Since x_0, y_0 are arbitrary in the neighbourhood, we deduce that every pair in $N(x_0, y_0)$ are commuting elements. Consider the elements

$$x_t = x_0 + t(x - x_0)$$
$$y_t = y_0 + t(y - y_0)$$

which for t small are again commuting, and identifying the coefficients of t we obtain that $xy = yx$.
The theorem is proved.

Theorem 1.2. *Let $\mathcal{A}$ be a Banach algebra such that for each $x, y \in \mathcal{A}$ there exists $j(x, y) > 1$, $n(x, y) > 1$, $m(x, y) > 1$ such that*

$$xy - yx = (x^n y^m - y^m x^n)^j ; \tag{2}$$

then $\mathcal{A}$ is commutative.

Proof. The proof is an easy generalization of the above proof with more computations and thus we omit it.
Let $\mathcal{X}$ be a Banach space and T, S be two bounded operators on $\mathcal{X}$. Suppose that for each x there exists $n(x) > 1$ such that

$$(TS - ST)^{n(x)} x = (TS - ST) x. \tag{3}$$

Our problem is the following: are the operators T, S commuting? As an application of the above technique we obtain that there exists an universal $n(x) = n_0$ such that the above equality holds. In a paper which is in preparation we intend to give another proof for some known commutativity theorems [3].

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Institute of Mathematics
Bucharest, România

ASUPRA UNOR TEOREME DE COMUTATIVITATE

Rezumat

Pornind de la o teoremă a lui Herstein, se demonstrează că dacă $\mathcal{A}$ este o algebră Banach şi dacă pentru orice $x, y \in \mathcal{A}$, există un întreg $n(x,y) > 1$ aşa ca (1) să fie satisfăcută, atunci $\mathcal{A}$ este comutativă. O generalizare a acestui rezultat se obţine înlocuind (1) cu (2), $m(x, y)$ şi $j(x, y)$ fiind alţi doi întregi. O conjectură asupra comutativităţii unei perechi de operatori mărginiţi într-un spaţiu Banach care satisfac (2) este formulată în încheiere.

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ON H-SMOOTH SETS IN LINEAR SPACES

BY

IOAN MUNTEAN and TEODOR PRECUPANU

The concept of H-smooth set was introduced in [3], where it is called a H-convex set; this concept is very useful in the study of Hilbertian semi-norms and in characterizing the topological linear pre-Hilbertizable spaces by a theorem of Kolmogoroff's type.

In this note we continue the study of H-smooth sets in real or complex linear spaces. We shall prove that the Minkowski functional associated to an absorbent symmetric and radially bounded set to be a Hilbertian norm depends only on the H-smoothness of the radial frontier of this set. In the end we shall make some remarks concerning the connection between H-smoothness, on the one hand, and convexity on the other hand.

1. Unless otherwise specified, we shall denote by X a linear space over the set of real numbers R.

A non-void subset M of X is called *H-smooth* if for any $\alpha > 0$, $\beta > 0$, $x \in \alpha M$ and $y \in \beta M$ there exist $\alpha_1 \geqslant 0$ and $\beta_1 \geqslant 0$, such that $\alpha_1^2 + \beta_1^2 \leqslant 2(\alpha^2 + \beta^2)$, $x + y \in \alpha_1 M$ and $x - y \in \beta_1 M$. The functional $p_M : X \to R$, corresponding to the absorbent subset M of X, defined by $p_M(x) = \inf\{\alpha > 0;\ x \in \alpha M\}$, $x \in X$, is called the *Minkowski functional* of M or the *Minkowski gauge* of M. The following properties are easy consequences of the definition:

$$\begin{aligned} &M \subset \{x \in X;\ p_M(x) \leqslant 1\}, \\ &p_M(\alpha x) = \alpha p_M(x), \text{ for } \alpha > 0. \end{aligned} \tag{1}$$

A subset M of X is said to be *radially bounded* if for each non-zero, $x \in X$, there is some $a_0 > 0$ with $ax \notin M$, whenever $a > a_0$. It is well known that p_M is a semi-norm of X if M is a convex symmetric subset (i.e. $-M = M$). Moreover, p_M is a norm if in addition M is a radially bounded subset of X ([4], p. 118, 120). A functional $p : X \to R$, which is a semi-norm (norm) on X, and also satisfies the parallelogram law

$$p^2(x + y) + p^2(x - y) = 2[p^2(x) + p^2(y)] \tag{2}$$

is called a *Hilbertian* semi-norm (norm) on X. We used this denomination because the mapping

$$(3)\qquad \langle x, y\rangle = \frac{1}{4}[p^2(x+y) - p^2(x-y)] \quad x, y \in X,$$

is an inner real semi-product [1], since it has the following properties :

$$(4)\qquad \langle x, x\rangle \geqslant 0 \text{ for each } x \in X,$$

$$(5)\qquad \langle a_1 x_1 + a_2 x_2, y\rangle = a_1 \langle x_1, y\rangle + a_2 \langle x_2, y\rangle \text{ for all } a_1, a_2 \in R,\ x_1, x_2 \in X,$$

$$(6)\qquad \langle x, y\rangle = \langle y, x\rangle \text{ for all } x, y \in X,$$

$$(7)\qquad p(x) = \langle x, x\rangle^{1/2} \text{ for each } x \in X.$$

Conversely, if a mapping, $\langle .,.\rangle : X \times X \to R$, satisfies the conditions (4), (5) and (6), then the functional $p : X \to R$, defined by (7), is a Hilbertian semi-norm on X.

2. Using a method of S. Kurepa [2], for the study of quadratic functionals, we shall obtain a theorem which completes an early result ([3], prop. 1), proved by the second author of this note.

Theorem 1. *Let M be an absorbent and H-smooth set of X. Then the Minkowski functional p_M is a Hilbertian semi-norm on X.*

Proof. First we prove the parallelogram law (2). Let $x, y, \in X$ and $\varepsilon > 0$. According to the definition of the Minkowski functional p_M, there exist $\alpha > 0$ and $\beta > 0$ with $x \in \alpha M$, $y \in \beta M$, $\alpha < p(x) + \varepsilon$ and $\beta < p(y) + \varepsilon$. Since M is H-smooth, there exist $\alpha_1 \geqslant 0$ and $\beta_1 \geqslant 0$ such that $\alpha_1^2 + \beta_1^2 \leqslant 2(\alpha^2 + \beta^2)$, $x + y \in \alpha_1 M$ and $x - y \in \beta_1 M$. From this and (1) we have $p(x+y) \leqslant \alpha_1$ and $p(x-y) \leqslant \beta_1$, which implies that $p^2(x+y) + p^2(x-y) \leqslant 2[p^2(x) + p^2(y)]$. Replacing $x+y$ and $x-y$ by x and y we obtain the contrary inequality. Hence (2) is proved. Putting $x = 0$ in the relation (2), we get $p(-y) = p(y)$, and therefore the mapping $\langle .,.\rangle$ defined by (3) has the properties (4) and (6). To prove (5) we note that the function $x \to \langle x, y\rangle$ is additive for every fixed $y \in X$. Indeed from (2) we have for any $x_1, x_2 \in X$ the relations :

$$p^2(x_1 + x_2 + y) + p^2(x_1 - y) = 1/2\,[p^2(2x_1 + x_2) + p^2(x_2 + 2y)],$$

$$p^2(x_1 + x_2 - y) + p^2(x_1 + y) = 1/2\,[p^2(2x_1 + x_2) + p^2(x_2 - 2y)].$$

Substracting these last relations it follows that

$$\langle x_1 + x_2, y\rangle - \langle x_1, y\rangle = 1/2 \langle x_2, 2y\rangle.$$

If we take $x_1 = 0$, we find $\langle x_2, y\rangle = 1/2 \langle x_2, 2y\rangle$, and hence $\langle x_1 + x_2, y\rangle = \langle x_1, y\rangle + \langle x_2, y\rangle$. In consequence, the function, $f : R \to R$,

defined by $f(t) = \langle tx, y\rangle$, $t \in R$, is additive for any $x, y \in X$. Moreover, f is bounded on the interval $[-1, 1]$ because from (2) we have

$$f(t) = \frac{1}{4}[p^2(tx + y) - p^2(tx - y)] = \frac{1}{2}[p^2(tx) + p^2(y) - p^2(tx - y)]$$

$$\leqslant \frac{1}{2}[p^2(x) + p^2(y)].$$

But then the Cauchy functional equation $f(t + s) = f(t) + f(s)$ has only solutions of the form $f(t) = ct$, where $c \in R$. Since $c = f(1) = \langle x, y\rangle$, we get $\langle tx, y\rangle = f(t) = tf(1) = t\langle x, y\rangle$ so that (5) is satisfied. Hence the functional $p_M = p$ given by (7) is a Hilbertian semi-norm on X and the theorem is completely proved.

Corollary 1. *On a real linear topological space X there exist continuous inner products if and only if the zero element of X possesses radially bounded H-smooth neighbourhoods.*

Indeed, if $\langle .,.\rangle$ is a continuous inner product on X, then $\{x : \langle x, x\rangle \leqslant 1\}$ is a convexe symmetric radially bounded neighbourhood of the zero element of X. We can verify easily (according to [3]) the H-smoothness of this neighbourhood. Conversely, if M is a H-smooth radially bounded neighbourhood of the zero element of X, then the Minkowski functional p_M is, according to theorem 1, a Hilbertian norm on X (see lemma 2). Since M is a neighbourhood of zero, from (1) we see immediately the continuity of p_M in $x = 0$ and, therefore on all the space X.

From corollary 1, we can find again the following theorem of Kolmogoroff's type for the pre-Hilbertizable linear topological spaces ([3], theorem A).

Corollary 2. *The topology of a real linear topological space X is generated by an inner product on X if and only if X is a Hausdorff space and there is a H-smooth bounded neighbourhood of the zero point in X.*

Indeed, if the topology of X is generated by an inner product $\langle .,.\rangle$, then $p(x) = \langle x, x\rangle^{1/2}$, $x \in X$, is a norm on X: therefore X is separated and $\{x : p(x) \leqslant 1\}$ is a H-smooth neighbourhood of zero. It is obvious that this neighbourhood is bounded. The converse statement follows from the corollary 1 and from the fact that the bounded sets are also radially bounded.

3. Let M be a subset of X. The totality of points $x \in X \setminus \{0\}$ with the property that for each $x_1 \in]0, x[$, we have $[x_1, x] \cap M \neq 0$ and $M \cap]x, \to[= 0$, where $]0, x[= \{tx : 0 < t < 1\}$, $[x_1, x_2] = \{tx_1 + (1 - t)x_2 : 0 \leqslant t \leqslant 1\}$ and $]x, \to[= \{tx : t > 1\}$, is said to be the radial *frontier* of M and it is denoted by fr$M$.

Lemma 1. *If M is absorbent, then* fr $M = \{x : p_M(x) = 1\}$.

Proof. For every $x \in$ fr M we have $M \cap]x, \to[= 0$, so that $tx \notin M$ for all $t > 1$, i.e. $p_M(x) \leqslant 1$. If we suppose that $p_M(x) < 1$ then there is a $\alpha > 0$ such that $x \in \alpha M$ and $\alpha < 1$, hence $1/\alpha\, x \in M \cap]x, \to[= 0$, and this is not possible. Conversely, if $x \in X$ so that $p_M(x) = 1$ we have $x \neq 0$

and $M \cap\,]x, \to [\, = 0$. Indeed, if $M \cap\,]x, \to [\, \neq 0$, there is a $t > 1$ such that $tx \in M$. But then $p_M(tx) \leqslant 1$ or $p_M(x) \leqslant 1/t < 1$, contrary to the hypothesis. Thus, if $x \in M$, we have in addition $[x_1, x] \cap M \neq 0$ for every $x_1 \in\,]0, x[$, hence $x \in \mathrm{fr}\, M$. Finally, we suppose that $x \notin M$. Since M is an absorbent set there exists $s > 0$ with $sx \in M$. But it follows from $s = sp_M(x) = p_M(sx) \leqslant 1$ that $s \leqslant 1$, hence $s_0 = \sup \{s > 0 ; sx \in M\} \leqslant 1$. Since $p_M(x) = \inf \{1/s ; x \in 1/s\, M\} = 1/s_0$ we have $s_0 = 1$. Now let $x_1 = tx$, with $0 < t < 1$. We can therefore find s such that $sx \in [x_1, x] \cap M$, with $t < s < 1$, i.e. $x \in \mathrm{fr}\, M$.

Lemma 2. *If M is an absorbent subset of X, the following conditions are equivalent:*

(i) *M is radially bounded.*

(ii) *$p_M(x) \neq 0$, for every $x \in X \setminus \{0\}$.*

(iii) *$\{0\} \cup \mathrm{fr}\, M$ is absorbent.*

Proof. (i) $\to$ (ii). Suppose that (i) is satisfied, but (ii) is not. Let be $a_0 > 0$ with $ax \notin M$ for $a > a_0$, and $p_M(x) = 0$. Then there is a $\alpha > 0$, with $\alpha < 1/a_0$ and $x \in \alpha M$. Thus $(1/\alpha)\, x \in M$ and $1/\alpha > a_0$, contradicting $(1/\alpha)\, x \notin M$.
(ii) $\to$ (i). Let be $x \in X \setminus \{0\}$, hence $p_M(x) > 0$. If $ax \in M$ we have $p_M(x) \leqslant 1/a$ and we, therefore, obtain $\alpha x \notin M$ for all $\alpha > 1/(p_M(x))$ i.e. M is radially bounded.
(ii) $\to$ (iii). If $x \in X \setminus \{0\}$, then by preceding lemma it follows that $1/(p_M(x))\, x \in \mathrm{fr}\, M$, hence $x \in p_M(x)\, M$ since $p_M(x) \neq 0$.
(iii) $\to$ (i). If $x \in X \setminus \{0\}$, then there is a $\alpha > 0$, with $x \in \alpha\, \mathrm{fr}\, M$. For $a > 1/\alpha > 0$ we find $ax \in\,]1/\alpha)\, x, \to [$, from which it follows that $ax \notin M$ since $(1/\alpha)\, x \in \mathrm{fr}\, M$.

Lemma 3. *If M is an absorbent and radially bounded subset of X, then $p_{\{0\} \cup \mathrm{fr} M} = p_M$.*

Proof. By lemma 1 and 2, for all $x \in X \setminus \{0\}$, we obtain

$$p_{\{0\} \cup \mathrm{fr} M}(x) = \inf \{\alpha > 0 : x \in \alpha(\{0\} \cup \mathrm{fr}\, M)\} =$$

$$\inf \{\alpha > 0 ;\ x \in \alpha\, \mathrm{fr}\, M\} = \inf \{\alpha > 0 ; p_M(x) = \alpha\} = p_M(x).$$

Theorem 2. *Let M be an absorbent and radially bounded set of X. If p_M is a Hilbertian norm on X then $\mathrm{fr}\, M$ is H-smooth. If in addition M is symmetric, the converse is also true.*

Proof. If p_M is a Hilbertian norm then p_M coincides with the Minkowski functional p_Y associated with an absorbent radially bounded set $Y = \{x \in X ; p_M(x) \leqslant 1\}$ (according to [4], p. 120). Let be $\langle ..., \rangle$ an inner product on X, with $p_M(x) = \langle x, x \rangle^{1/2}$, $x \in X$. We can easily see that Y is H-smooth. From lemma 1 we find:

$$\mathrm{fr} M = \{x \in X : p_M(x) = 1\} = \{x \in X : p_Y(x) = 1\} \subset Y.$$

If $\alpha > 0$, $\beta > 0$, $x \in \alpha\, \mathrm{fr}\, M$ and $y \in \beta\, \mathrm{fr}\, M$, then $x \in \alpha Y$ and $y \in \beta Y$ and since Y is H-smooth it results that there are $\alpha_1 \geqslant 0$ and $\beta_1 \geqslant 0$ with $\alpha_1^2 + \beta_1^2 \leqslant 2(\alpha^2 + \beta^2)$, $x + y \in \alpha_1 Y$ and $x - y \in \beta_1 Y$; therefore $p_M(x + y) \leqslant \alpha_1$ and $p_M(x - y) \leqslant \beta_1$. According to theorem 1, $p_M = p_Y$ is a Hilbertian semi-norm and from (2) we find $\alpha_1^2 + \beta_1^2 \geqslant p_M^2(x + y) + p_M^2(x - y) = 2[p_M^2(x) + p_M^2(y)] = 2(\alpha^2 + \beta^2)$, which involves $p_M(x + y) = \alpha_1$ and $p_M(x - y) = \beta_1$; hence $x + y \in \alpha_1\, \mathrm{fr}\, M$ and $x - y \in \beta_1\, \mathrm{fr}\, M$. Therefore $\mathrm{fr}\, M$ is H-smooth.

Conversely, we suppose that $\mathrm{fr}\, M$ is H-smooth and that M is symmetric. Following lemma 2, the set $\{0\} \cup \mathrm{fr}\, M$ is absorbent. Furthermore, this set is H-smooth. Indeed, if $\alpha > 0$, $\beta > 0$, $x \in \alpha(\{0\} \cup \mathrm{fr}\, M)$ and $y \in \beta \cdot (\{0\} \cup \mathrm{fr}\, M)$, and if we admit that $x = 0$ and $y \neq 0$, then $p_M(x) = \beta$ and, choosing $\alpha_1 = p_M(y)$ and $\beta_1 = p_M(-y) = p_M(y)$, we find $\alpha_1^2 + \beta_1^2 < 2(\alpha^2 + \beta^2)$, $0 + y \in \alpha_1\, \mathrm{fr}\, M$ and $0 - y \in \beta_1\, \mathrm{fr}\, M$. By the theorem 1 we find that $p_{\{0\} \cup \mathrm{fr} M}$ is a Hilbertian semi-norm and by lemmas 1 and 3 we have $p = p_{\{0\} \cup \mathrm{fr} M}$ is a Hilbertian norm on X.

4. We shall make some remarks concerning the connection between H-smoothness on the one hand and the convexity and strictly convexity on the other hand.

The set $\{(x, y) \in R^2 ;$ with $|x| + |y| < 1\}$ is absorbent, convex, symmetric and radially bounded in the euclidean space R^2 without being H-smooth.

The set $\{(x, y) \in R^2 ;$ with $|x| < 1$ or $|x| = 1$ and y rational$\}$ is absorbent, symmetric and H-smooth in the space R^2, but it is not convex because the segment defined by the points (1,1) and (1,0) from M does not belong to M.

However, applying the theorem 3 from below, the situation mentioned in the preceding example cannot take place if the set is radially bounded.

An absorbent subset M of X is said to be *strictly convex*, if for all $x, y \in M$ with $x \neq y$ and each $t \in (0, 1)$ we have $p_M(tx + (1 - t)\, y) < 1$.

Theorem 3. *Each absorbent, H-smooth and radially bounded subset M of X is strictly convex.*

Proof. From theorem 1a nd lemma 2 we see that p_M is a Hilbertian norm on X. If now $x, y \in M$ with $x \neq y$ and $0 < t < 1$, then $p_M(x) \leqslant 1$, $p_M(x) \leqslant 1$ and $p_M(tx + (1 - t)\, y) \leqslant tp_M(x) + (1 - t)\, p_M(y) \leqslant 1$. But from a well-known property of the Hilbertian norms, in the last inequality we must have only a strict inequality.

Remarks. a) From theorem 3 we see that each absorbent, circled, H-smooth and radially bounded subset M of X is convex.

b) From the following example one can see that there exist absorbent, symmetric, radially bounded and strictly convex sets which are not H-smooth. Let be $1 \leqslant p < 2$. We consider the plane R^2 endowed with the norm $\|\cdot\|_p$ given by $\|(x, y\|_p = (|x|^p + |y|^p)^{1/p}$ where $(x, y) \in R^2$. The set $M_p = \{(x, y) \in R^2 : \|(x, y)\|_p \leqslant 1\}$ is absorbent, convex, symmetric and radially bounded, but it is not H-smooth since for $\alpha = \beta = 1$, the relations

$(1.0) + (0.1) \in \alpha_1 M_p$ and $(1.0) - (0.1) \in \beta_1 M_p$ would imply $\alpha_1 \geqslant 2^{1/p}$ and $\beta_1 \geqslant 2^{1/p}$. We obtain the contradiction $4 < 2.2^{1/p} \leqslant \alpha_1^2 + \beta_1^2 \leqslant 2.2$. However for $1 < p < 2$, the set M_p is strictly convex.

e) The preceding results are also true in the case when X is a linear space on the field C of complex numbers. In this case we shall ask that the equality (5) to be verified for all $a_1, a_2 \in C$ and the equality (6) to be replaced by $(x, y) = \overline{(y, x)}$, where the complex inner product $(...)$ is connected with the real inner product $<...>$ by the equality $(x, y) = <x, y> - i <ix, y>$. Furthermore we suppose that the set M verifies the equality $\alpha M = M$ for all $\alpha \in C$ with $|\alpha| = 1$.

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ASUPRA MULȚIMILOR *H*-NETEDE DINTR-UN SPAȚIU LINIAR

Rezumat

Se dau cîteva noi proprietăți ale mulțimilor *H*-netede introduse în [3] sub denumirea de mulțimi *H*-convexe. Printre altele, se arată că pentru mulțimile absorbante, simetrice și radial mărginite, proprietatea funcționalei lui Minkowski de a fi normă hilbertiană depinde numai de *H*-netezimea frontierei radiale (teorema 2). De asemenea, se stabilesc corelații între noțiunile de *H*-netezime, convexitate și strictă convexitate.

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TRANSITIVE QUASI-UNIFORMITIES AND ZERO DIMENSIONAL BITOPOLOGICAL SPACES

BY

T. BÎRSAN

1. Introduction. The concepts of transitive quasi-uniformity and of covering quasi-uniformity was introduced by P. Fletcher [3], which initiated a systematic study of this topics.

A quasi-uniformity $\mathcal{U}$ on a set X is a *transitive quasi-uniformity* if $\mathcal{U}$ has a base (or a subbase) $\mathcal{B}$ satisfying the condition that for each $B \in \mathcal{B}$, $B \circ B = B$. In the special case of the uniformities, this concept coincides with the concept of *non-archimedean uniformity* which was considered previously by A. F. Monna [7].

Let (X, τ) be a topological space. A collection (cover) $\mathcal{C}$ of open sets is a *Q*-collection (*Q*-cover) if for each $x \in X$ $\cap\{C \in \mathcal{C}; x \in C\} \in \tau$. Let $x \in A \in \tau$. An open cover $\mathcal{C}$ of X is a cover about (x, A) if there exists $C \in \mathcal{C}$ such that $x \in C \subset A$. Every collection $\mathcal{A}$ of *Q*-covers of X such that, for each $A \in \tau$ and each $x \in A$, $\mathcal{A}$ contains a cover about (x, A) leads in a natural way [3, Theorem 1] to a compatible quasi-uniformity on (X, τ) which is called a *covering quasi-uniformity* for (X, τ). If $\mathcal{A}$ is the collection of all point (locally) finite open covers of X, then the corresponding covering quasi-uniformity is called the *point (locally) finite covering quasi-uniformity* for (X, τ). In [3] the equivalence of concepts of a transitive quasi-uniformity and of a covering quasi-uniformity is proved : a compatible quasi-uniformity $\mathcal{U}$ for (X, τ) is a covering quasi-uniformity if and only if $\mathcal{U}$ is a transitive quasi-uniformity.

In section 2 and 3 we give two new characterizations of transitive quasi-uniformities. In section 4 it is proved that a bitopological space (X, τ_1, τ_2) is pairwise zero dimensional if and only if it is transitive quasi-uniformizable. This latter result is an improvement of the result of I. L. Reilly [10, Theorem 4]. Some of the results of this paper generalize the corresponding results for non-archimedean uniformities [7].

2. Transitive quasi-uniformities and non-archimedean quasi-pseudometrics. A quasi-pseudo-metric p on a set X is a *non-archimedean quasi-pseudometric* if p satisfies the inequality