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ON SOME BOUNDARY VALUE PROBLEMS OF NONLOCAL ELASTICITY

BY

S. CHIRIȚĂ

1. Introduction. The incorporation of nonlocal interatomic forces into continuum mechanics has only a recent origin. However, the literature on this aspect of the field theory is extensive [1], [6], [7], [8], [10]. A review of various models of elastic bodies is given in [9], [13]. The nonlocal theories of continua take into account the distant effects of atomic, molecular and granular interactions. The papers [1], [3] prove that the nonlocal theory has many advantages for the study of molecular and atomic-scale effects.

The main purpose of the present paper is to establish the existence, uniqueness and general properties of regularity of solution for some boundary value problems in linear theory of nonlocal elastostatics. In this context, the existence theory of Fichera [4], [5] is extended to the linear integro-differential operator of nonlocal elasticity. Then, these results are used to prove the existence and uniqueness of solutions of the boundary value problems from the linear theory of nonlocal elastiticity.

A boundary value problem for the onedimensional nonlocal theory of elasticity was considered in [14]. A formulation of various boundary value problems in nonlocal elasticity is given in [12].

2. General existence theorems. Throughout this section we shall use basic concepts and results from [4]. For some additional concepts which we shall use in the paper, we shall give specific definitions.

Let A be a bounded domain of the real r-dimensional cartesian space X^r. Without any loss of generality we may assume that $\overline{A}$ is contained in the open square $Q: |x_k| < \pi$ $(k = 1, \ldots, r)$.

Let m and n be non-negative integers. If $\varphi(x, z)$ is a function on $A \times A$, we say that $\varphi(x, z)$ is of class $C^{m,n}$ on $A \times A$, if $\varphi(x, z)$ is continuous on $A \times A$ and the functions

$$D_x^p D_z^q \varphi, \quad 0 \leqslant |p| \leqslant m, \quad 0 \leqslant |q| \leqslant n, \quad |p| + |q| \leqslant \max(m, n),$$

exist and are continuous on $A \times A$. The symbol D_x denote the differentiation vector with respect to the variable x. Finally, we write C^n for $C^{n,n}$ and C^∞ for $n \to \infty$.

Suppose that the $n \times n$ complex matrices $a_s(x)$ $(0 \leqslant |s| \leqslant 2m)$ be defined in A and that $b_s(x, z)$ $(0 \leqslant |s| \leqslant m)$ be defined in $A \times A$. Consider the linear matrix integro-differential operator

$$Lu \equiv \sum_{|s|=0}^{2m} a_s(x) D^s u + \sum_{|s|=0}^{m} \int_A b_s(x, z)\, D_z^s\, u(z) dz,$$

where D_z is the differentiation vector with respect to the variable z. This operator is an elliptic operator in A if, for any real non-zero r-vector ξ the following condition is satisfied

$$\det \sum_{|s|=2m} a_s(x)\, \xi^s \neq 0, \quad \forall x \in A.$$

If $a_s(x) \equiv \| a_s^{hk}(x) \|_{n\times n}$, $b_s(x, z) \equiv \| b_s^{hk}(x, z) \|_{n\times n}$, we denote by $\overline{a}_s(x) \equiv \| \overline{a}_s^{kh}(x) \|_{n\times n}$, $\overline{b}_s(x, z) \equiv \| \overline{b}_s^{kh}(z, x) \|_{n\times n}$ the adjoint matrices, respectively.

We define as adjoint operator of the operator L the operator

$$L^*u \equiv \sum_{|s|=0}^{2m} (-1)^{|s|} D^s(\overline{a}_s(x)u) + \sum_{|s|=0}^{m} (-1)^{|s|} \int_A (D_z^s \overline{b}_s(x, z))u(z)dz.$$

The function $u \in \mathcal{L}^2(A)$ is said to be a weak solution of the integro-differential system $Lu = f$, where $f \in \mathcal{L}^2(A)$, if for any function $v \in \overset{\circ}{C}{}^{\infty}(A)$, we have

$$\int_A uL^*v dx = \int_A fv dx. \tag{1}$$

We can prove the following interior regularity theorem for the weak solution of (1).

Theorem 2.1. *Let u be a weak solution in A of the elliptic system $Lu = f$. Let f be a function belonging to $H_{1-2m+l}(A)$ $\{$to $H_0(A)$ if $1 - 2m + l < 0\}$, $a_s(x) \in C^{|s|+l}(A)$, $b_s(x, z) \in C^{1-2m+l,|s|}(A)$ $\{$to $C^{0,|s|}(A)$ if $1 - 2m + l < 0\}$. If x^0 is any point of A, there exists a $\delta > 0$ such that $u \in H_{l+1}(\Gamma_\delta)$ and*

$$\| u \|^2_{l+1, \Gamma_\delta} \leq c(\| f \|^2_{1-2m+l, \Gamma_{2\delta}} + \| u \|^2_{l, \Gamma_{2\delta}}),$$

where Γ_ρ is the ball of center x^0 and radius ρ and c is a positive constant depending only on L and x^0.

The *proof* is readily obtained by an easy modification of the corresponding proof in [4].

As a consequence of this theorem, it follows that if $f \in C^\infty(A)$, $a_s(x) \in C^\infty(A)$, $b_s(x, z) \in C^\infty(A)$, then $u \in C^\infty(A)$ and hence u is a solution in the classical sense.

Let us now consider in X^r the integro-differential operator

$$Lu \equiv \sum_{|p|,|q|=0}^{m} D^p(a_{pq}(x)D^q u) + \sum_{|s|=0}^{m} \int_A D_x^s b_s(x, z) D_z^s u(z)dz. \tag{2}$$

The adjoint operator is

$$L^*u \equiv \sum_{|p|,|q|=0}^{m} (-1)^{|p|+|q|} D^q(\overline{a}_{pq}(x)D^p u) + \sum_{|s|=0}^{m} \int_A D_x^s \overline{b}_s(x, z)\, D_z^s\, u(z)dz,$$

and the ellipticity condition is then written in the form

$$\det \sum_{|p|,|q|=m} a_{pq}(x)\, \xi^p \xi^q \neq 0, \quad \forall \xi = (\xi_1, \dots, \xi_r) \neq 0.$$

To the operator (2) we associate the following bilinear form

$$B(u, v) = \sum_{|p|,|q|=0}^{m} (-1)^{|p|} \int_A a_{pq}(x) D^q u(x) D^p v(x) dx + \tag{3}$$
$$+ \sum_{|s|=0}^{m} (-1)^{|s|} \int_A \int_A b_s(x, z)\, D_x^s v(x) D_z^s u(z) dx dz, \quad \forall u, v \in H_m(A).$$

Let us now consider the following boundary value problem

(4) $Lv = f$, on A, (5) $Mu = 0$, on ∂A,

where Mu is an integro-differential operator of order m in u.

Let us denote by V the linear variety consisting of all n-vector valued functions belonging to $H_m(A)$ such that the system (4) with the boundary conditions (5) implies

$$B(u, v) = \int_A fv dx, \tag{6}$$

for any $v \in V$.

We assume the following hypotheses

(i) $\overset{\circ}{H}_m(A) \subset V \subset H_m(A)$;

(ii) for every $v \in V$, the following inequality holds

$$| Re\, B(v, v) | \geq c_0 \| v \|^2_m, \quad c_0 > 0, \quad c_0 = \text{const.} \tag{7}$$

We are lead to the following generalized formulation of the problem (4), (5): Find a function $u \in V$ such that (6) is satisfied for any $v \in V$.

As in [4], we can prove

Theorem 2.2. *A sufficient condition in order that there exist a unique solution of (6) is that (7) be satisfied.*

Let $x \equiv (y_1, y_2, \dots, y_{r-1}, t) \equiv (y, t) \in X^r$. Let Σ^+ be the closed semiball $\{ x = (y, t) \in X^r;\ t \geq 0,\ | y |^2 + t^2 \leq 1 \}$. We suppose that the domain A is C^∞-smooth [4]. Let us denote by $\xi \equiv (y, t)$ the point of the (y, t) space and let $\xi = \xi(x)$ be the C^∞-homeomorphism that maps $J = \overline{I}(x^0) \cap \overline{A}$ onto Σ^+, where $I(x^0)$ is a neighborhood of $x^0 \in \partial A$. We denote by W the variety spanned by $v[x(\xi)]$ when $v \in V$ and is such that $v \equiv 0$ in $A \setminus J$. It follows that (7) can be written in the new coordinates in the form

$$| Re\, B(v, v) | = | \mathrm{Re} \{ \sum_{|p|,|q|=0}^{m} \int_{\Sigma^+} \alpha_{pq}(\xi)\, D^q v D^p v d\xi + \tag{8}$$
$$+ \sum_{|s|=0}^{m} \int_{\Sigma^+} \int_{\Sigma^+} \beta_s(\zeta, \xi) D_\xi^s v(\zeta) D_\zeta^s v(\zeta)\ d\zeta d\xi \} | \geq c_0 \| v \|^2_{m, \Sigma^+}, \quad v \in W,$$

where the meaning of the symbols α_{pq}, β_s is self-explanatory.

From (6) it follows

$$B(u, v) = (F, v)_0, \ \forall v \in W,\ F(\xi) = f[x(\xi)] \cdot \left| \frac{\partial x}{\partial \xi} \right|. \tag{9}$$

Using the relations (8), (9) and a similar proof as in [4], we obtain

Theorem 2.3. *Let $0 < \delta < 1$ and $\Gamma^+_\delta = \{ t > 0, | y |^2 + t^2 < \delta \}$. Suppose that $a_{pq}(x) \in C^{l+1}(\overline{A})$, $b_p(x, z) \in C^{|p|, l+1}(\overline{A})$, $f \in H_{1-m+l}(A)$ $\{ f \in \mathcal{L}^2(A)$ if $1 - m + l < 0 \}$. Let $s \equiv (s_1, \dots, s_{r-1})$ be an arbitrary $(r-1)$ — multiindex such that $| s | \leq l + 1$. Then the function u is such that $D^s_y u \in H_m(\Gamma^+_\delta)$. Moreover*

$$\| D^s_y\, u \|^2_{m, \Gamma^+_\delta} \leq c_1\, (\| F \|^2_{|s|-m, \Sigma^+} + \| u \|^2_{m, \Sigma^+}), \quad c_1 = \text{const.}, \tag{10}$$

where c_1 for any given s depends only on α_{pq}, β_s and on δ.

This theorem proves the existence of tangential derivatives of the solution.

Theorem 2.4. *Suppose that $a_{pq}(x) \in C^{l+1}(\overline{A})$, $b_p(x, z) \in C^{|p|, l+1}(\overline{A})$ $f \in H_{1-m+l}(A)$ $\{ f \in \mathcal{L}^2(A)$, if $1 - m + l < 0 \}$. Let $0 < \delta < 1$ and $\Gamma^+_\delta = \{ t > 0, | y |^2 + t^2 < \delta \}$, $s = (s_1, \dots, s_{r-1})$ an arbitrary $(r-1)$ — multiindex and $i > 0$ such that $| s | + i \leq m + l + 1$. Then the solution u has the $\mathcal{L}^2$-strong derivative $\partial^{m+i}/\partial t^{m+i} D^s_y u$ in any Γ^+_δ. There is a constant c_2, depending only on α_{pq}, β_p and δ, such that*

$$\left\| \frac{\partial^{m+t}}{\partial t^{m+t}} D_y^s u \right\|^2_{0,\Gamma_\delta^+} \leqslant c_2 \left(\| F \|^2_{|s|+i=m,\Sigma^+} + \| u \|^2_{m,\Sigma^+} \right). \tag{11}$$

The method of proof is the same as in [4].

3. Nonlocal elasticity. Uniqueness theorems. In what follows a rectangular coordinate system (x_1, x_2, x_3) is used.

We consider a bounded regular region A, occupied by an elastic material, whose boundary is ∂A. We suppose that the elastic material is anisotropic and nonhomogeneous. The basic equations in the static theory of linear nonlocal elasticity, are [2], [6]

— the equilibrium equations

$$t_{ji,j} + f_i = 0, \text{ in } A, \tag{12}$$

— the constitutive equations

$$t_{ij} = C_{ijkl} e_{kl} + \int_A G_{ijkl}(x,z) e_{kl}(z) dz, \text{ in } A, \tag{13}$$

— the geometrical equations

$$e_{ij} = (1/2)(u_{i,j} + u_{j,i}), \tag{14}$$

where we have used the following notations: t_{ij} — components of the stress tensor, f_i — components of the body force vector, u_i — components of the displacement vector, e_{ij}-components of the strain tensor, $C_{ijkl}(x)$ — the local characteristics of the material and $G_{ijkl}(x, z)$ — the nonlocal characteristics of the material. Moreover, we assume the symmetry relations [2]

$$C_{ijkl}(x) = C_{jikl}(x) = C_{klij}(x), \quad G_{ijkl}(x,z) = G_{jikl}(x,z) = G_{ijlk}(x,z) = G_{klij}(x,z). \tag{15}$$

To the equations (12)–(14) we adjoin one of the boundary conditions:

— the first boundary value problem

$$u_i = \tilde{u}_i, \quad \text{on } \partial A, \tag{16}$$

— the second boundary value problem

$$t_{ji} n_j = \tilde{t}_i, \quad \text{on } \partial A, \tag{17}$$

where n_i are components of the unit vector of the outward normal to ∂A, $\tilde{u}_i$, $\tilde{t}_i$ being the prescribed functions on ∂A. The total energy has two parts: one due to local effects and another to nonlocal effects. As in classical theory of elasticity, we assume that the internal energy density due to local effects is a positive definite quadratic form, i. e.

$$C_{ijkl} e_{ij} e_{kl} \geqslant c_3 e_{ij} e_{ij}, \quad c_3 = \text{const.}, \; c_3 > 0. \tag{18}$$

Moreover we assume that the global energy due to nonlocal effects is positive. From (12)–(15) we obtain the field equations of the nonlocal elasticity in the form

$$\frac{\partial}{\partial x_j}\left(C_{ijkl} \frac{\partial u_k}{\partial x_l} \right) + \frac{\partial}{\partial x_j} \int_A G_{ijkl}(x, z) \frac{\partial u_k(z)}{\partial z_l} dz + f_i = 0, \quad \text{in } A. \tag{19}$$

Let us denote by $Lu \equiv (L_i u)$ the operator of the nonlocal elastostatics given by

$$-L_i u \equiv \frac{\partial}{\partial x_j}\left(C_{ijkl}(x) \frac{\partial u_k}{\partial x_l} \right) + \frac{\partial}{\partial x_j} \int_A G_{ijkl}(x, z) \frac{\partial u_k(z)}{\partial z_l} dz. \tag{20}$$

Then, we can write (19) in the form

$$Lu = f, \quad \text{in } A, \tag{21}$$

where $f \equiv (f_i)$. If we introduce the notations

$$u \equiv (u_i), \; \tilde{u} \equiv (\tilde{u}_i), \quad \tilde{t} \equiv (\tilde{t}_i),$$

$$t(u) \equiv (t_i(u)) \equiv (C_{ijkl} u_{k,l} n_j + n_j \int_A G_{ijkl}(x, z) u_{k,l}(z) dz),$$

from (16), (17) we obtain

$$u = \tilde{u}, \text{ on } \partial A, \tag{22}$$

$$t(u) = \tilde{t}, \text{ on } \partial A. \tag{23}$$

Let us establish the uniqueness theorems for the boundary value problems (21), (22) and (21), (23). Consider the body subjected to two different systems of elastic loadings $\{ f_i^{(\rho)}, \tilde{t}_i^{(\rho)} \}$, $(\rho = 1, 2)$, and the two corresponding elastic states $\{ u_i^{(\rho)}, e_{ij}^{(\rho)}, t_{ij}^{(\rho)} \}$, $(\rho = 1, 2)$. Using (12)–(15), we obtain:

$$U_{\alpha\beta} \equiv U(u^{(\alpha)}, u^{(\beta)}) = \int_A f_i^{(\alpha)} u_i^{(\beta)} dx + \int_{\partial A} \tilde{t}_i^{(\alpha)} u_i^{(\beta)} dx; \quad U_{\alpha\beta} = U_{\beta\alpha}, \tag{24}$$

$$(\alpha, \beta = 1, 2; \quad \alpha \neq \beta),$$

where

$$U_{\alpha\beta} \equiv \int_A C_{ijkl} e_{ij}^{(\alpha)} e_{kl}^{(\beta)} dx + \int_A \int_A G_{ijkl}(x, z) e_{ij}^{(\alpha)}(x) e_{kl}^{(\beta)}(z) dx dz. \tag{25}$$

Taking $u = u^{(1)}$, $v = u^{(2)}$, $U(u, v) = U_{12}$, from (24) we deduce

$$U(u, v) = \int_A vLu dx + \int_{\partial A} vt(u) dx, \tag{26}$$

$$\int_A (vLu - uLv) dx = \int_{\partial A} [ut(v) - vt(u)] dx, \tag{27}$$

$$U(u, u) = \int_A uLu dx + \int_{\partial A} ut(u) dx. \tag{28}$$

Let $u_i^{(\rho)}$ be two solutions of the boundary value problems considered. If $u_i^{(0)} = u_i^{(1)} - u_i^{(2)}$, according to the linearity of the problem, the functions $u_i^{(0)}$ satisfy the equation $Lu^0 = 0$, in A, with the boundary condition $u^{(0)} = 0$, on ∂A, or $t(u^{(0)}) = 0$, on ∂A.

From (25), (28) it follows that $U(u^{(0)}, u^{(0)}) = 0$, and hence, in view of (18) and the assumption on the global energy due to nonlocal effects, we obtain

$$e_{ij}(u^{(0)}) = 0. \tag{29}$$

Then, we find

$$u_i^{(0)} = a_i + \varepsilon_{ijk} b_j x_k, \tag{30}$$

where ε_{ijk} is the alternating symbol, while a_i, b_i are arbitrary constants. Thus we get

Theorem 3.1. *The first boundary value problem* (21), (22) *admits at most one solution.*

Theorem 3.2. *The solution of the second boundary value problem* (21), (23) *is determined to within an additive rigid displacement* (30).

4. Nonlocal elasticity. Existence theorems. Using the results of section 2, in what follows we study existence and regularization of solution of the boundary value problems considered, under the assumption that the boundary conditions (22), (23) are homogeneous. Thus, we consider the system

(31) $$Lu = f, \quad \text{in} \quad A,$$

with the boundary condition

(32) $u = 0$, on ∂A, or (33) $t(u) = 0$, on ∂A.

We consider here only homogeneous boundary conditions. Other boundary conditions could be considered. However we shall restrict ourselves to the two above considered cases. Let us consider the boundary value problem defined by (31), (32). In order to prove the existence and the regularization of the solution of this problem, we take as V the space $\mathring{H}_1(A)$. We must prove the inequality (7), for any $v \in \mathring{H}_1(A)$. In this particular case the inequality (7) becomes

(34) $$\int_A C_{ijkl}(x)\, e_{ij}(v)\, e_{kl}(v) dx + \int_A \int_A G_{ijkl}(x, z) e_{ij}(v(x)) e_{kl}(v(z)) dx dz \geq c_4 \| v \|_1^2, \quad \forall v \in \mathring{H}_1(A).$$

Taking into account the first Korn inequality [4]

(35) $$\int_A e_{ij}(v) e_{ij}(v) dx \geq c_5 \| v \|_1^2, \quad \forall v \in \mathring{H}_1(A).$$

the inequality (34) will follows from the hypothesis (18) and the assumption on the global energy due to nonlocal effects. Thus we get the following theorem:

Theorem 4.1. *Suppose $C_{ijkl}(x) \in C^{l+1}(\bar{A})$, $G_{ijkl}(x, z) \in C^{l+1}(\bar{A})$. Given $f \in H_l(A)$, there exists one and only one solution of the boundary value problem (31), (32) which belongs to $H_{l+1}(A)$. In particular, when C_{ijkl}, $G_{ijkl} \in C^\infty$ and $f \in C^\infty$, using the Sobolev lemma, it folows that the solution belongs to $C^\infty(\bar{A})$.*

Let us consider the second boundary value problem (31), (33). In order to prove the existence theorem for (31), (33), let us consider the system

(36) $$L_i u + p_0 u_i = f_i,$$

where p_0 is any positive constant. We consider the problem (36), (33) and we take as V the space $H_1(A)$. It is easily seen that the inequality to be proved in this case is the following

(37) $$\int_A C_{ijkl}\, e_{ij}(v)\, e_{kl}(v) dx + \int_A \int_A G_{ijkl}(x, z) e_{ij}(v(x)) e_{kl}(v(z)) dx dz + p_0 \int_A | v |^2 dx \geq c_6 \| v \|_1^2, \quad \forall v \in H_1(A).$$

Using the second Korn inequality [5]

(38) $$\int_A e_{ij}(v) e_{ij}(v) dx + \int_A |v|^2 dx \geq c_7 \| v \|_1^2, \quad \forall v \in H_1(A),$$

the inequality (37) follows from (18) and the assumption on the global energy due to nonlocal effects.

In view of (27) and (33), the integro-differential operator is formally self-adjoint. Then a solution $u \in H_{l+1}(A)$ of the following system

(39) $$Lu + p_0 u - \lambda u = f,$$

with the boundary conditions (33), exists when and only when

(40) $$\int_A f_i \varphi_i dx = 0,$$

where φ_i is any solution belonging to $H_{l+1}(A)$ of (39) with $f_i \equiv 0$. In the case when $\lambda = p_0$, the only solutions of the homogeneous system are (30). Thus, from (30) and (40) it follows

(41) $$\int_A f_i\, dx = 0, \quad \int_A \varepsilon_{ijk} x_j f_k\, dx = 0.$$

So, we get

Theorem 4.2. *Suppose $C_{ijkl}(x) \in C^{l+1}(\bar{A})$, $G_{ijkl}(x, z) \in C^{l+1}(\bar{A})$. Given $f \in H_l(A)$, the boundary value problem (31), (33) has solutions belonging to $H_{l+1}(A)$ if and only if f satisfies the compatibility conditions (41). In particular, when C_{ijkl}, $G_{ijkl} \in C^\infty$ and $f \in C^\infty$ satisfy (41), the solution is of C^∞.*

Let us observe that the solution of the first and second boundary value problems which we have obtained, minimize the functional

$$F(u) = \int_A u L u dx - 2 \int_A f u dx,$$

in the space $\mathring{H}_1(A)$ and $H_1(A)$, respectively.

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Faculty of Mathematics
University of Jassy
România