$$\nabla_{\dot{\gamma}}(\dot{\gamma}) = \lambda\dot{\gamma}, \tag{52}$$

c'est à dire elles sont les courbes considérées d'habitude comme géodésiques de la connexion linéaire Γ.

Nous terminons ce travail avec la remarque que la méthode utilisée ici peut être appliquée à l'étude des autres structures et connexions subordonnées respectivement à la structure et aux connexions projectives sur une variété différentiable.

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ON THE COVERING SPACES OF VECTOR AND GEOMETRIC BUNDLES

BY

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1. In this paper there are given some results on the morphisms $P = (p, p') : (M', \pi, M) \to (N', \pi', N)$ where p and p' are covering maps. It is proved that $\xi = (M', \pi, M)$ and $\xi' = (N', \pi', N)$ are vector bundles and $p'_x : \pi^{-1}(x) \to \pi'^{-1}(p(x))$, $x \in M$, are linear isomorphisms, then some extra structures (distributions, orientations, connexions, etc.) of ξ' can be lifted at ξ. Some examples are given.

2. Let $\xi = (M', \pi, M, F)$ and $\xi' = (N', \pi', N, F')$ be two vector bundles. Let $p : M \to N$ and $p' : M' \to N'$ be two maps such that the couple $P = (p, p')$ be a bundle map. We say that P is a *bundle covering morphism* if p and p' are covering maps. In this case ξ' is called *covering vector bundle of* ξ'.

Examples. 1. Let $p : M \to N$ and $\varphi : F \to F'$ be two covering maps. Then,

$$P = (p, p \times \varphi) : (M \times F, \mathrm{pr}_1, M, F) \to (N \times F', \mathrm{pr}_1, N, F')$$

is a bundle covering morphism.

2. Let be the diagram

$$\begin{tikzcd} & M \arrow[d, "p"] \\ N' \arrow[r, "\pi'"] & N \end{tikzcd}$$

with $\xi' = (N', \pi', N)$ a vector bundle and p a covering map. Then, the pull-back of ξ' over p is a covering vector bundle of ξ'.

3. Let be the commutative diagram

$$\begin{tikzcd} M' \arrow[r, "\pi"] \arrow[d, "p'"'] & M \arrow[d, "p'"] \\ N' \arrow[r, "\pi'"] & N \end{tikzcd}$$

with p and p' two covering maps having the same covering index. Then π is a vector bundle and, therefore $P = (p, p')$ is a bundle covering morphism.

Particularly, let $\pi' : N' \to N$ be a vector bundle with N', N connected path connected spaces. Let be $x'_0 \in N'$ and H' a subgroup of the group $\pi(N', x'_0)$. Then, $H = \pi_{\#} H'$ is a subgroup of the group $\pi(N, x_0)$ where $x_0 = \pi(x'_0)$. Let $\mathcal{U}$ be an open covering of N such that $\pi(\mathcal{U}, x_0) \subset H$. Then, $\mathcal{U}' = \{\pi^{-1}(U) / U \in \mathcal{U}\}$ is an open covering of N' and $\pi(\mathcal{U}', x'_0) \subset H'$. From the classifying theorem of the covering spaces, there exists $p : (\tilde{N}, \tilde{x}_0) \to (N, x_0)$ and $p' : (\tilde{N}', \tilde{x}'_0) \to (N', x'_0)$, covering maps, such that $p_{\#}\, \pi(\tilde{N}, \tilde{x}_0) = H$ and $p'_{\#}\, \pi(\tilde{N}', \tilde{x}'_0) = H'$. The pair $P = (p, p')$ is a bundle covering morphism.

Proposition 1. *If $P = (p, p') : \xi \to \xi'$ is a bundle map such that p be a covering map and $p'_x = p / M'_x : M'_x = \pi^{-1}(x) \to N'_{p(x)} = \pi'^{-1}(p(x))$ be a linear isomorphism, for every point $x \in M$, then, $p' : M' \to N'$ is also a covering map, that is P is a bundle covering morphism.*

Proof. Let $\pi'^{*}N' = \{(x', x) \in N' \times M / p(x) = \pi'(x')\}$. Then, $\omega' : \pi'^{*} N' \to N'$ with $\omega'(x', x) = x'$, for $(x', x) \in \pi'^{*} N'$ is a covering map and we have the commutative diagram

$$\begin{tikzcd} \pi'^{*} N' \arrow[r, "\pi_1"] \arrow[d, "\omega'"'] & M \arrow[d, "p"] \\ N' \arrow[r, "\pi'"'] & N \end{tikzcd}$$

where $\pi_1(x', x) = x$. It is obvious that $(\pi'^{*} N', \pi_1, M, F')$ is a vector bundle because this is the pull-back of ξ' over p. Now, we consider the diagram

$$\begin{tikzcd} \pi'^{*} N' \arrow[r, "\pi_1"] \arrow[d, "h"'] & M \\ M' \arrow[ur, "\pi'"'] & \end{tikzcd}$$

where the map h is defined by $h(x') = (\pi(x'), p'(x'))$, for $x' \in M'$. The map h is a M-morphism of vector bundles. Moreover, we have $h_x = h/\pi^{-1}(x) = \alpha \circ p'_x$ where $\alpha : \pi'^{-1}(p(x)) \to \pi_1^{-1}(x)$ is defined by $\alpha(y') = (x, y')$, for $y' \in \pi'^{-1}(p(x))$. By assumption, h_x is a linear isomorphism, for every $x \in M$. It results that h is a M-isomorphism of vector bundles, that is h is a diffeomorphism. From the equality $\omega' h = p'$ and since ω' is a covering map, it results that p' is a covering map.

Remarks. 1. The covering map $p' : M' \to N'$ has the same covering index as the covering map p.

2. If $P = (p, p') : \xi \to \xi'$ satisfies the conditions of the proposition 1 we say that P a *strong bundle covering morphism* and ξ a *strong covering vector bundle of* ξ'.

Examples. 1. Let $p : M \to N$ be a smooth covering map with M and N connected manifolds. The derivative $T(p) = dp : TM \to TN$ is a covering map, that is $P = (p, T(p))$ is a strong bundle covering morphism.

2. Let $p : M \to N$ be a smooth covering map with M and N connected manifolds. Then, the map

$$\overset{m}{\oplus} T(p) : \underbrace{T(M) \oplus \ldots\ldots \oplus TM}_{m \text{ times}} \to \underbrace{TN \oplus \ldots\ldots \oplus TN}_{m \text{ times}}$$

with $\oplus$ Whitney sum, is a covering map, that is $(p, \overset{m}{\oplus} T(p))$ is a strong bundle covering morphism.

3. Let $P = (p, p') : \xi \to \xi'$ be a strong bundle covering morphism. We consider the vector bundles $L_2(\pi) : L_2(M', R) \to M'$ and $L_2(\pi') : L_2(N', R) \to N'$ and the map $L_2(p') : L_2(M', R) \to L_2(N', R)$ defined by $L_2(p')(g)(n'_x, n''_x) = g(p'^{-1}_x(n'_x), p'^{-1}_x(n''_x))$, for $g \in L_2(M'_x, R)$ and $n'_x, n''_x \in N'_{p(x)}$. Then, the couple $(p, L_2(p'))$ is a strong bundle covering morphism.

4. Let $P = (p, p') : \xi \to \xi'$ be a strong bundle covering morphism. Let $\Lambda^p \xi'$ and $\Lambda^p \xi'$ be the p-th exterior power of ξ and respectivelly ξ', having the total spaces $\Lambda^p M'$ and respectivelly $\Lambda^p M$. The map $\Lambda^p(p') : \Lambda^p(M') \to \Lambda^p(M)$ is obviously defined. In these conditions the couple $(p, \Lambda^p(p'))$ is a strong bundle covering morphism.

Proposition 2. *If $P = (p, p')$ is a bundle covering map and p is a regular covering map, then p' is a regular covering map. If M' is a connected manifold, then, the reciprocal statement is also true.*

Proof. We have $p_{\#} \pi'_{\#} = \pi_{\#} p'_{\#} : \pi(M', x') \to \pi(N, p\,\pi(x'))$ for every point $x' \in M'$. If $x'_1, x'_2 \in M'$ such that $p'(x'_1) = p'(x'_2)$, it follows that $p_{\#}(M, \pi(x'_1)) = p_{\#} \pi(M, \pi(x'_2))$. Because $\pi_{\#}$ and $\pi'_{\#}$ are isomorphisms, we have $\pi((M, \pi(x'_1)) = \pi_{\#} \pi(M', x'_1)$. Therefore, we obtain $\pi'_{\#} p'_{\#} \pi(M', x'_1) = \pi'_{\#} p'_{\#} \pi(M', x'_2)$ which implies that $p'_{\#} \pi(M', x'_1) = p'_{\#} \pi(M', x'_2)$, that is p' is a regular covering map.

Now, we suppose that M' is a connected manifold. It follows that M' is a path connected space. If p' is regular, then $p'_{\#} \pi(M', x')$ is a normal subgroup of the group $\pi(N', p'(x'))$ for every $x' \in M'$. If $x \in M$, let be $x' \in \pi^{-1}(x)$. We have that $\pi'_{\#} p'_{\#} \pi(M', x')$ is a normal subgroup of the group $\pi(N, p(x))$, that is $p_{\#} \pi(M, x)$ is isomorph with the subgroup $\pi'_{\#} p'_{\#} \pi(M', x')$ which proves that $p_{\#} \pi(M, x)$ is a normal subgroup of the group $\pi(N, p(x))$.

Analogously it is proved:

Proposition 3. *If $P = (p, p')$ is a bundle covering morphism, M' is a connected manifold, ξ has a cross-section and p is an universal covering map, then p' is an universal covering map.*

Proposition 4. *Let $P = (p, p') : \xi \to \xi'$ be a bundle covering morphism. If M' is a connected manifold and M, N are paracompact manifolds, then, for every cross-section $s' : N \to N'$ of the vector bundle ξ' and every point $x'_0 \in M'$ such that $p'(x'_0) = s'p\,\pi(x'_0)$ there exists, uniquely defined, a cross-section $s : M \to M'$ of the vector bundle ξ such that $p's = s'p$ and $s\pi(x'_0) = x'_0$.*

Proof. Let be the diagram

$$\begin{tikzcd} M' \arrow[r, "p'"] & N' \\ & M \arrow[u, "s'p"'] \end{tikzcd}$$

We have $\pi'_{\#} s'_{\#} p_{\#} \pi(M, \pi(x'_0)) = p_{\#} \pi(M, \pi(x'_0))$. Because M is a paracompact manifold the vector bundle (M', π, M) is a Hurewicz fibre space and, since the fibres of this are contractible spaces, it results $\pi_{\#} \pi(M', x'_0) = \pi(M, \pi(x'_0))$. Therefore $\pi'_{\#} s'_{\#} p_{\#} \pi(M, \pi(x'_0)) = p_{\#} \pi_{\#} \pi(M', x'_0) = \pi'_{\#} p'_{\#} \pi(M', x'_0)$. But $\pi'_{\#}$ is also an isomorphism and, therefore, $s'_{\#} p_{\#} \pi(M, \pi(x'_0)) = p'_{\#} \pi(M', x'_0)$. Because p' is a covering map and since M' is a connected (and locally path connected) space, it follows that there exists, uniquelly, the map $s : M \to M'$ such that $s(\pi(x'_0)) = x'_0$ and $p's = s'p$. It results $p\pi s = \pi' p' s = \pi' s' p = p$ and $\pi s(\pi(x'_0)) = \pi(x'_0)$ Because p is a covering map it follows $\pi \circ s = 1_{M'}$, that is s is a cross-section of the vector bundle ξ.

Remark. If $P = (p, p')$ is a strong bundle covering morphism, then, there exist points $x'_0 \in M'$ such that $p'(x'_0) = s'p\,\pi(x'_0)$. In fact, let be $x_0 \in M$. Then, $s'p(x_0) \in \pi'^{-1}(p(x_0)) = N'_{p(x_0)}$ and, because p'_{x_0} is an isomorphism, there exists $x'_0 \in M'_{x_0}$, uniquelly, such that $p'_{x_0}(x'_0) = s'p(x_0)$. We have $s'p\,\pi(x'_0) = s'p\,\pi p'^{-1}_{x_0}(s'p(x_0)) = s'\pi' p\,p'^{-1}_{x_0}(s'p(x_0)) = s'\pi'(s'p(x_0)) = s'p(x_0) = p'(x'_0)$.

Corollary 1. *Let $p : M \to N$ be a smooth covering map with M and N two connected manifolds of dimension n. Then, every distribution α on N, of dimension $p \leqslant n$, has a lift on M : there exists a p-dimensional distribution β on M such that $p_{*.x}\beta_x = \alpha_{p(x)}$, $x \in M$.*

Particularly, we find that, if N is a parallelizable manifold then M, is also parallelizable.

If α is an involutive distribution, then β is also an involutive distribution. Particularly, if N is a foliated manifold then M is a foliated manifold.

If α is homotopically distribution then β is also homotopically. Particularly, if TN/α has a reduction to a finite group, then, TM/β has a reduction to a finite group, [1].

Proposition 5. *If ξ is a strong covering vector bundle of the orientable vector bundle ξ', then ξ is an orientable vector bundle and every orientation of ξ' can be lifted to an orientation of ξ.*

(This assumption is the generalisation of the case of covering spaces of the orientable manifolds).

Proof. Let ξ' be a vector bundle of rank r, with dual bundle ξ'^{*}. If ξ is a strong covering vector bundle of ξ', then ξ has also the rank r and ξ^{*} is a strong covering vector bundle of ξ'^{*}. From the example 4, we have that $\Lambda^r \xi^{*}$ is a strong covering vector bundle of $\Lambda^r\xi'^*$. Then, from the Proposition 4, every cross-section Δ' of the vector bundle $\Lambda^r\xi'^*$ can be lifted at a cross-section Δ of the vector bundle $\Lambda^r\xi'^*$. Because the bundle map $P^* = (p, p'^*) : \xi^* \to \xi'^*$ is a bundle map inducing linear isomorphisms on the fibres, if $\Delta'(y) \neq 0$, for $y \in N$, then $\Delta(x) \neq 0$, for every $x \in M$.

Proposition 6. *Every strong bundle covering morphism $P = (p, p') : \xi \to \xi'$ is an isometry.*

Proof. From the example 3 and the Proposition 4, every Riemannian metric of ξ' can be lifted to a Riemannian metrics of ξ.

Proposition 7. *If ξ is a strong covering vector bundle of ξ', then, for every connection of the vector bundle ξ' there exists a cannonical connection of the vector bundle ξ.*

Prof. We note by $J^1\xi'$ the bundle of jets of first order of the vector bundle ξ'. The total space of this is $J^1N' = \{s'_{*,y}/s' \in \Gamma(N'; N),\ y \in N\}$. We consider the pull-back of $J^1\xi$, over p. We obtain the commutative diagram:

$$\begin{tikzcd}
p^*J^1N' \arrow[r, "\pi_1"] \arrow[d, "p'_1"'] & M \arrow[d, dashed, "p"] \\
J^1N' \arrow[r, "J^1\pi"] & N
\end{tikzcd}$$

where $p^*J^1N' = \{(x, s'_{*,p(x)})/s' \in \Gamma(N', N)\}$, $\pi_1(x, s'_{*,p(x)}) = x$, $p'_1(x, s'_{*,p(x)}) = s'_{*,p(x)}$ and $J^1\pi'(s'_{*,y}) = y$. We prove that the pull-back (p^*J^1N', π_1, M) is isomorphical with ξ. Let s' be a cross-section of ξ'. From the proposition 4, there exists s, a cross-section of ξ such that $p's = s'p$ and $s\pi(x'_0) = x'_0$, where $x'_0 = p'^{-1}_{p(x_0)}(s'p(x_0))$. We define $h : p^*J^1N' \to J^1M'$ by $h(x, s'_{*,p(x)}) = s_{*,x}$. We have $(J^1\pi)(h(x, s'_{*,p(x)})) = (J^1\pi)(s_{*,x}) = x = \pi_1(x, s'_{*,p(x)})$, and h_x is an isomorphism for every $x \in M$. From the Propositon 1 and from the commutative diagram

$$\begin{tikzcd}
p^*J^1N' \arrow[r, dashed, "p'_1"] \arrow[d, dashed, "h"'] & J^1N' \\
J^1M' \arrow[ur, "J^1p'"'] &
\end{tikzcd}$$

it results that the map J^1p' defined by $(J^1p')(s_{*,x})(Y_{p(x)}) = p'_{*,s(x)}s_{*,x}(p^{-1}_{*,x}(Y_{p(x)}))$, for $Y_{p(x)} \in T_{p(x)}(N)$, $x \in M$, is a covering map. Let be the maps $\varphi : J^1M' \to M'$, $\psi : J^1N' \to N'$ with $\varphi(_{*,x}) = s(x)$ and, respectivelly $\psi(s'_{*,y}) = s'(y)$. We have $p' \circ \varphi = \psi \circ J^1p'$. In fact, $\psi(J^1p')(s_{*,x}) = s'p(x) = p's(x) = p'\varphi(s_{*,x})$. Then, we have the commutative diagram:

$$\begin{tikzcd}
O \arrow[r] & M' \otimes T(M)^* \arrow[r] \arrow[d, "p' \otimes p^*"] & J^1M' \arrow[r, "\varphi"] \arrow[d, "J^1p'"] & M' \arrow[r] \arrow[d, "p"] & O \\
O \arrow[r] & N' \otimes T(N)^* \arrow[r] & J^1N' \arrow[r, "\psi"] & N' \arrow[r] & O
\end{tikzcd}$$

If the inferior sequence splits, then the superior sequence also splits.

3. Let $\xi = (M', \pi, M, F)$ and $\xi' = (N', \pi', N, F')$ be two vector bundles such that M and N are pointed spaces, with base points $*$ and respectivelly $*'$. We suppose that N is a weakly locally contractible space, [3]. We remember, [2], that a good family of paths in N is a family of paths $P = \{f : I = [0, 1] \to N\}$ with following conditions:

1) If $f_1, f_2 \in P$ and $f_1(1) = f_2(0)$, then the path

$$(f_1 * f_2)(t) = \begin{cases} f_1(t) & \text{if } 0 \leqslant 2t \leqslant 1 \\ f_2(t) & \text{if } 1 \leqslant 2t \leqslant 2 \end{cases}$$

also is in P.

2) If $f \in P$ and $\Phi : I \to I$ is a piecewise linear continuous map, then $f \circ \Phi \in P$.

3) For each point $x \in N$ there exists an open neighbourhood of x, U, and a continuous map $h : U \times I \to N$ such that, if $x \in U$, then $h(x, 0) = *$ and $h(x, 1) = x$. The path $h^x(t) = h(x,t)$ is in P.

Lémma 1. *If $p : M \to N$ is a covering map such that M is a locally path connected space, N is a connected, locally path connected and weakly locally contractible space, then M also a weakly locally contractible space and every good family of paths in N has a lift at a good family of paths in M.*

Proof. Let be $x \in M$. Then $p(x) \in N$ has a neighbourhood $U_{p(x)}$ which we suppose is connected, such that there exists $F' : U_{p(x)} \times I \to N$ with $F'(y', 0) = y'$, $F'(y, 1) = *'$. Moreover, we can suppose that $U_{p(x)}$ is distinct by p. Let be $\tilde{U}_x \subset p^{-1}(U_{p(x)})$ such that $x \in \tilde{U}_x$ and $p : \tilde{U}_x \to U_{p(x)}$ is a homeomorphism. We consider the diagram

$$\begin{tikzcd}
M \arrow[r, "p"] & N \\
\tilde{U}_x \arrow[u, dashed, "i"] & \tilde{U}_x \times I \arrow[u, "F'(p \times 1)"']
\end{tikzcd}$$

We have $F'(p \times 1)(x, 0) = F'(p(x), 0) = p(x)$. There exists $G : \tilde{U}_x \times I \to M$, uniquely defined, such that $G(y, 0) = y$ and $pG = F'(p \times 1)$. It results $pG(y, 1) = *'$. Because $\tilde{U}_x$ is connected and since $p^{-1}(*')$ is a discrete space, it follows $G(y, 1) = \text{constant} = y_0 \in p^{-1}(*')$. Because $p(y_0) = p(*) = *'$ there exists ω a path closed in $*'$ such that $p_{\#}\pi(M, y_0) = h_{[\omega]}p_{\#}\pi(M, *)$. It follows that $p_{\#}\pi(M, y_0)$ is conjugated to $p_{\#}\pi(M, *)$. Therefore, there exists a homeomorphism $f : M \to M$ such that $f(y_0) = *$. We define $F : \tilde{U}_x \times I \to M$ by $F(y, t) = f^{-1}(G(f(y), t))$. Then $F(y, 0) = f^{-1}(G(f(y), 0) = f^{-1}(f(y)) = y$ and $F(y, 1) = f^{-1}(y_0) = *$. Therefore M is weakly locally contractible.

Now, let P' be a good family of paths in N and let be $f \in P'$ and $x_f \in p^{-1}(f(0))$. Then, there exists, uniquely, $g : I \to M$ such that $g(0) = x_f$ and $pg = f$. We note $P = \{g/pg = f, f \in P, g(0) = x_f\}$ for a choice of the points x_f such that if $f(0) = *'$, then $x_f = *$. If $g_1, g_2 \in P$, it follows $g_1 * g_2 \in P$. If $g \in P$ and $\Phi : I \to I$ is a piecewise linear continuous map, then $p(g \circ \Phi) = pg \circ \Phi$, and, therefore $g \circ \Phi \in P$. Let be $x \in M$ and let U' be an open neighbourhood of $p(x)$, $h' : U' \times I \to N$ a map such that $h'(x', 0) = *'$, $h(x', 1) = x'$, for $x' \in U'$ and such that U' is connected and distinct by p. If $x \in \tilde{U}_x$ with $p/\tilde{U}_x : \tilde{U}_x \to U'$ a homeomorphism, it follows that there exists $F : \tilde{U}_x \times I \to M$ with $F(y, 1) = y$ and $pF(y, 0) = *'$. It follows $F(y, 0) = y_0$, $pF = f'(p \times 1)$. We obtain $f : M \to M$, a diffeomorphism, such that $f(y_0) = *$. Let be $h(y, t) = f^{-1}(F(f(y), t))$. We have $h(y, 0) = *$, $h(y, 1) = y$ and $ph^x = h'^{pf(x)}$ with $h^x(0) = h(x, 0) = *$. Therefore $h^x \in P$ and P is a good family of pathsin N.

We remember, after [2], that if $\pi : N' \to N$ is a continuous map and P' is a good family of paths in N, it is noted $P'(\pi') = \{(y', f') \in N' \times P'/\pi'(y') = f'(0)\}$. A P'-connection of the map π' is a continuous map $C' : P'(\pi') \to (N')^I$ such that:

1) $C'(y', f')(0) = y'$, $\pi'C'(y', f')(t) = f'(t)$, for $(y', f') \in P'(\pi')$ and $t \in I$.

2) If Φ is a continuous piecewise linear map, $(y', f') \in P'(\pi')$ and $y'_1 = C'(y', f')(\Phi(0))$, then $C'(y', f') \circ \Phi = C'(y'_1, f' \circ \Phi)$.

Proposition 8. *Let be the commutative diagram:*

$$\begin{tikzcd}
M' \arrow[r, "\pi"] \arrow[d, "p'"'] & M \arrow[d, "p"] \\
N' \arrow[r, "\pi'"] & N
\end{tikzcd}$$

with p and p' covering maps.

If P' is a good family of paths in N and P is a lift of P' at M (Lemma 1), then, every P'-connection C' of π' can be lifted at a P-conection C of π, that is $P = (p', p)$ is a morphism of geometric bundles, [2]. If $\xi' = (N', \pi', N, P', C')$ is a geometric vector bundle, [2], and $p'_* : M'_* \to N'_*$, is a bijection, then $\xi = (M', \pi, M, P, C)$ is also a geometric vector bundle.

Proof. Let be $P(\pi) = \{(y, f) \in M' \times P / \pi(y) = f(0),\ pf = f' \in P'\}$. If $(y, f) \in P(\pi)$, then $(p'(y), pf) \in P'(\pi')$ and, therefore, $C'(p,(y), pf) \in (N')^I$. Let $C'_1 : P'(\pi') \times I \to N'$ be the map $C'_1((y', f'), t) = C'((y', f'))(t)$ and let be the diagram

$$\begin{tikzcd} M' \arrow[r, "p'"] & N' \\ P(\pi) \arrow[u, "j"] & P'(\pi') \times I \arrow[u, "C'_1"'] \\ & P(\pi) \times I \arrow[u, "h"'] \end{tikzcd}$$

with $j(y, f) = y$, $h((y, f), t) = ((p'(y), pf), t)$. There exists a continous map $C_1 : P(\pi) \times I \to \to M'$ such that $C_1((y, f), 0) = y$ and $p'C_1((y,f), t) = C'_1 h((y, f), t) = C'_1((p'(y), pf), t) = = C'((p'(y), pf)(t)$. We define $C : P(\pi) \to (M')^I$ by $C((y,f))(t) = C_1((y,f), t)$. Then, C is a continuous map and $C((y,f))(0) = C_1((y, f), 0) = y, p'C(y,f) = C'(p'(y), pf)$. We have $p\pi C(y, f)(t) = \pi' p' C(y, f)(t) = \pi' C'(p'(y), pf)(t) = pf(t)$. Moreover, $\pi C(y, f)(0) = \pi(y) = f(0)$. Because p is a covering map it results $\pi(C(y, f)) = f$, for $(y, f) \in P(\pi)$.

Let be $(y, f) \in P(\pi)$ and $y_1 = C(y, f)(\Phi(0))$, for $\Phi : I \to I$ a continuous piecewise linear map. We have $p'C(y_1, f \circ \Phi)(t) = C'(p'(y_1), p(f \circ \Phi))$, $p'C(y, f)(\Phi(t)) = C'(p'(p'(y), pf)(\Phi(t))$, that is, $p'C(y, f) \circ \Phi = C'(p'(y), pf) \circ \Phi$. Let be $y'_1 = p'C(y,f)(\Phi(0)) = C'(p'(y), pf)(\Phi(0))$; $C'(p'(y), pf) \circ \Phi = C'(y'_1, p(f \circ \Phi))$. We have $p'y_1 = p'C(y, f)(\Phi(0)) = y'_1$. It follows $C'(p'(y), pf) \circ \Phi = C'(p'(y_1), p(f \circ \Phi))$, that is $p'C(y_1, f \circ \Phi)(t) = p'(C(y, f) \circ \Phi))(t)$. Because $C(y_1, f \circ \Phi)(0) = y_1 = (C(y, f) \circ \Phi)(0)$ and p' is a covering map, we have $C(y_1, f \circ \Phi) = = C(y, f) \circ \Phi$. Therefore C is a P-connection of π.

If $N'_* = \pi'^{-1}(*')$ has a structure of vector space, then $M'_* = \pi^{-1}(*)$ is also vector space. Moreover, we have the commutative diagram:

$$\begin{tikzcd} M'_* \times P_{**} \arrow[r, "\varphi"] \arrow[d, "p'_* \times p"'] & M'_* \arrow[d, "p'"] \\ N'_{*'} \times P'_{*'*'} \arrow[r, "\varphi'"] & N'_{*'} \end{tikzcd}$$

with $P_{**} = \{f \in P / f(0) = f(1) = *\}$, $P'_{*'*'} = \{f' \in P' / f'(0) = f'(1) = *'\}$, $\varphi(y, f) = C(y, f)(1)$ and $\varphi'(y', f') = C'(y', f')(1)$. In fact, $p'\varphi(y,f) = p' C(y,f)(1) = C'(p'(y), pf)(1) = \varphi'(p'(y), pf) = \varphi'(p'_* \times p)(y, f)$. It follows that if $\varphi''_f : N'_{*'} \to N'_{*'}$, $y' \to \varphi'(y', f')$, is a linear automorphism, for every $f' \in P'_{*'*'}$, then $\varphi_f : M'_* \to M'_*$ is also a linear automorphism, for every $f \in \in P_{**}$.

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COHOMOLOGY OF LIE ALGEBROIDS

BY

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Let V be a manifold of C^∞ class, $A(V)$ be the algebra of the real differential mappings on V, (M, π, V) be a vector bundle with the base space V and $\Gamma(\pi)$ be the $A(V)$-module of the differential cross sections of π.

If the module $\Gamma(\pi)$ has a structure of Lie pseudoalgebra then (M, π, V) is called a *Lie algebroid* [5]. A structure of *Lie pseudoalgebra* on $\Gamma(\pi)$ is given [5] by:

1) a mapping $p^{\Gamma(\pi)} \in \mathrm{Hom}_{A(V)}(\Gamma(\pi), \mathcal{D}(A(V)))$ where $\mathcal{D}(A(V))$ is the $A(V)$-module of the derivations of $A(V)$;

2) a bracket operation $\lambda^{\Gamma(\pi)} : \Gamma(\pi) \times \Gamma(\pi) \to \Gamma(\pi)$ such that

$$p^{\Gamma(\pi)} \lambda^{\Gamma(\pi)}(X, Y) = [p^{\Gamma(\pi)} X, p^{\Gamma(\pi)} Y] \text{ and} \tag{1}$$

$$\lambda^{\Gamma(\pi)}(X, uY) - u\lambda^{\Gamma(\pi)}(X, Y) = (p^{\Gamma(\pi)} X.\, u)\, Y, \tag{2}$$

for all $u \in A(V)$ and $X, Y \in \Gamma(\pi)$.

$$J^{\Gamma(\pi)}(X, Y, Z) = \sum_{Cyrc} \lambda^{\Gamma(\pi)}(\lambda^{\Gamma(\pi)}(X, Y), Z) = 0 \quad \forall X, Y, Z \in \Gamma(\pi). \tag{3}$$

If a vector bundle (M, π, V) has the property that $\pi^{-1}(x)$ is a Lie algebra for all $x \in V$ then obviously it is a Lie algebroid with $p^{\Gamma(\pi)} = 0$; particularly, the trivial vector bundle $V \times g$ where g is a Lie algebra, is also a Lie algebroid.

Let (M, π, V) be a Lie algebroid and $Q_p(\Gamma(\pi), A(V))$ be the set of all alternating p-linear mappings of $\Gamma(\pi)$ into $A(V)$. Obviously, $Q_p(\Gamma(\pi), A(V))$ is a $A(V)$-module for each p and $Q_0(\Gamma(\pi), A(V)) = A(V)$. We define on the set

$$Q(\Gamma(\pi), A(V)) = \sum_{p \in z} Q_p(\Gamma(\pi), A(V)) \tag{4}$$

a structure of a differential graduate $\mathcal{D}(A(V))$-algebra ([2], [7]) in the following manner

$$\begin{aligned}(\theta^p_X \omega)(X_1, \dots X_p) &= p^{\Gamma(\pi)}X(\omega(X_1, \dots, X_p)) + \\ &+ \sum_{i=1}^{p} (-1)^i \omega(\lambda^{\Gamma(\pi)}(X, X_i), X_1, \dots, \check{X}_i, \dots, X_p)\end{aligned} \tag{5}$$

$$(i^p_X \omega)(X_1, \dots, X_{p-1}) = \omega(X, X_1, \dots, X_{p-1}); \tag{6}$$

$$\begin{aligned}d^p\omega(X_1, \dots, X_{p+1}) &= \sum_{i=1}^{p+1} (-1)^{i+1} p^{\Gamma(\pi)}X_i\, \omega(X_1, \dots \check{X}_i, \dots X_{p+1}) + \\ &+ \sum_{i<j} (-1)^{i+j} \omega(\lambda^{\Gamma(\pi)}(X_i, X_j), \dots \check{X}_i, \dots \check{X}_j, \dots X_{p+1}).\end{aligned} \tag{7}$$