots$, have been proposed by J. D. Cowan and R. B. Stein (cf. [17]) respectively to describe the neural activities in a neural net. In (I.1) and (I.2), $x_i=x_i(t)$ denotes Cowan's „sensitivity" of the i-th cell, $0<x_i<1$,

*) This work was partly supported by the National Research Council of Canada under Grant NRC—A 4345 through the University of Alberta.

τ denotes the mean interspike interval, γ_i is the external input to the i-th cell, β_{ij} represent the strength of influence of the j-th cell to the i-th cell, $\beta_{ii} = 0$, and α_i represent the strength of self inhibition in the i-th cell.

It has been shown in [17] that Eqs (I.1) and (I.2) admit unique solutions for $t > 0$ which satisfy the initial conditions

$$\text{(I.3)} \qquad x_i(0) = x_{i,0} \qquad (0 < x_{i,0} < 1)$$

$i = 1, 2, 3,\ldots$ and these solutions are monotonic in sufficiently small intervals.

II. Numerical solutions of finite (I.1) nets. In this section we deal with the numerical solutions of a finite net (I.1). To this end, we put once more

$$\text{(II.1)} \qquad \begin{aligned} u_i &= \log \frac{x_i}{1 - x_i}, \quad x_{i,r}(u_{i,r}) = \frac{e^{u_{i,r}}}{1 + e^{u_{i,r}}}, \\ u_{i,0} &= \log \frac{x_{i,0}}{1 - x_{i,0}}, \quad u_{i,r} = u_i(rh), \quad (r = 0, 1, 2, \ldots), \end{aligned}$$

where h is a sufficiently small positive number. We rewrite Eq (I.1) and (I.3) in the form

$$\text{(II.2)} \qquad (1 + \tau D)\, u_i = a_i + \sum_{j=1}^{N} b_{ij}\, x_j, \quad u_i(0) = u_{i,0} \ (i = 1, 2, \ldots, N),$$

where

$$\text{(II.3)} \qquad a_i = \gamma_i/\tau, \quad b_{ij} = \beta_{ij}/\tau \ (b_{ii} = 0).$$

We wish to compute $u_{i,r}$ to find $x_{i,r}$. We shall use the single step method by Taylor expansion. By Taylor's formula, we have

$$\text{(II.4)} \qquad u_{i,1} = u_i(h) = u_{i,0} + \frac{h}{1!} u'_{i,0} + \frac{h^2}{2!} u''_{i,0} + \frac{h^3}{3!} u'''_{i,0} + \ldots + \frac{h^s}{s!} u^{(s)}_{i,0} + \ldots,$$

where

$$\text{(II.5)} \qquad u^{(s)}_{i,0} = \left. \frac{d^s u_i}{dt^s} \right|_{t=0}.$$

Note that

$$\text{(II.6)} \qquad u^{(1)}_{i,0} = \frac{1}{\tau} \left[-u_{i,0} + a_i + \sum_{j=1}^{N} b_{ij}\, x_{j,0} \right].$$

We wish to find $u^{(s)}_{i,0}$ in terms of $u^{(s-1)}_{j,0}, \ldots, u^{(1)}_{j,0}, u_{j,0}$, $j = 1, 2, \ldots, N$. For this purpose, consider the polynomials $P_r(\theta)$ defined by the recurrence relations

$$\text{(II.7)} \qquad P_0(\theta) = 0, \ P_r(\theta) = (\theta - \theta^2) \frac{d}{d\theta} P_{r-1}(\theta) = P_1(\theta) \frac{d}{dt} P_{r-1}(\theta).$$

We have

$$\text{(II.8)} \qquad \begin{cases} P_1(\theta) = \theta - \theta^2, \\ P_2(\theta) = \theta - 3\theta^2 + 2\theta^3, \\ P_3(\theta) = \theta - 7\theta^2 + 12\theta^3 - 6\theta^4, \\ P_4(\theta) = \theta - 15\theta^2 + 50\theta^3 - 60\theta^4 + 24\theta^5, \\ P_5(\theta) = \theta - 31\theta^2 + 180\theta^3 - 390\theta^4 + 360\theta^5 - 120\theta^6, \\ \ldots\ldots\ldots \end{cases}$$

It is not difficult to show that the coefficients of the polynomials $P_r(\theta) = \sum_{s=1}^{r+1} a_{r,s}\, \theta^s$ satisfy the recurrence relations

$$\text{(II.9)} \qquad a_{r,s} = s a_{r-1,s} - (s-1)\, a_{r-1,s-1}; \ a_{1,1} = 1, \ a_{1,2} = -1.$$

Obviously we have $P_r(0) = P_r(1) = 0$ for $r = 1, 2, 3, \ldots$

By direct computations we can show that

$$\text{(II.10)} \qquad \frac{dx_j}{du_j} = x_j - x_j^2 \quad (j = 1, 2, \ldots, N)$$

and

$$\text{(II.11)} \qquad \begin{aligned} \frac{dx_j}{dt} &= P_1(x_j) \frac{du_j}{dt}, \\ \frac{d^2 x_j}{dt^2} &= P_1(x_j) \frac{d^2 u_j}{dt^2} + P_2(x_j) \left(\frac{du_j}{dt} \right)^2, \\ \frac{d^3 x_j}{dt^3} &= P_1(x_j) \frac{d^3 u_j}{dt^3} + P_2(x_j) \frac{d^2 u_j}{dt^2} \frac{du_j}{dt} + P_3(x_j) \left(\frac{du_j}{dt} \right)^3, \\ \frac{d^4 x_j}{dt^4} &= P_1(x_j) \frac{d^4 u_j}{dt^4} + 4 P_2(x_j) \frac{d^3 u_j}{dt^3} \frac{du_j}{dt} + 3 P_2(x_j) \left(\frac{d^2 u_j}{dt^2} \right)^2 \\ &\quad + 6 P_3(x_j) \frac{d^2 u_j}{dt^2} \left(\frac{du_j}{dt} \right)^2 + P_4(x_j) \left(\frac{du_j}{dt} \right)^4, \\ \frac{d^5 x_j}{dt^5} &= P_1(x_j) \frac{d^5 u_j}{dt^5} + 5 P_2(x_j) \frac{d^4 u_j}{dt^4} \frac{du_j}{dt} + 10 P_2(x_j) \frac{d^3 u_j}{dt^3} \frac{d^2 u_j}{dt^2} \end{aligned}$$

$$+ 10\, P_3(x_j) \frac{d^3 u_j}{dt^3} \left(\frac{du_j}{dt} \right)^2 + 12\, P_3(x_j) \left(\frac{d^2 u_j}{dt^2} \right)^2 \frac{du_j}{dt}$$

$$+ 10\, P_4(x_j) \frac{d^2 u_j}{dt^2} \left(\frac{du_j}{dt} \right)^3 + P_5(x_j) \left(\frac{du_j}{dt} \right)^5,$$

. . . .

Thus, putting

$$P^{(j)}_{r,0} = P_r(x_{j,0}),\ u^{(s)}_{i,0} = \left. \frac{d^s u_i}{dt^s} \right|_{t=0}, \tag{II.12}$$

we find

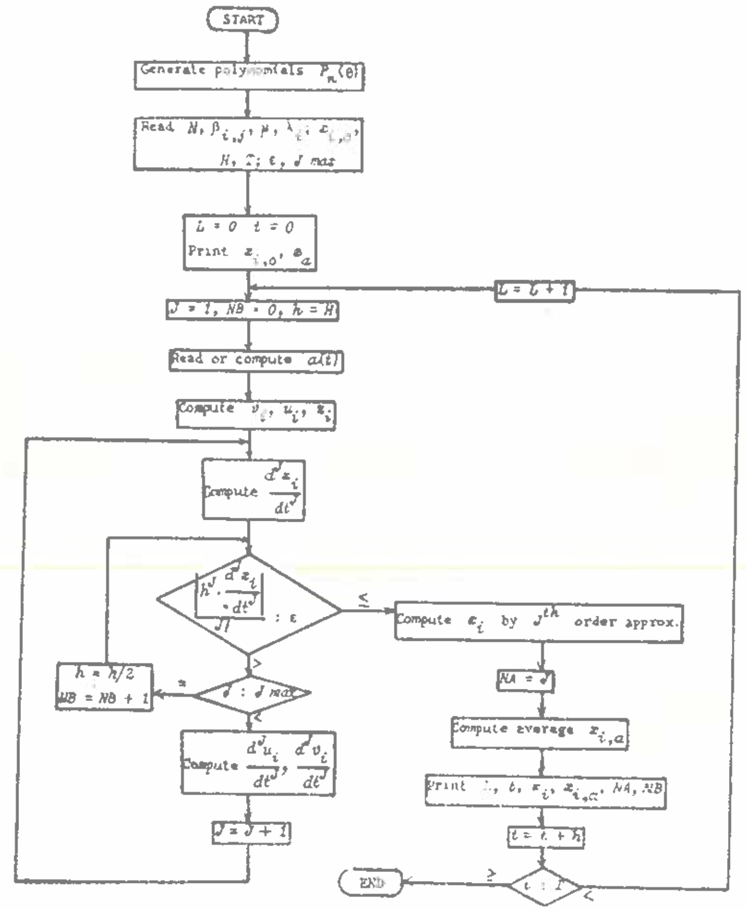
$$\left\{ \begin{aligned} u^{(1)}_{i,0} &= \frac{1}{k} \left\{ -u_{i,0} + a_i + \sum_{j=1}^{N} b_{ij}\, P^{(1)}_{0,0} \right\}, \\ u^{(2)}_{i,0} &= \frac{1}{k} \left\{ -u^{(1)}_{i,0} + \sum_{j=1}^{N} b_{ij} \left[P^{(j)}_{1,0}\, u^{(1)}_{j,0} \right] \right\}, \\ u^{(3)}_{i,0} &= \frac{1}{k} \left\{ -u^{(2)}_{i,0} + \sum_{j=1}^{N} b_{ij} \left[P^{(j)}_{1,0}\, u^{(2)}_{j,0} + P^{(j)}_{2,0} (u^{(1)}_{j,0})^2 \right] \right\}, \\ u^{(4)}_{i,0} &= \frac{1}{k} \left\{ -u^{(3)}_{i,0} + \sum_{j=1}^{N} b_{ij} \left[P^{(4)}_{1,0}\, u^{(3)}_{j,0} + 3\, P^{(j)}_{2,0}\, u^{(2)}_{j,0}\, u^{(1)}_{j,0} + P^{(j)}_{3,0} (u^{(1)}_{j,0})^2 \right] \right\}, \\ u^{(5)}_{i,0} &= \frac{1}{k} \left\{ -u^{(4)}_{i,0} + \sum_{j=1}^{N} b_{ij} \left[P^{(j)}_{1,0}\, u^{(4)}_{j,0} + 4\, P^{(j)}_{2,0}\, u^{(3)}_{j,0}\, u^{(1)}_{j,0} + 3\, P^{(j)}_{2,0} (u^{(2)}_{j,0})^2 + \right. \right. \\ &\qquad \left. \left. + 6\, P^{(j)}_{3,0}\, u^{(2)}_{j,0} (u^{(1)}_{j,0})^2 + P^{(j)}_{4,0} (u^{(1)}_{j,0})^4 \right] \right\}. \end{aligned} \right. \tag{II.13}$$

. . . .

Substituting (II.13) into (II.4) we obtain $u_{i,1} = u_i(h)$ which yields $x_i(h)$. Note that, the error of x is less than the error of u, since $|\Delta x| \cong \cong x(1 - x) |\Delta u| \leqslant \frac{1}{4} |\Delta u|$.

The flowchart of the above single-step method with variable step size and variable order of approximations for the numerical solutions of Eqs (I.1) with N-cells is given below. The program listing for this process is given in Appendix A.

FLOWCHART FOR EQS (I.2)



III. Numerical solutions of finite (I.2) nets. In this section we deal with the numerical solutions of a finite system of equations (I.2). To this end, we first put

$$\lambda_i = \frac{\alpha_i}{\tau},\ \mu = \frac{1}{\tau},\ u_i(v_i) = \frac{1}{1 + e^{-v_i}},\ v_i = v_i(t) = \gamma_i + \sum_{j=1}^{N} \beta_{i,j}\, x_j(t). \tag{III.1}$$

Then, we rewrite Eqs (I.2), (I.3) in the form

$$\frac{dx_i}{dt} + \lambda_i x_i = \mu u_i [v_i(t)],\ x_i(0) = x_{i,0}. \tag{III.2}$$

We wish to find $x_{i,1} = x_i(h)$, where h is a sufficiently small positive number. By Taylor's formula, we have

$$x_{i,1} = x_i(h) = x_{i,0} + \frac{h}{1} x_{i,0}^{(1)} + \frac{h^2}{2!} x_{i,0}^{(2)} + \frac{h^3}{3!} x_{i,0}^{(3)} + \frac{h^4}{4!} x_{i,0}^{(4)} + \frac{h^5}{5!} x_{i,0}^{(5)} + \dots, \tag{III.3}$$

where

$$x_{i,0}^{(5)} = \left.\frac{d^s x_i}{dt^3}\right|_{t=0}. \tag{III.4}$$

We now introduce the following sequence of polynomials $Q_n(\theta) = \sum_{m=0}^{n-1} a_{n,m} \theta^m$,

$$Q_1(\theta) = 1, \quad Q_n(\theta) = (2\theta - 1)\, Q_{n-1}(\theta) + (\theta - 1)\, Q'_{n-1}(\theta). \tag{III.5}$$

The coefficients $a_{m,n}$ satisfy the recurrence relationships

$$\begin{aligned} a_{n,m} &= (m+1)(a_{n-1,\, m-1} - a_{n-1,\, m}), \quad a_{n,\,-1} = 0, \quad a_{1,0} = 1, \\ a_{n,m} &= 0 \quad \text{if} \quad m \geq n. \end{aligned} \tag{III.6}$$

We have

$$\begin{aligned} Q_1(\theta) &= 1, \\ Q_2(\theta) &= -1 + 2\theta, \\ Q_3(\theta) &= 1 - 6\theta + 6\theta^2, \\ Q_4(\theta) &= -1 + 14\theta - 36\theta^2 + 24\theta^3, \\ Q_5(\theta) &= 1 - 30\theta + 150\theta^2 - 240\theta^3 + 120\theta^4 \end{aligned} \tag{III.7}$$

. . . .

We can easily show that

$$\frac{du_i}{dv_i} = u_i z_i Q_1(z_i), \quad z_i = z_i(v_i) = u_i(v_i)\, e^{-v_i} \tag{III.8}$$

and

$$\frac{dz_i}{dv_i} = z_i(z_i - 1). \tag{III.9}$$

Similarly we have

$$\frac{d^2 u_i}{dv_i^2} = u_i z_i Q_2(z_i), \quad \frac{d^3 u_i}{dv_i^3} = u_i z_i Q_3(z_i), \dots, \frac{d^n u_i}{dv_i^n} = u_i z_i Q_n(z_i) \tag{III.10}$$

for any n. Note that

$$\frac{d^r v_i}{dt^r} = \sum_{j=1}^{N} \beta_{ij} \frac{d^r x_j}{dt^r}. \tag{III.11}$$

Hence, putting $u_{i,0}^{(r)} = \left.\frac{d^r u_i}{dv_i^r}\right|_{t=0}$ and $v_{i,0}^{(r)} = \left.\frac{d^r v_i}{dt^r}\right|_{t=0}$, we find

$$\begin{aligned} x_{i,0}^{(1)} &= -\lambda_i x_{i,0} + \mu\, u_{i,0}^{(1)} v_{i,0}^{(1)}, \\ x_{i,0}^{(2)} &= -\lambda_i x_{i,0}^{(1)} + \mu\, u_{i,0}^{(1)} v_{i,0}^{(1)}, \\ x_{i,0}^{(3)} &= -\lambda_i x_{i,0}^{(2)} + \mu\, [u_{i,0}^{(2)} (v_{i,0}^{(1)})^2 + u_{i,0}^{(1)} v_{i,0}^{(2)}], \\ x_{i,0}^{(4)} &= -\lambda_i x_{i,0}^{(3)} + \mu\, [u_{i,0}^{(3)} (v_{i,0}^{(1)})^3 + 3\, u_{i,0}^{(2)} v_{i,0}^{(1)} v_{i,0}^{(2)} + u_{i,0}^{(1)} v_{i,0}^{(3)}], \\ x_{i,0}^{(5)} &= -\lambda_i x_{i,0}^{(4)} + \mu\, [u_{i,0}^{(4)} (v_{i,0}^{(1)})^4 + 6\, u_{i,0}^{(3)} (v_{i,0}^{(1)})^2 v_{i,0}^{(2)} \\ &\quad + 3\, u_{i,0}^{(2)} (v_{i,0}^{(2)})^2 + 4\, u_{i,0}^{(2)} v_{i,0}^{(1)} v_{i,0}^{(3)} + u_{i,0}^{(1)} v_{i,0}^{(4)}]. \end{aligned} \tag{III.12}$$

. . . .

Thus, combining Eqs (III.3), (III.8), (III.10), (III.11) and (III.12), we find the value of $x_i(t)$ at $t = h$.

The flowchart for the above single-step method with variable step size and variable order of approximation is given below, and the program listing is given in Appendix B.

Let us note that the numerical methods presented in this and in the preceding section can be easily extented to neural nets where the constant inputs, γ_i's, are replaced by sufficiently smooth functions of t.

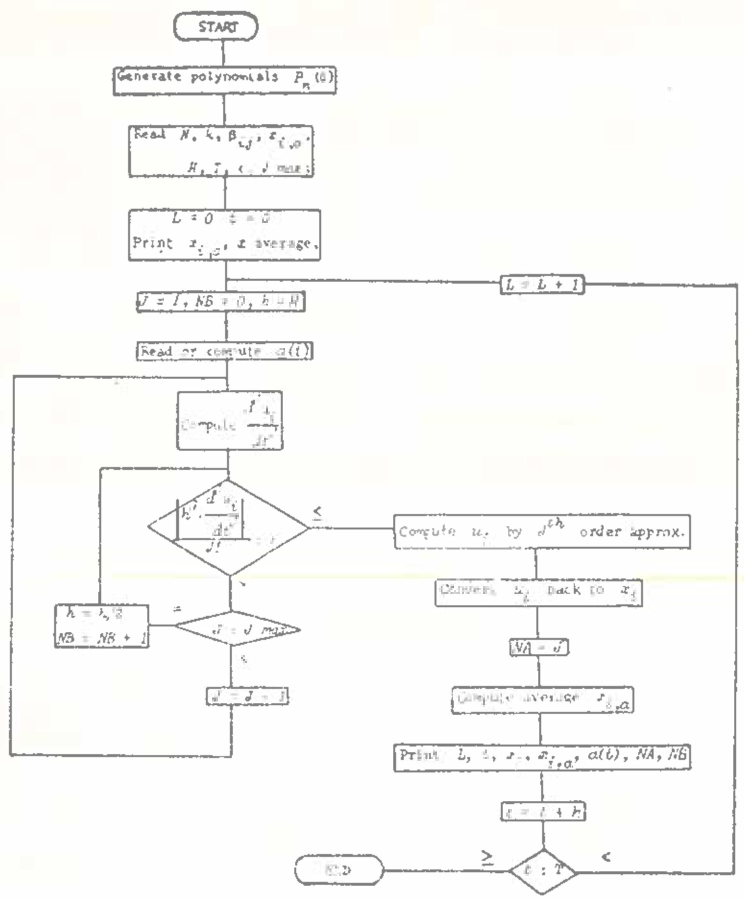
To illustrate this case we refer to Appendix C.

In our investigations we observed that both neural models (I.1) and (I.2) are very sensitive to the variations in the inputs when they are in the neighborhood of their steady-state positions. In both models the $x_i(t)$'s generally approach very fast to their steady-state position from any arbitrarily given initial position whenever they are stable. An apparent pattern adaptation around the steady-state solutions can be observed easily by periodic inputs. Two examples are given in Appendix C and Appendix D.

VII. Description of the programs. In both FORTRAN programs for Eqs (I.1) and (I.2) the following parameters are used:

N	:	number of equations in the systems,
NA	:	order of approximation used in the calculation,
NB	:	number of times that the normal step size is halved,
TC	:	the time t,
H	:	normal step size of the calculation,
T	:	the time that calculation is terminated,
B (I, J)	:	the matrix β_{IJ},
A (I)	:	the input vector,
Eps	:	a small positive number.

FLOWCHART FOR EQS (I.1)



Furthermore, in the program for Eqs (I.2) we have the following two parameters:

$ZL(I)$: the quantities λ_i,
ZM : the quantity u.

In both programs, the single-step method by Taylor's expansion is used. For flexibility, a method of variabile order of approximation and variable step size is employed. To do this, the contributions from various order derivatives of the solutions x_i are checked in each iteration, starting from the second order derivative. If the contribution from the j-th order derivative, say, is less than ε, a very small positive number, then the solutions x_i will

be determined by the j-th order approximation formula. If the contribution is greater than ε, the contribution from the $(j+1)$-st order derivative is checked and so on. If J_{max} is reached and the contribution from the J_{max}-th derivative is still greater than ε, then the normal step size will be halved and the contribution is cheked again, and the same procedure is repeated until the contribution is less than ε. In this case, the x_i are determined by the J_{max}-th order approximation formula.

For both of the systems (I.1) and (I.2) program listings are written in Fortran and are run under FORTRAN IV G, in the University of Alberta IBM 360/67 Computer. Numerical examples are given in Appendices A, B, C and D.

APPENDIX A. — Numerical solutions of equations (I.1) with a constant input vector.

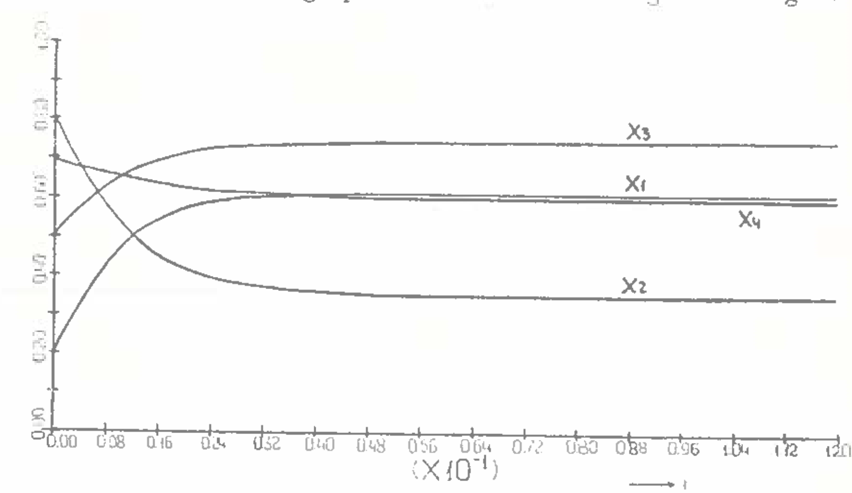
Two examples are considered. In the first example, the sum of the elements in each row or column of the matrix $B(I, J) = (\beta_{ij})$ is less than 1, and the parameters used are:

$$N = 4 \qquad H = 0.4 \times 10^{-3}$$

$$TC = 0.12 \qquad Eps = 0.1 \times 10^{-5}$$

$$A(I) = \begin{pmatrix} 0.4 \\ -0.5 \\ 0.9 \\ 0.6 \end{pmatrix}, \quad x_{i,0} = \begin{pmatrix} 0.2 \\ 0.8 \\ 0.5 \\ 0.7 \end{pmatrix}, \quad B(I,J) = \begin{pmatrix} 0 & 0.4 & -0.2 & 0.1 \\ -0.4 & 0 & 0.3 & -0.15 \\ 0.2 & -0.3 & 0 & 0.26 \\ -0.1 & 0.15 & -0.26 & 0 \end{pmatrix}.$$

The solutions are monotonically increasing or decreasing to their respective steady-state solutions. The graphs of the solutions are given in Fig. I.



In the second example, the sum of the elements in each row or column of B_{ij} is of the order of 100. The parameters used are the same as in the first example except that $B(I, J)$ in this case is:

$$B(I, J) = \begin{pmatrix} 0 & 45 & 30 & -20 \\ -45 & 0 & 25 & -10 \\ -30 & -25 & 0 & 40 \\ 20 & 10 & -40 & 0 \end{pmatrix}.$$

The solutions first oscillate and then approach to steady-state solutions. The graphs of the solutions are given in Fig. II.

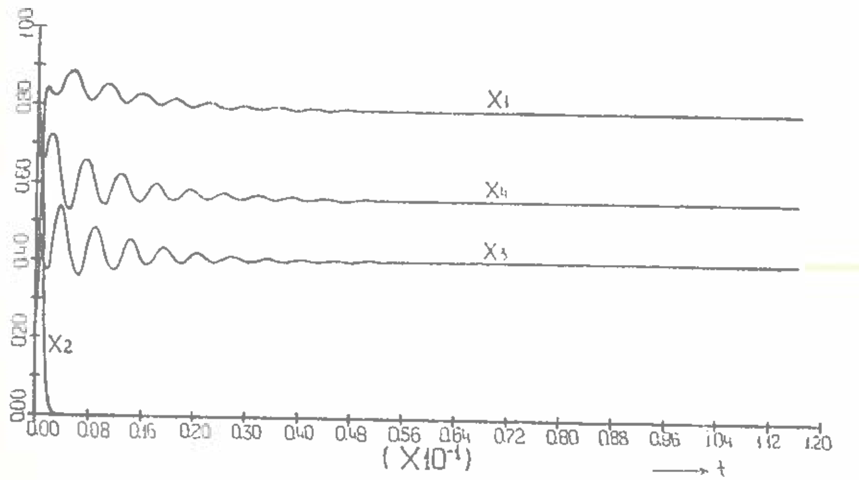


Fig. 2

APPENDIX B. — Numerical solutions of equations (I.2) with a constant input vector.

Two examples are given here, in the first one, the matrix $B(I, J)$ is chosen such that $\sum_{i \text{ or } j} |\beta_{ij}| \leqslant 1$. The parameters used are identical with the ones used in example I of Appendix A, together with

$$ZM = 80$$

$$ZL(I) = \begin{pmatrix} 90 \\ 95 \\ 80 \\ 85 \end{pmatrix}.$$

The solutions are monotonically increasing or decreasing to their steady-state solutions. The graphs are given in Fig. III.

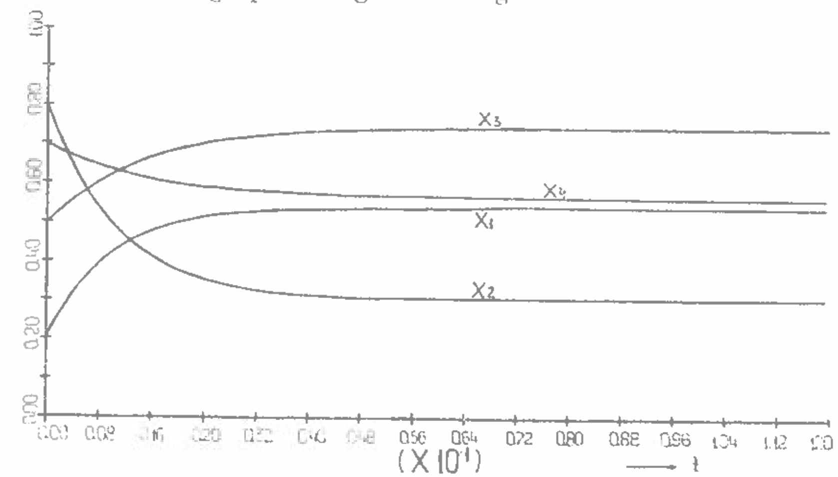


Fig. 3

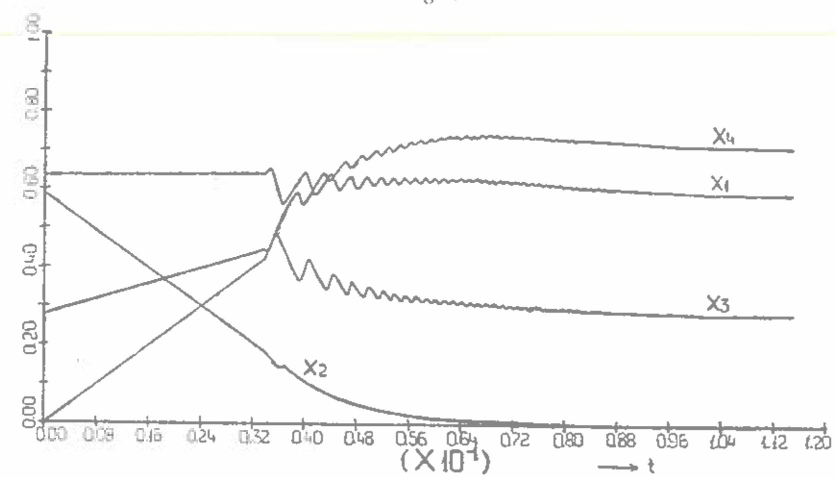


Fig. 4

In the second example, the condition $\sum_{i \text{ or } j} |\beta_{ij}| \approx 1000$ is used:

$$B(I, J) = \begin{pmatrix} 0 & 400 & -250 & 100 \\ -400 & 0 & 320 & 150 \\ 250 & -320 & 0 & -210 \\ -100 & -150 & 210 & 0 \end{pmatrix}$$

All the other parameters are the same as above. The solutions first oscillate and finally come to steady-state solutions. The graphs are shown in Fig. IV.

APPENDIX C. – Numerical solutions of equations (I.1) with an oscillating input vector

Two examples are given here. In the first one, the input vector $A(1)$ is oscillating while $A(2)$ and $A(3)$ are kept constant. All other parameters are the same as in example I of Appendix A, except $N = 3$ in the present case.

$A(1)$ oscillates with a magnitude of 0.8 and a period of 10 steps (iterations).

$$A(2) = -0.5 \qquad\qquad A(3) = 0.9$$

The solution oscillates and show the characteristic of pattern adaptation. The graphs are shown in Fig. V.

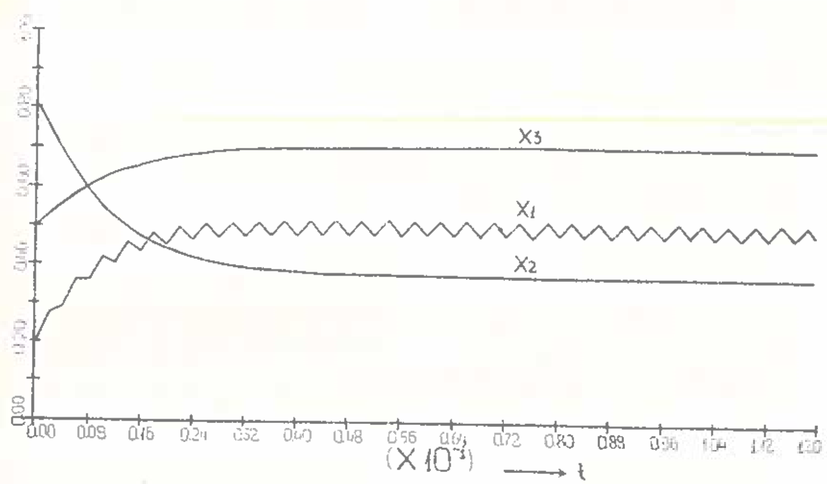


Fig. 5

In the second example, the input vector $A(1)$ is given an abrupt change (impulse) at the 100^{th} step of calculation while at the rest of the time it is kept constant. The magnitude of the impulse is 10. All other parameters are the same as in the first example. Due to the presence of the impulse, the solution x_1 increases abruptly, passes the value of the steady-state solution and then decreases monotonically back to it. The graphs are shown in Fig. VI.

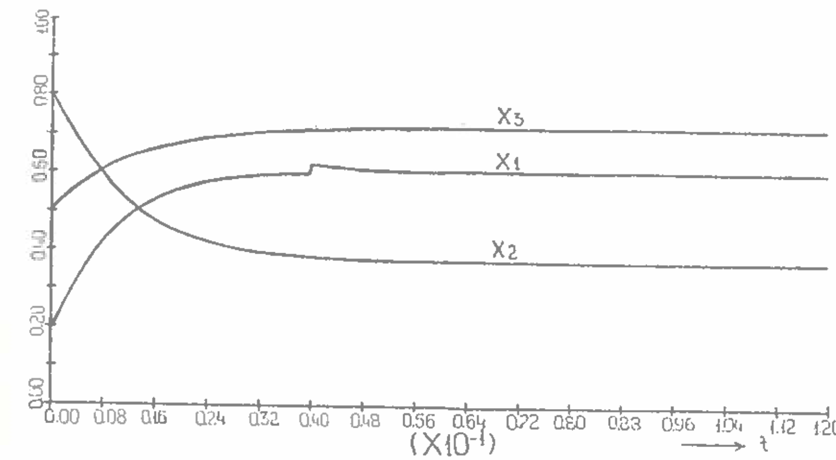


Fig. 6

APPENDIX D. – Numerical solution of equations (I.2) with an oscillating input vector

In the example given here, the input vector $A(1)$ is oscillating while $A(2)$ and $A(3)$ are kept constant. All other parameters are the same as in the first example of Appendix B, except $N = 3$ in this case.

$A(1)$ oscillates with a magnitude of 0.7 and period of 12 steps.

$$A(2) = -0.5 \qquad\qquad A(3) = 0.9$$

The solutions oscillate and show the characteristic of pattern adaptation. The graphs are shown in Fig. VII.

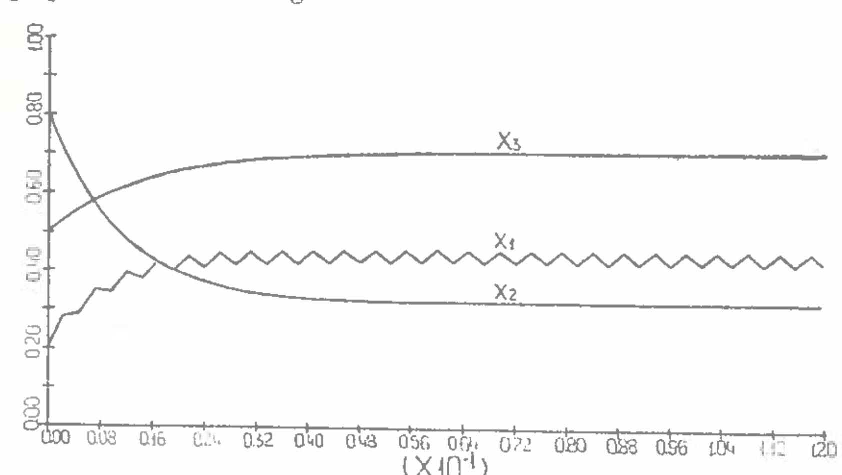


Fig. 7

VIII. Remarks. We would like to note that the results in Section III for the stability of the infinite systems (I.1) and (I.2) are considerably restrictive. We feel that a more comprehensive investigation can be made in the framework of Refs [7] and [8]. Further, for finite (I.1) and (I.2) systems the powerful analytic and asymptotic methods of Refs [12] and [13] can be used to investigate oscillatory solutions. For systems with relatively small number of equations certain specific results of Refs [5], [11], [18], and [23] can be employed to describe more accurately the behavior of the solutions to these equations.

In Sections V–VII, we used freely Refs [3], [9] and [16].

For the physiological interpretations of the results given in this paper we refer to Refs [1], [4], [6], [10], [14], [17], [19]–[22] and [24].

In a forthcoming paper in this series we shall present more results for the neural nets (I.1) and (I.2) and for their extensions.

IX. Acknowledgements. It is the authors' very pleasant duty to express their sincere thanks to Mrs. M. Willard of the University of Alberta, for her help in some of the computer simulations. We would also like to thank Professor V. Lakshmikantham for his valuable comments and to the University of Alberta for use if its computing facilities.

REFERENCES

1. Ashby W. R. — *Properties of complex dynamic systems.* Biological Computer Lab., Univ. Illinois, Urbana, Ill., 72. 2 (1972), 21—24.
2. Caianello, E. R., Ed. — *Neural networks.* Proc. School on Neural Networks, June 1967 in Ravello. Springer-Verlag, New York, 1968.
3. Edwards, R. E. — *Functional analysis. Theory and applications.* Holt-Rinehart-Winston, New York, 1965.
4. Gear, C. W. — *Numerical initial value problems in ordinary differential equations.* Prentice-Hall, Englewood Cliffs, 1971.
5. Harmon, L. D. — *Neuronimes; Action of a reciprocally inhibitory pairs.* Science, 146 (1964), 1323—1325.
6. Hayashi, C. — *Nonlinear Oscillations in Physical Systems.* McGraw-Hill, New York, San Francisco, Toronto and London, 1964.
7. Kling, U., Szekely, G. — *Simulation of rhythmic nervous activities.* Kybernetik, 5 (1968), 89—103.
8. Kokjer K. — *A display technique for neurophysiological data.* Biological Computer Lab. Univ. Illinois, Urbana, Ill., No. 69. 2 (1969), 189—193 and No. 70.2 (1970), 308—316.
9. Ladas, G. E., Lakshmikantham, V. — *Differential equations in abstract spaces.* Academic Press, New York and London, 1972.
10. Lakshmikantham, V., Leela, S. — *Differential and integral inequalities. Vol. I.* Academic Press, New York and London, 1969.
11. Ludovici, M. A. S. — *Programmazione di metodi per il calcolo numerico.* Editrice Veschi, Roma, 1970.
12. Matthews, P. B. C., Stein, R. B. — *The sensitivity of muscle spindle afferents to small sinusoidal changes of length.* J. Physiol., (1969) 723—743.
13. Minorski, N. — *Nonlinear oscillations.* D. Van Nostrand, Princeton, 1962.
14. Митропольский Ю. А. *Метод осреднения в нелинейной механике.* Изд. Наукова Думка, Киев, 1971.
15. Митропольский Ю. А. *Задачи асимптотической теории неустановившихся колебаний.* Изд. Наукова Думка. Киев, 1964.

16. Morishita, I., Yajima, A. — *Analysis and simulation of mutually inhibiting neurons* Kibernetik, 11 (1972). 154—165.
17. Oguztoreli M. N. — *On the neural equations of Cowan and Stein.* Utilitas Mathematica, 2 (1972), 305—317.
18. Ortega, J. M., Rheinboldt, W. C. — *Iterative solutions of nonlinear equations in several variables.* Academic Press, New York and London, 1970.
19. Позин Н. В. *Моделирование нервных структур.* Изд. Наука, Отд. физ.-мат. лит., Москва, 1970.
20. Reissig, R., Sansone, G., Conti, R. — *Nichtlineare Differentialgleichungen höherer Ordnung.* Edizioni Cremonese, Roma, 1969.
21. Reihardt, W. von, Ginitie, G. M. — *Zur Theorie der lateralen Inhibition.* Kybernetik, 1 (1962), 155—165.
22. Saxena, K., Foester, H. von, Wolf, D. — *Two Theorems regarding interspike-interval Histograms.* Biologic Computer Lab., Univ. Illinois Urbana, Ill., Report No. 72.2 (1972), 127—141.
23. Schellart, N. A. M., Sperkreise, H. — *Dynamic characteristics of retinal ganglion cell responses in goldfish.* J. Gen. Physiol., 59 (1972), 1—21.
24. Stein, R. B., French, A. S., Holden, A. V. — *The frequency response coherence and information capacity of two neuronal models.* Biophys. J., 12 (1972), 295—322.
25. Suzuki, R. — *Dynamics of „Neuron Ring".* Kybernetik, 8 (1971), 39—45.
26. Urabe, M. — *Nonlinear autonomous oscillations-analytical theory.* Academic Press, New York and London, 1967.
27. Wilkins, M. — *Mathematical modelling of interactions in neurons and neural nets.* Biological Comp. Lab., Univ. Illinois, Urbana, Ill.,, Report No. 68.2 (1968), 143—145; No. 69.2 (1969), 98—112.
28. Wilkins, M. — *Neural modelling: methodology, techniques and a multilinear model for information processing.* Biological Comp. Lab. Univ. Illinois, Urbana, Ill., Report No. 70.2 (1970), 81—84.
Wilson, H. R., Cowan, J. D. — *Exitatory and inhibitory interactions in localized populations of model neurons.* Biophys. J., 12 (1972), 1—24.
30. Wolf, D. — *Sensory-motor interactions. The „Critter": A Model.* Biological Computer Lab., Univ. Illinois, Urbana, Ill., Report No. 71. 2 (1971), 115—134.

1) Department of Computing Science. University of Alberta, Edmonton Alberta, Canada.
2) Polytechnic Institute of Jassy, Iași, Romania. At present: Department of Computer Science, Sir George Williams University, Monreal, Quebec, Canada.
3) Department of Mathematics, University of Alberta, Edmonton, Alberta, Canada.
5. Department of Physiology, University of Alberta, Edmonton, Alberta, Canada.

SOLUȚII NUMERICE A DOUĂ MODELE NEURONALE ȘI PROGRAMAREA FORTRAN CORESPUNZĂTOARE

Rezumat

În cadrul actual al dezvoltării dinamice multilaterale a BIONICII, autorii, continuînd studiile proprii, consacrate determinării soluțiilor ecuațiilor ce modelează rețelele neuronale, examinării stabilității acestor soluții și confruntării rezultatelor teoretice obținute cu experiențe adecvate de laborator, expun analiza numerică a soluțiilor modelelor matematice a rețelelor neuronale (I.1) și (I.2), în ipoteza rețelelor finite, și prezintă schițe de programare FORTRAN referitoare la metodele de pas și de ordin de aproximație variabil. Un număr de grafice ilustrative, corespunzătoare unor modele concrete este reprodus la finele prezentei lucrări.