Applying the definition of π_t we have $\alpha_t(s') = (\bar{s}, s\ )$, $\alpha_t(s'') = (\bar{s}, s_1'')$, if $(s', s'') \in \pi_t$. Thus, we obtain $\alpha_{t+1}(f_t(s', i)) = F_t((\bar{s}, s_1'), i) = (\varphi(\bar{s}, i), \mathrm{pr}_1\, \gamma(\bar{s})(s_1', i))$ and similarly $\alpha_{t+1}(f_t(s'', i)) = (\varphi(\bar{s}, i), \mathrm{pr}_1\, \gamma(\bar{s})(s_1', i))$. Therefore $\mathrm{pr}_1\, \alpha_{t+1}(f_t(s', i)) = \mathrm{pr}_1\, \alpha_{t+1}(f_t(s'', i))$, hence $(f_t(s',)i, f_t(s'', i)) \in \pi_{t+1}\ \forall i, I$ and π is a congruence.

Conversely, let π be a congruence on the t. v. a $A = (I, S, O, f_t, g_t)$, $\pi = (\gamma_t, \ldots, \pi_t, \ldots)$. Using the same technique as in Th. 5 we can construct on every set S_t an equivalence τ_t which is orthogonal on π_t, $\pi_t \wedge \tau_t = \iota_{S_t}$.

We shall use the t. v. a. $A_\tau = (I, S_\tau, O, f_{\tau t}, g_{\tau t})$, whree $\tau = (\tau_1, \ldots, \tau_t, \ldots)$, $S^\tau = (S/\tau_1, \ldots, \ldots, S_t/\tau_t, \ldots)$, the fonctions $f_{\tau t}$ and $g_{\tau t}$ being defined in the sequel. Suppose that the actual state of A_τ is the block $B \in S_t/\tau_t$. Since τ is generally not a congruence, different states of this block are carried in A by a certain symbol i in different blocks of the set S_{t+1}, τ_{t+1}. As monitor, we shall take the Moore automaton $M = (I, \bigcup_{t \in N_1} S_t/\pi_t, O_1, \varphi, \gamma)$, where $\varphi(K\ i) = K'$, $K \in S/\pi_t$, $K' \in S_{t+1}/\pi_{t+1}$ and $f_t(K, i) \subseteq K'$.

The correctness of this definition for M follows from the fact that π is a congruence. Since $\pi_t \wedge \tau_t = \iota_{S_t}$ every state $s \in S_t$ is contained in an unique pair of blocks $(K, B) \in S_t/\pi_t \times S_t/\tau_t$. We shall define α_t as $\alpha_t(s) = (K, B)$ and, if $\gamma(K) = (f_{\tau t}, g_{\tau t})$ we take:

$$f_{\tau t}(B, i) = \begin{cases} B', f_t(s, i) \in B' \in S_{t+1}/\tau_{t+1} \ \textit{and}\ K \cup B = \{ s \}, \\ \textit{arbitrary, if}\ K \cap B = \emptyset \end{cases}$$

and

$$g_{\tau t}(B, i) = \begin{cases} g_t(s, i),\ \textit{where}\ \{ s \} = K \cup SB, \\ \textit{arbitrary, if}\ K \cap B = O. \end{cases}$$

The t. v. a. related to the pair (A_τ, M) is $\hat{A} = (I, (\bigcup_{t \in N_1} S_t/\pi_t) \times S^\tau, O, F_t, G_t)$, where $F_t((K, B), i) = (\varphi(K, i), \mathrm{pr}_1\, \gamma(K)(B, i))$, $G_t((K, B), i) = \mathrm{pr}_2\, \gamma(K)(B, i)$.

In this special case we can take in (1) and (2) $h = 1_I$ and $k = 1_9$ and through a straightforward calculation we obtain:

$$k(G_t(\alpha_t(s), h(i))) = G_t(\alpha_t(s), i) = G_t((K, B), i) = \mathrm{pr}_2\, \gamma(K)(B, i) = g_{\tau t}(B, i) = g_t(s, i);$$

$$F_t(\alpha_t(s), h(i)) = F_t(\alpha_t(s), i) = F_t((K, B), i) = (\varphi(K, i), \mathrm{pr}_1\, \gamma(K)(B, i)) = (\varphi(K, i), f_{\tau t}(B, i)) = (K', B'),$$

where $f_t(s, i) \in K' \cup B'$, and $\alpha_t(s) = (K, B)$,

which means that $\hat{A}$ is able to simulate. A. The same proof (with obvious modifications) works also for m, n-decompositions.

Corollary. *The following three assertions are equivalent for a t. v. a A.*

i) there exists a congruence on A;

ii) there exists a serial decomposition of A;

iii) there exists a monitorial decomposition of A.

These assertions hold also if „congruence" is replaced with „m, n-congruence" and „decomposition" with „m n-decomposition".

Remark. If $(f_t(s, i_1), f_t(s, i_2)) \in \pi_{t+1}$, for every $i_1, i_2 \in I$ the monitor M is a pure one.

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PERMUTATION AUTOMATA WITH A TIME-VARIANT STRUCTURE

BY

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1. Introduction. Permutation automata have been studied by G. Thierrin [6]. Agasandjan [1, 2] and Arto Salomaa [5] have introduced the notion of automaton with a time-variant structure and solved some of the basic problems concerning time-variant automata.

In this paper the notion of permutation automaton with a time-variant structure is introduced. The automata with a time-variant structure (not only finite) are studied; from this study one obtains the characterization for the finite automata with a time-variant structure and for some families of permutation automata with a time-variant structure. The inclusions between different families of languages accepted by the permutation automata with a time-variant structure are considered. Some operations, under which these families of languages are closed, are determined.

2. Automata with a Time-Variant Structure. Definition 2.1. *A time-variant automaton (shortly t. v. a.) is defined by:*

$$A = < X, S_n, M_n^{n+1}, s_0, F_n, n = 0, 1, 2, \ldots >$$

where:

1) X is a finite nonempty set of inputs,

2) S_n is a finite nonempty set of states at the n-th time instant,

3) M_n^{n+1} is a function from $S_n \times X$ into S_{n+1} called the direct function from the n-th time instant into $(n+1)$-th time instant,

4) S_0 is the initial state and $s_0 \in S_0$,

5) F_n is the set of final states at the moment n, $F_n \subseteq S_n$. We recursively define the functions M_n^m, with $n \leq m$, by

$$M_n^n(s, e) = e \text{ for every } s \in S_n,\ n = 0, 1, 2, \ldots$$

$$M_n^m(s, p_n^m) = M_{n+1}^m[M_n^{n+1}(s, x_{n+1}),\ p_{n+1}^m]$$

for every $s \in S_n$ and $p_n^m = x_{n+1} \cdots x_m \in X^*$.

Definition 2.2. *Let A be a t. v. a.*

$$T(A) = \{ p_0^n \in X^* / M_0^n(s_0, p_0^n) \in F_n,\ n = 0, 1, 2, \ldots \}$$

is called the set of tapes accepted by the automaton A. A language $L \subseteq X^*$ is accepted by a t. v. a. (shortly X-t. v. a. a.) iff there exists a t. v. a. A such that $L = T(A)$.

Agasandjan [1, 2] and A. Salomaa [5] define time-variant automata for which $S_n = S$, $n = 0, 1, 2, \ldots$ and S is a finite set. This automaton will be denoted by f. t. v. a.

In the following, we denote the set of the natural numbers by N, $N' = N \cup \{ 0 \}$, the language accepted by an automaton of the type α by αa. The following two definitions appear in [6]:

Definition 2.3. *An equivalence relation R on X^* is said to be right cancellative iff $pq \equiv \bar{p}q(R)$ implies that $p \equiv \bar{p}(R)$.*

Definition 2.4. *Let U be a subset of X^*. We define $p \equiv \overline{p}(R^U)$ iff $\{q/q \in X^*$ and $pq \in U\} = \{q'/q' \in X^*$ and $\overline{p}q' \in U\}$.*

For every $h \in N$, the relation R restricted on X^h will be denoted by R_h. The number of equivalence classes of R_h will be denoted by $n(R, h)$; $n(R)$ is the number of equivalence classes of R. If R is an equivalence relation on X^*, then for every $h \in N$, R_h is an equivalence relation on X^h.

Definition 2.5. [4]. *An equivalence relation R on X^* is said to be a right congruence iff $p \equiv \overline{p}(R)$ implies that $pq \equiv \overline{p}q(R)$ for every $q \in X^*$.*

Definition 2.6. *An equivalence relation R on X^* is said to be a right congruence on X^h iff $p_0^h \equiv \overline{p}_0^h$ (R) implies that $p_0^h c \equiv \overline{p}_0^h c(R)$ for every $c \in X^*$.*

It is known that R^U is a right congruence, hence R^U is a right congruence on X^h, for every $h \in N$.

Theorem 1. *Let U be a subset of X^*. The following conditions are equivalent:*

(1) U is X-t. v. a. a.;

(2) there is an equivalence relation R on X^*, which is a right congruence on X^h and either $U \cap X^h$ is the union of some classes of R_h or $U \cap X^h = \emptyset$, for every $h \in N$.

(3) There is an equivalence relation R on X^*, which is a right congruence on X^h and $R_h \subseteq R_h^U$ for every $h \in N$.

Proof. Condition (2) *implies* (3). If $a \equiv b(R_h)$, then $a = p_0^h$, $' = q_0^h$, $p_0^h \equiv q_0^h (R_h)$. If $p_0^h r_0^k \in U$, since $p_0^h r_0^k \equiv q_0^h r_0^k (R_{h+k})$ and $U \cap X^{h+k}$ is the union of some classes of R_{h+k} then $q_0^h r_0^k \in U$, i. e. $p_0^h \equiv q_0^h (R^U)$. hence $p_0^h \equiv q_0^h (R_h^U)$. (3) implies (2). Let p_0^h be from U and $p_0^h \equiv q_0^h (R_h)$, since $R_h \subseteq R_h^U$ we have $p_0^h \equiv q_0^h (R_h^U)$, i. e. $p_0^h \equiv q_0^h (R^U)$. We have $p_0^h e \in U$, therefore $q_0^h e = q_0^h \in U$. (1) *implies* (2). There is a t. v. a. A such that $U = T(A)$. Let R be the equivalence relation defined on X^* by: $p_0^h \equiv q_0^k (R)$ iff $M_0^h (s_0, p_0^h) = M_0^k (s_0, q_0^k)$ If $p_0^h \in U$ and $p_0^h \equiv q_0^h (R)$, then $M_0^h (s_0, q_0^h) = M_0^h (s_0, p_0^h) \in F_h$, hence $q_0^h \in T(A) = U$. It follows that either $U \cap X^h$ is the union of some classes of R_h or $U \cap X^h = \emptyset$. If $p_0^h \equiv q_0^h (R)$, we have $M_0^h(s_0, p_0^h) = M_0^h (s_0, q_0^h)$ what implies $M_0^{h+k} (s_0, p_0^h r_0^k) = M_0^{h+k} (s_0, q_0^h r_0^k)$, $p_0^h r_0^k \equiv q_0^h r_0^k (R)$, hence R is a right congruence over X^h. (2) *implies* (1). We denote by $[p]$ the class which contains p, where $p \in X^*$. Consider $s_0 = [e]$, $S_0 = \{s_0\}$, $S_h = \{[p_0^h]/p_0^h \in X^h\}$, $h \in N$; $F_h = \{[p_0^h]/p_0^h \in U\}$, $h \in N'$. For every $h \in N'$ we define $M_h^{h+1}([p_0^h], x) = [p_0^h x]$. Let us to show that M_h^{h+1} is a function. If $p_0^h \equiv q_0^h (R)$, then $p_0^h x \equiv q_0^h x(R)$, i. e. $[p_0^h] = [q_0^h]$ implies that $[p_0^h x] = [q_0^h x]$. For the t. v. a. $A = < X, S_h, M_h^{h+1}, s_0, F_h$ $h \in N' >$ it easily follows that $T(A) = U$: $p_0^h \in U$ iff $M_0^h (s_0, p_0^h) = M_0^h ([e], p_0^h) = [p_0^h] \in F_h$ iff $p_0^h \in T(A)$.

Remark 2.1. R and R^U may be of infinite index on X^*.

Remark 2.2. For every $U \subseteq X^*$ there is an equivalence relation R on X^* which is a right congruence on X^h and $R_h \subseteq R_h^U$ for every $h \in N$ (see Theorem 2.2). It follows that every subset of X^* is a X-t. v. a. a.

Remark 2.3. Consider a f. t. v. a. A. Then R in Theorem 2.1 is of finite index on X^*.

Theorem 2.2. *Let U be a subset of X^*. The following conditions are equivalent:*

(4) U is X-f. t. v. a. a.,

(5) Condition (2) where R is of finite index on X^*,

(6) Condition (3) where R is of finite index on X^*,

(7) There exists $k \in N$ such that $n(R^U, h) \leqslant k$, for every $h \in N$.

Proof. It suffices to show that (6) and (7) are equivalent, because the equivalence of (4), (5), (6) follows as in Theorem 1.

From $R_h \subseteq R_h^U$ it follows that $n(R^U, h) \leqslant n(R, h)$; R being of finite index on X^*, we have $n(R, h) \leqslant k$, for every $h \in N$, where $k = n(R)$. Conversely, if (7) holds, consider the equivalence classes of R_h^U arranged in some order:

$$X^h = C_1^h \cup C_2^h \cup \ldots \cup C_{n(R^U, h)}^h \text{for every } h \in N'.$$

Now, we define R by: $p_0^h \equiv q_0^k (R)$ iff there is $l \in N$, such that $p_0^h \in C_l^h$, $q_0^k \in C_l^k$. It is clear that R is an equivalence relation on X^* and a right congruence on X^h, for every $h \in N$, because R^U is a right congruence on X^*. Moreover, by definition $R_h = R_h^U$ for every $h \in N$.

Remark 2.4. The proof of the equivalence of (4) and (7) is given in Theorem 1 of A g a s a n d j a n [1, 2] and S a l o m a a [5].

3. Permutation Automata with a Time-Variant Structure. Definition 3.1. *Let A be a t. v. a. A is said to be a permutation automaton (shortly p. t. v. a.) iff for every $h \in N'$, $x \in X$ and $s', s'' \in S_h$, $s' \neq s''$ implies that $M_h^{h+1}(s', x) \neq M_h^{h+1}(s'', x)$.*

It is obvious that the following conditions are equivalent:

(8) A is p. t. v. a.,

(9) for every $h \in N'$, $p_0^k \in X^*$ and $s', s'' \in S_h$, $s' \neq s''$ implies that $M_h^{h+k}(s', p_0^k) \neq$ $\neq M_h^{h+k}(s'', p_0^k)$,

(10) for every $h \in N'$, $k \in N$ and $p_0^k \in X^* \mid M_h^{h+k}(S_h, p_0^k) \mid = \mid S_h \mid$.

Definition 3.2. *An equivalence relation R on X^* is said to be right cancellative (left cancellative) on X^h iff $p_0^h r \equiv q_0^h r(R)(rp_0^h \equiv rq_0^h$ $(R))$ implies that $p_0^h \equiv q_0^h$ (R).*

Remark 3.1. If A is p. t. v. a., then we have: $\mid S_h \mid \leqslant \mid S_{h+1} \mid$ for every $h \in N'$.

Theorem 3.1. *Let U be a subset of X^*. The following conditions are equivalent:*

(11) U is X-p. t. v. a. a.,

(12) There is an equivalence relation R on X^*, which is a right congruence on X^h and right cancellative on X^h and either $U \cap X^h$ is the union of some classes of R_h or $U \cap X = \emptyset$ for every $h \in N$.

(13) There is an equivalence relation R on X^*, which is a right congruence on X^h and right cancellative on X^h and $R_h \subseteq R_h^U$ for every $h \in N$.

Proof. We shall follows the proof of Theorem 1. If $U = T(A)$, where A is p. t. v. a., then the relation R is right cancellative on X^h for every $h \in N$. Indeed, if $p_o^h r_o^k \equiv q_o^h r_o^k (R)$, we have $M_o^{h+k} (s_0, p_o^h r_o^k) = M_o^{h+k} (s_0, q_o^h r_o^k)$: since A is p. t. v. a. this implies that M_o^h $(s_0, p_o^h) = M_o^h (s_0, q_o^h)$, i. e. $p_o^h \equiv q_o^h$ (R). Thus, (11) implies (12). If $[p_o^h] \neq [q_o^h]$, then $p_o^h \not\equiv q_o^h (R)$, this implies that $p_o^h x \not\equiv q_o^h x(R)$, i. e. $[p_o^h x] \neq [q_o^h x]$, because R is right cancellative on X^h. Hence $M_h^{h+1} ([p_o^h], x) \neq M_h^{h+1} ([q_o^h], x)$ and (12) implies (11).

Corollary 3.1. *If R^U is right cancellative on X^h for every $h \in N$, then U is p. t. v. a. a.*

A p. t. v. a. A with $S_n = S$ for every $n \in N'$ and S finite set will be denoted by X-f. p. t. v. a.

Corollary 3.2. *Let U be a subset of X^*. The following conditions are equivalent:*

(14) U is X-f. p. t. v. a. a.,

(15) Condition (12), where R is of finite index,

(16) Condition (13), where R is of finite index,

Corollary 3.3. *If R^U is of finite index on X^* and right cancellative on X^h for every $h \in N$, then U is X-f. p. t. v. a. a.*

Corollary 3.4. *For the relation R in Corollary 3.2. there is $m \in N$ such that $n(R, m) =$ $= n(R, m + l)$ for every $l \in N$.*

Theorem 3.2. *Let U be a subset of X^*. There is an equivalence relation R on X^* depending on U, which is a right congruence on X^h right cancellative on X^h and $R_h \subseteq R_h^U$ for every $h \in N$; moreover, if the equivalence relation R' on X^* is a right congruence on X^h right cancellative on X^h and $R'_h \subseteq R_h^U$ for every $h \in N$, then $R'_h \subseteq R_h$ for every $h \in N$.*

Proof. Define the relation R_h by induction:

$$p_0^1 \equiv q_0^1 (R_1) \text{ iff } p_0^1 \equiv q_0^1 (R^U).$$

We assume that R_h is known for $h \geqslant 1$. Let $C_1^h, C_2^h, \ldots, C_{n_h}^h$ be the equivalence classes determined by R_h (on X^h), ordered in any way. If $X = \{x_1, x_2, \ldots, x_n\}$, we consider the family of sets $\mathscr{A} = \{C_i^h x_j, \ 1 \leqslant i \leqslant n_h, \ 1 \leqslant j \leqslant n\}$, which is a partition of X^{h+1}. Let P_{h+1} be the partition of X^{h+1} defined by:

$$(17) \qquad C^{h+1} \in P_{h+1} \text{ iff } C^{h+1} = C_{i_1}^h x_{j_1} \cup C_{i_2}^h x_{j_2} \cup \ldots \cup C_{i_t}^h x_{j_t},$$

where for $1 \leqslant k \neq k' \leqslant t$ we have $j_k \neq j_{k'}$ and there exist $p_{t_j} \in C^h_{i_j}$ for every $1 \leqslant j \leqslant t$, such that $p_{t_1} x_{j_1} \equiv p_{t_2} x_{j_2} \equiv \dots \equiv p_{t_t} x_{j_t} (R^U)$ and there is no $p_m x_l$ with $l \neq j_k$, $1 \leqslant k \leqslant t$, such that $p_{t_1} x_{j_1} \equiv p_m x_l(R^U)$. We define: $p_0^{h+1} \equiv q_0^{h+1} (R_{h+1})$ iff there is $C^{h+1} \in P_{h+1}$ such that $p_0^{h+1}, q_0^{h+1} \in C^{h+1}$. Now $p_0^k \equiv q_0^h (R)$ iff there is l such that $p_0^k \in C_l^k$ and $q_0^h \in C_l^h$.

By definition we have $t \leqslant n$ and $n_h \leqslant n_{h+1} \leqslant n \cdot n_h$. Let us show that $R_h \subseteq R_h^U$ for every $h \in N$.

We have $R_1 = R_1^U$. Assume that $R_h \subseteq R_h^U$. Consider $p_0^h x_i \equiv q_0^h x_j (R_{h+1})$. If $i = j$, then $p_0^h \equiv q_0^h (R_h)$ what implies that $p_0^h \equiv q_0^h (R_h^U)$; $p_0^h x_i \equiv q_0^h x_i (R^U_{h+1})$, since R^U s a right congruence on X^h. If $i \neq j$, $p_0^h \in C^h_{i_k}$, $q_0^h \in C^h_{i_l}$, $i_k \neq i_l$ and there are $\bar p_0^h \in C^h_{i_k}$, $\bar q_0^h \in C^h_{i_l}$ such that $\bar p_0^h x_i \equiv \bar q_0^h x_j(R^U)$, then from $p_0^h \equiv \bar p_0^h (R_h)$, $q_0^h \equiv \bar q_0^h (R_h)$ it follows that $p_0^h \equiv \bar p_0^h (R_h^U)$ and $q_0^h \equiv \bar q_0^h (R_h^U)$ hence $p_0^h x_i \equiv \bar p_0^h x_i (R^U_{h+1})$, $q_0^h x_j \equiv \bar q_0^h x_j (R^U_{h+1})$ and $p_0^h x_i \equiv q_0^h x_j (R^U_{h+1})$. Thus, we have $R_{h+1} \subseteq R^U_{h+1}$. Let us now show that R is right cancellative on X^h, for every $h \in N$. Consider $p_0^h c_0^k \equiv q_0^h c_0^k(R_{h+k})$. By the definition of R_{h+k} it follows that $p_0^h c_0^{k-1} \equiv q_0^h c_0^{k-1} (R_{h+k-1})$ and by $k-1$ steps we obtain $p_0^h \equiv q_0^h(R_h)$. If $p_0^h \equiv q_0^h (R_h)$, then there is C_i^h such that p_0^h, $q_0^h \in C_i^h$, this implies that $p_0^h x$, $q_0^h x \in C_i^h x$ and $p_0^h x \equiv q_0^h x(R_{h+1})$; in the same way we have $p_0^h c_0^k \equiv q_0^h c_0^k(R_{h+k})$ for every $k \in N$ and $c_0^k \in X^k$. Therefore R is a right congruence on X^h for every $h \in N$.

Now, let R' be an equivalence relation on X^* with the properties stated above. Consider $p_0^h \equiv q_0^h(R'_h)$, $p_0^h = x_{i_1} x_{i_2} \dots x_{i_k} x_{i_{k+1}} \dots x_{i_h}$, $q_0^h = x_{j_1} x_{j_2} \dots x_{j_k} x_{i_{k+1}} \dots x_{i_h}$ with $i_k \neq j_k$ and $1 \leqslant k \leqslant h$. Since R' is right cancellative on X^k we have $x_{i_1} x_{i_2} \dots x_{i_k} \equiv x_{j_1} x_{j_2} \dots x_{j_k} (R'_k)$; $R'_k \subseteq R_k^U$ implies that $x_{i_1} x_{i_2} \dots x_{i_k} \equiv x_{j_1} x_{j_2} \dots x_{j_k} (R_k^U)$. Since $i_k \neq j_k$ by definition of R_k we have $x_{i_1} x_{i_2} \dots x_{i_k} \equiv x_{j_1} x_{j_2} \dots x_{j_k}(R_k)$ what implies $p_0^h \equiv q_0^h (R_h)$, hence $R'_h \subseteq R_h$ for every $h \in N$.

In the following we mean by R the relation in Theorem 3.2.

Remark 3.2. Because $n(R) = \max\limits_{h \in N} n_h$, R is of finite index iff there exists $k \in N$ such that $n_h \leqslant k$ for every $h \in N$.

Remark 3.3. If there is an equivalence relation R' of finite index on X^*, which is a right congruence on X^h, right cancellative on X^h and $R'_h \subseteq R_h^U$ for every $h \in N$, then R is finite index on X^*.

Theorem 3.3. *Let U be a subset of X^* and $|X| \geqslant 2$. The following conditions are equivalent:*

(18) *R is of finite index,*

(19) *There exists $m \in N$ such that for every $h \geqslant m$, $1 \leqslant i \neq j \leqslant n$ and $p_0^h \in X^h$, there exists $q_0^h \in X^h$ with the property that $p_0^h x_i \equiv q_0^h x_j (R^U)$.*

Proof. If R is of finite index on X^*, by Remark 3.2 there is $k \in N$ such that $n_h \leqslant k$, for every $h \in N$; because $n_h \leqslant n_{h+1}$ for every $h \in N$, there is $m \in N$ such that $n_m = n_{m+l}$ for every $l \in N$. It folows that for every $l \in N$ and $C_i^{m+l} \in P_{m+l}$ $t = n$ (in (17)). By (17) and $R_h \subseteq R_h^U$ for every $h \in N$ we have (19).

If (19) holds, we have $t = n$ in (17), hence $n_m = n_{m+l}$ for every $l \in N$. Therefore R is of finite index on X^*.

Remark 3.4 If $i \neq j$, $p_0^h x_i \equiv q_0^h x_j(R^U)$ and $\bar p_0^h x_i \equiv \bar q_0^h x_j(R^U)$, then $p_0^h \equiv \bar p_0^h (R_h)$ iff $q_0^h \equiv \bar q_0^h(R_h)$.

Indeed, we have $p_0^h \equiv \bar p_0^h (R_h)$ iff $p_0^h x_i \equiv \bar p_0^h x_i (R_{h+1})$ and $q_0^h x_j \equiv \bar q_0^h x_j (R_{h+1})$ iff $q_0^h \equiv \bar q_0^h (R_h)$, since $p_0^h x_i \equiv q_0^h x_j(R_{h+1})$ and $\bar p_0^h x_i \equiv \bar q_0^h x_j(R_{h+1})$.

Corollary 3.5. *Let U be a subset of X^* and $|X| \geqslant 2$. Condition (19) is equivalent with the following condition:*

(20) *There exists $m \in N$ such that for every $h \geqslant m$, $1 < j \leqslant n$ and $p_0^h \in X^h$, there exists $q_0^h \in X^h$ with the property that $p_0^h x_1 \equiv q_0^h x_j(R^U)$.*

The proof follows from Remark 3.4.

Corollary 3.6. *Every subset of X^* is a X-p. t. v. a. a. (in general, it is not X-f. p. t. v. a.a.)*

Corollary 3.7. *Let U be a subset of X^* and $|X| \geqslant 2$. Conditions (14) and (19) are equivalent.*

The statement follows from Corollary 3.2, Theorems 3.2, 3.3, and Remark 3.3.

Definition 3.2. *Let A be a t. v. a. A is said to be connected (shortly c. t. v. a.) iff for every $h \in N$ and $s \in S_h$, there is $p_0^h \in X^*$ such that $M_0^h(s_0, p_0^h) = s$.*

Remark 3.5. Let A be a f. p. t. v. a. A is connected iff for each $s \in S$ there is $x \in X$ such that $M_0^1(s_0, x) = s$. A connected p. t. v. a. A will be denoted by c. p. t. v. a.

Remark 3.6. Because the t.v. a. A constructed in Theorem 2.1 is connected, it follows that every subset U of X^* is accepted by a c. t. v. a. By Theorems 3.1, 3.2, 2.1 we have that every subset U of X^* is X-c. p. t. v. a.

Theorem 3.4. *Let U be a subset of X^*. The following conditions are equivalent:*

(21) *U is X-c. f. p. t. v. a. a.,*

(22) *Condition (19) with $m = 1$,*

(23) *Condition (20) with $m = 1$.*

Definition 3.3. *Let A be a t. v. a. and $h, l \in N$. A is said to be $(h, h+l)$-strongly connected (for short $(h, h+l)$-s. c. t. v. a.) iff for every $s \in S_h$, $M_l^{h+l}(s, X^l) = S_{h+l}$.*

Definition 3.4. *Let p_1 and p_2 be two functions from $\bigcup\limits_{h=1}^{\infty} (X^h \times X^h)$ into X^* defined by: $p_1(p_0^h, q_0^h) = x_{i_1} x_{i_2} \dots x_{i_k}$, $p_2(p_0^h, q_0^h) = x_{j_1} x_{j_2} \dots x_{j_k}$ iff $p_0^h = x_{i_1} x_{i_2} \dots x_{i_k} x_{i_{k+1}} \dots x_{i_h}$, $q_0^h = x_{j_1} x_{j_2} \dots x_{j_k} x_{i_{k+1}} \dots x_{i_h}$ and $i_k \neq j_k$ where $1 \leqslant k \leqslant h$. If $p_0^h = q_0^h$, then $p_1(p_0^h, q_0^h) = p_2(p_0^h, q_0^h) = e$*

Theorem 3.5. *Let U be a subset of X^* with $|X| \geqslant 2$. The following conditions are equivalent:*

(24) *U is X-$(h, h+1)$-s. c. p. t. v. a. a.,*

(25) *For every $p_0^h x \in X^*$ and $q_0^h \in X^*$ such that $p_0^h \neq q_0^h$ and $p_1(p_0^h, q_0^h) \not\equiv p_2(p_0^h, q_0^h)(R^U)$ there x eists $\bar x \in X$, $\bar x \neq x$ with the property that $p_0^h x \equiv q_0^h \bar x(R^U)$.*

Proof. Following Theorem 3.1, Condition (24) is equivalent with the condition:

(26) There is an equivalence relation R' on X^*, which is a right congruence on X^l, right cancellative on X^l and $R'_l \subseteq R_l^U$ for every $l \in N$; for every $p_0^{h+1} \in X^*$ and $q^h \in X^*$ there exists $\bar x \in X$ such that $p_0^{h+1} \equiv q_0^h \bar x(R'_{h+1})$.

From $R'_{h+1} \subseteq R_{h+1}$ it follows that the relation R satisfies (26), i. e. for $p_0^h x \in X^*$ and $q_0^h \in X^*$, there exists $\bar x \in X$ such that $p_0^h x \equiv q_0^h \bar x(R_{h+1})$. Let $p_0^h x$, q_0^h be from X^* such that $p_0^h \neq q_0^h$ and $p_1(p_0^h, q_0^h) \not\equiv p_2(p_0^h, q_0^h) (R^U)$; by definition of R it follows that $p_1(p_0^h, q_0^h) \not\equiv p_2(p_0^h, q_0^h) (R_k)$, where k is the length of $p_1(p_0^h, q_0^h)$, hence $p_0^h \not\equiv q_0^h (R_h)$. We have $\bar x \neq x$, since from $\bar x = x$ and $p_0^h x \equiv q_0^h x(R_{h+1})$ it follows that $p_0^h \equiv q_0^h(R_h)$, which is a contradiction.

From $R_{h+1} \subseteq R^U_{h+1}$ it follows that $p_0^h x \equiv q_0^h \bar x(R^U)$, hence (26) implies (25).

Now, we assume that R^U satisfies (25). Let $p_0^h x$ and q_0^h be from X^*. If $p_0^h = q_0^h$ then $p_0^h x = q_0^h x$, whence $p_0^h x \equiv q_0^h x(R_{h+1})$. If $p_0^h \neq q_0^h$ and $p_1(p_0^h, q_0^h) \equiv p_2(p_0^h, q_0^h) (R^U)$, then by the definition of R it follows that $p_0^h \equiv q_0^h(R_h)$, whence $p_0^h x \equiv q_0^h x(R_{h+1})$. If $p_0^h \neq q_0^h$ and $p_1(p_0^h, q_0^h) \not\equiv p_2(p_0^h, q_0^h) (R^U)$, by (25) there exists $\bar x \in X$, $\bar x \neq x$ such that $p_0^h x \equiv q_0^h \bar x (R^U)$ what implies that $p_0^h x \equiv q_0^h \bar x(R_{h+1})$. Therefore (25) implies (26).

Theorem 3.6. *Let U be a subset of X^*, $|X| \geqslant 2$ and $h \in N$. The following conditions are equivalent:*

(27) *U is X-$(h, h+1)$-s. c. f. p. t. v. a. a.,*

(28) *Conditions (25) and (20);*

(29) *Conditions (25) and (19).*

Remark 3.7. Let A be a f. p. t. v. a. and $h \in N$. A is $(h, h+1)$-strongly connected iff A is $(h, h+l)$-strongly connected for every $l \in N$.

Definition 3.5. *Let A be a t. v. a. A is said to be strongly connected (shortly s. c. t. v. a.) iff for every $n \in N$ and $s' \in S_n$, $s'' \in S_{n+1}$ there is $x \in X$ such that $M_n^{n+1}(s', x) = s''$; for every $s \in S_1$ there is $x \in X$ such that $M_0^1(s_0, x) = s$.*

Remark 3.8. Let A be a t. v. a. A is strongly connected iff A is $(h, h+1)$-strongly connected for every $h \in N$ and $M_0^1(s_0, X) = S_1$.

Theorem 3.7. *Let U be a subset of X^* and $|X| \geq 2$. The following conditions are equivalent:*

(30) U is X-s. c. p. t. v. a. a.,

(31) For every $h \in N$, $p_0^h x \in X^*$ and $q_0^h \in X^*$ such that $p_0^h \neq q_0^h$ and $p_1(p_0^h, q_0^h) \not\equiv p_2(p_0^h, q_0^h)$ (R^U), there exists $\bar{x} \in X$, $\bar{x} \neq x$ with the property that $p_0^h x \equiv q_0^h \bar{x}(R^U)$.

Theorem 3.8. *Let U be a subset of X^* and $|X| \geq 2$. The following conditions are equivalent:*

(32) U is X-s. c. f. p. t. v. a. a.,

(33) Conditions (22) and (31),

(34) Conditions (23) and (31).

4. The Inclusion Relations between the Families of Subsets of X^* introduced in 3. Theorems 3.3—3.8 give methods for the construction of languages of different types; moreover they give the inclusion relations between some families of the accepted languages of X^*. The following Propositions point out these relations:

Proposition 4.1. *If $|X| \geq 2$ and $X_1 \subset X$, then $U = X_1^*$ is not X-f. p. t. v. a. a.*

Proof. For every $p_0^h \in X^*$, $x_i \in X_1$ and $x_j \in X - X_1$ there is no $q_0^h \in X^*$ such that $p_0^h x_i \equiv q_0^h x_j(R^U)$, because for every $r_0^k \in X_1^*$ $p_0^h x_i r_0^k \in X_1^* = U$ and $q_0^h x_j r_0^k \notin U$.

Proposition 4.2. *If $X = \{x_1, x_2\}$ and P is the set of the prime numbers, then $U = x_1 \cup x_2 \cup (x_1x_1 \cup x_2x_1)(\bigcup_{p \in P} X^p)$ is a X-f. p. t. v. a. a., but it is not X-c. f. p. t. v. a. a.*

Proof. We have $m = 2$; for every $h \geq 2$ and $p_0^h \in X^h$ there exists $q_0^h \in X^h$ such that $p_0^h x_1 \equiv q_0^h x_2(R^U)$.

Moreover $x_1x_1 \not\equiv x_2x_2(R^U)$ and $x_1x_1 \not\equiv x_1x_2(R^U)$, hence $m \neq 1$.

Proposition 4.3. *We consider $X = \{x_1, x_2\}$ and $U = x_1 \cup x_2 \cup (x_1x_2 \cup x_2x_1)X^*$. U is X-f. p. t. v. a. a., but for $h \geq 2$ U is not X-$(h, h+1)$-s. c. p. t. v. a. a., what implies that U is not X-$(h, h+1)$-s. c. f. p. t. v. a. a.*

Proof. We have $x_1x_1 \not\equiv x_1x_2(R^U)$. If $h \geq 2$, for $x_1x_1p_0^{h-2}x_1$ and $x_1x_2p_0^{h-2}$ there is no x_2 such that $x_1x_1p_0^{h-2}x_1 \equiv x_1x_2p_0^{h-2}x_2(R^U)$, because $x_1x_1p_0^{h-2}x_1 \in U$ and $x_1x_2p_0^{h-2}x_2 \notin U$.

Proposition 4.4. *Let U be a subset of X^*. If U is a X-s. c. p. t. v. a. a., then U is X-f. p. t. v. a. a.*

Proposition 4.5. *Let $X = \{x_1, x_2\}$ and $U = x_1 \cup (x_1x_2^*x_1 \cup x_2x_2 \cup x_2x_1x_2^*x_1)$. U is a X-(1, 2)-s. c. p. t. v. a. a., but it is not X-(1, 2)-s. c. f. p. t. v. a. a.*

Proof. We have $x_1 \not\equiv x_2(R^U)$, $x_1x_2 \equiv x_1x_2(R^U)$, $x_1x_2 \equiv x_2x_1(R^U)$. It follows that (25) holds for $h = 1$. For every $h \in N$ we have $x_2x_1x_2^hx_1 \in U$ and $p_0^{h+2}x_2 \notin U$ what implies that

$$x_2x_1x_2^hx_1 \not\equiv p_2^{h+2}x_2(R^U)\,.\,(x_2^h = \underbrace{x_2x_2 \cdots x_2}_{h\text{-times}}).$$

In the same manner, for every $h \geq 2$, there can be constructed languages, which are X-$(h, h+1)$-s. c. p. t. a. v. a., but they are not X-$(h, h+1)$-s. c. f. p. t. v. a. a.

Proposition 4.6. *Let $X = \{x_1, x_2\}$. $U = x_1X^*$ is a X-f. p. t. v. a. a., but it is not X-(1, 2)-s. c. p. t. v. a. a.*

Proof. Condition (19) holds for $m = 1$. Moreover $x_1 \not\equiv x_2(R^U)$ and $x_1x_2 \not\equiv x_2x_1(R^U)$, hence U is not X-(1, 2)-s. c. p. t. v. a. a. It follows that U is not X-(1, 2)-s. c. f. p. t. v. a. a.

Proposition 4.7. *There is a X-s. c. p. t. v. a. a., which is not X-s. c. f. p. t. v. a. a.*

Proof. Consider $X = \{x_1, x_2\}$. Define U recursively:

1) $x_1, x_2, x_1x_1, x_2x_1 \in U$

2) for $h \geq 2$, if $X^h = P_1^h \cup P_2^h$, where $P_1^h = U \cap X^h$ and $P_1^h \cap P_2^h = \emptyset$, then $P_2^hx_1 \cup P_2^hx_2 \in U$.

It is easy to show that U holds our statement.

Proposition 4.8. *Let $|X| \geq 2$ and U be a finite subset of X^*. U is a X-f. p. t. v. a. a. iff there are integers $k, n_1, n_2, \ldots, n_k$, $0 \leq n_1 < n_2 < \ldots < n_k$, such that $U = \bigcup_{i=1}^{k} X^{n_i}$.*

Proof. It is obvious that $U = \bigcup_{i=1}^{k} X^{n_i}$ is a X-f. p. t. v. a. a. If U is a X-f. p. t. v. a. a., then by Corollary 3.4 there is $m \in N$ such that $n(R, m) = n(R, m+l)$ for every $l \in N$. Since U is finite, there is $h \in N$ such that $X^{h+l} \notin U$ for every $l \in N$. It follows that $n(R, l) = 1$ for every $l \in N$ and by Theorem 3.1 we obtain the second part of the statement.

Proposition 4.9. *If $|X| = 1$ every subset $U \subseteq X^*$ is X-s. c. f. p. t. v. a. a.*

The following two Propositions specify the inclusions between the family of X-f. p. t. v. a. a. (FPV), the family of the X-languages accepted by the finite automata with time-variant structure (FV) and the family of X-languages accepted by the finite permutation automata (FP).

Proposition **4.10.** *If $X = \{x_1, x_2\}$ and $U = \{x_1x_2x_1x_2^2,\ x_1x_2x_1x_2^2x_1x_2^3x_1x_2^4,\ x_1x_2x_1x_2^2x_1x_2^3 x_1x_2^4x_1x_2^5x_1x_2^6, \ldots\}$, then U is a X-f. t. v. a. a., but it is not X-f. p. t. v. a. a.*

Proof. It is known [5] that U is X-f. t. v. a. a. For every odd $k \in N$ and $p_0^h = x_1x_2x_1x_2^2 \ldots x_1x_2^k$ there is no q_0^h such that $p_0^hx_1 \equiv q_0^hx_2(R^U)$, because $p_0^hx_1x_2^{k+1} \in U$, but $q_0^hx_2^{k+2} \notin U$. $(h = k(k+3)/2)$.

Proposition 4.11. *Let $X = \{x_1, x_2\}$ and $U = x_1 \cup x_2 \cup (x_1x_1 \cup x_2x_1)X^*$. U is a X-f. p. t. v. a. a., but it is not accepted by any finite permutation automata over X.*

Proof. Condition (19) holds for the relation R^U. But the relation R^U is not right cancellative on X^*, since $x_1x_2 \equiv x_2^3(R^U)$ implies that $x_1 \not\equiv x_2^2(R^U)$. Therefore U is not X-f. p. a. a. [6].

It is known that there is a language over X, which is not X-t. v. a. a. [5].

Consider that X is fixed. Let V be the family of the X-p. t. v. a. a., PFV be the family of X-f. p. t. v. a. a., and so on.

For $|X| \geq 2$ we have the diagram of fig. 1, where the inclusions are indicated by sags The inclusions specified by Propositions 4.10 and 4.11 are the following

$$FP \rightarrow FPV \rightarrow FV \rightarrow V \leftarrow PV = CPV.$$

5. Operations on Languages. Proposition 5.1. *Every family of subsets of X^* introduced in 3 is closed under complementation.*

Proof. Denote by $\bar{U}$ the complement of U. We have $R^U = R^{\bar{U}}$, hence R^U and $R^{\bar{U}}$ have the same properties.

Proposition 5.2. *The family FPV of X^* is a Boolean algebra of sets.*

Proof. If $|X| = 1$, the statement follows from Proposition 4.9. Let U_1 and U_2 be two X-f. p. t. v. a. a. with $|X| \geq 2$. For R^{U_1} and R^{U_2} we have (19), with m_1 and m_2 respectively. For every $m \geq \max(m_1, m_2)$, $h \geq m$, $1 \leq i \neq j \leq n$ and $p_0^h \in X^*$ there are $q_0^h, \bar{q}_0^h \in X^*$ such that $p_0^hx_i \equiv q_0^hx_j(R^{U_1})$ and $p_0^hx_i \equiv \bar{q}_0^hx_j(R^{U_2})$. For $q_0^h \in X^*$ there is $\bar{p}_0^h \in X^*$ such that $q_0^hx_j \equiv \bar{p}_0^hx_i(R^{U_2})$. By Remark 3.4 it follows that $p_0^h \equiv \bar{p}_0^h(R_h)$, $q_0^h \equiv \bar{q}_0^h(R_h)$, where R is the relation in Theorem 3.2, corresponding to U_2. We have $p_0^hx_i \equiv \bar{p}_0^hx_i(R_h)$, $\bar{q}_0^hx_j \equiv q_0^hx_j(R_h)$, $p_0^hx_i \equiv \bar{p}_0^hx_i(R^{U_2})$, $\bar{q}_0^hx_j \equiv q_0^hx_j(R^{U_2})$, $p_0^hx_i \equiv q_0^hx_j(R^{U_2})$. Therefore $p_0^hx_i \equiv q_0^hx_j(R^{U_1} \cap R^{U_2})$. From $R^{U_1} \cap R^{U_2} \subseteq R^{U_1 \cap U_2}$ it follows that (19) hold for $R^{U_1 \cap U_2}$.;

Proposition 5.3. *Let h be a mapping from X into X_1, $\tilde{h}$ be the homomorphism from X^* into X_1^* defined by*

$$\tilde{h}(e) = e,\ \tilde{h}(\alpha\beta) = \tilde{h}(\alpha)\tilde{h}(\beta),\ \alpha,\ \beta \in X^*,\ \tilde{h}(x) = h(x)$$

and $X_2 = \{h(x)/x \in X\}$.

If U is a X-f. p. t. v. a. a., then $\tilde{h}(U)$ is a X_2-f. p. t. v. a. a.

Proof. $\widetilde{h}(U) = \{ q \in X_2^* / \exists p \in X^*, \widetilde{h}(p) = q \}$. Let us consider $\overline{p}_0^m \in X_2^*, \overline{x}_k, \overline{x}_l \in X_2$, $k \neq l$. There are $p_0^m \in X^*$, $x_i, x_j \in X$ such that $\widetilde{h}(p_0^m) = \overline{p}_0^m$, $\widetilde{h}(x_i) = \overline{x}_k$, $\widetilde{h}(x_j) = \overline{x}_l$. Because U is X-f. p. t. v. a. a. there is $m_0 \in N$ such that for $m \geq m_0$, $p_0^m x_i \in X^*$, $1 \leq i \neq j \leq n$ there is q_0^m with the property that $p_0^m x_i \equiv q_0^m x_j (R^U)$, this implies that $h(p_0^m x_i) \equiv h(q_0^m x_j) (R^{h(U)})$, whence $\overline{p}_0^m \overline{x}_k \equiv \overline{q}_0^m \overline{x}_l (R^{h(U)})$.

If $|X_2| = 1$, $h(U)$ is X_2-f. p. t. v. a. a. by Proposition 4.9.

Proposition 5.4. *Let h be a mapping from X onto X_1, and $\widetilde{h}$ be the corresponding homomorphism from X^* onto X_1^*. If U is a X_1-f. p. t. v. a. a., then $\widetilde{h}^{-1}(U)$ is a X-f. p. t. v. a. a.*

Proof. $\widetilde{h}^{-1}(U) = \{ p \in X^* / \exists q \in X_1^*, \widetilde{h}(p) = q \in U \}$.

Consider $p_0^m \in X^*$, $x_i, x_j \in X$, $i \neq j$, $\widetilde{h}(p_0^m) = \overline{p}_0^m$, $\widetilde{h}(x_i) = \overline{x}_i$, $\widetilde{h}(x_j) = \overline{x}_j$. Since U is a X_1-f. p. t. v. a. a. there exists $m_0 \in N$ such that for every $m \geq m_0$, $\overline{p}_0^m \in X_1^*$, there is $\overline{q}_0^m \in X_1^*$ with the property that $\overline{p}_0^m \overline{x}_i \equiv \overline{q}_0^m \overline{x}_j (R^U)$ what implies that $p_0^m x_i \equiv q_0^m x_j (R^{\widetilde{h}^{-1}(U)})$, where $q_0^m \in \widetilde{h}^{-1}(\overline{q}_0^m)$.

Proposition 5.5. *If U is a X-f. p. t. v. a. a. and V is a subset of X^*, then:*

$$U_1 = \{ p / \in X^*, pV \cap U \neq \emptyset \}$$

is a X-f. p. t. v. a. a.

Proof. There is $m_0 \in N$ such that for every $m \geq m_0$, $p_0^m \in X^*$, $x_i, x_j \in X$, there is $q_0^m \in X^*$ with the property that $p_0^m x_i \equiv q_0^m x_j (R^U)$. We shall prove that $p_0^m x_i \equiv q_0^m x_j (R^{U_1})$. If $p_0^m x_i r_0^k \in U_1$, then there is $v \in V$ such that $p_0^m x_i r_0^k v \in U$, this implies that $q_0^m x_j r_0^k v \in U$, hence $q_0^m x_j r_0^k \in U_1$. Therefore $p_0^m x_i \equiv q_0^m x_j (R^{U_1})$.

Proposition 5.6. *The family of X-f. p. t. v. a. a. is not closed under the operation of transposition.*

Proof. Let $X = \{ x_1, x_2 \}$ and $U = x_1 \cup x_2 \cup (x_1 x_1 \cup x_2 x_1) (\bigcup_{p \in P} X^p)$. Let U^T be the transpose of U. U is X-f. p. t. v. a. a. by Proposition 4.2. $U^T = x_1 \cup x_2 \cup (\bigcup_{p \in P} X^p) (x_1 x_1 \cup x_1 x_2)$. For $k \geq 2$ we have $p_0^k x_1 \not\equiv q_0^k x_2 (R^{U^T})$ since $p_0^k x_1 x_1 \in U^T$, but $q_0^k x_2 x_1 \notin U^T$.

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RECENZII

E. OESER: *Wissenschaft und Information*. Band I, Wissenschaftstheorie und empirische Wissenschaftsforschung. R. Oldenbourg Verlag. Wien. München, 1976, 158 p.

Această lucrare este un eseu asupra dezvoltării cercetării științifice și a istoriei științei. Concepția care stă la baza întregii lucrări, este cea a conceperii științei ca un sistem informațional. Se poate evalua astfel imediat rolul dezvoltării mijloacelor de comunicare în procesul de dezvoltare a științei.

Primul volum al unei serii mai ample este consacrat teoriei științei și cercetării științifice empirice. Principalele teme studiate aici sînt: teoria științei ca sistem explicativ reconstructiv, istoria descriptivă a științei, teoria demonstrației, euristică, teoria motivării și teoria verificării, psihologia și sociologia științei, evaluarea și statistica științei, știința ca problemă de informație, evoluția științei și explicația ei cu ajutorul teoriei informației.

Dan Simovici

GYÖRGY BIZÁM, JÁNOS HERSZEG: *Logik macht Spass*. Akadèmiai Kiadó, Budapest, Ernst Klett Verlag, Stuttgart (1976), 390 p.

Cartea conține enunțurile și rezolvările a 85 probleme de tipul celor cunoscute în literatură ca probleme de logică, întrucît nu fac apel la cunoștințe ci, doar la raționamente. Prin tematica lor, problemele prezintă situații întîlnite în viața de toate zilele (unele chiar în criminologie). Esența matematică a lor este următoarea: fiind date două sau mai multe mulțimi finite, disjuncte, echivalente și un număr de relații între elementele lor, să se determine o corespondență biunivocă între aceste mulțimi. Meritul autorilor este de a fi înlocuit, în rezolvare, improvizația printr-o tehnică de rezolvare aproape generală: se prezintă prin tabele bi sau tridimensionale, precum și prin grile, produsul cartezian al mulțimilor considerate și se suprimă succesiv, în tabele sau în grilă, acele perechi, triplete etc. care în baza relațiilor date, nu pot contribui la rezolvarea corespondenței biunivoce. Prin această tehnică, rezolvarea celor mai multe probleme de acest tip poate fi efectuată de oricine, cu o economie apreciabilă de efort și de timp. Se analizează separat și minuțios, cazuri cînd problema pusă are mai multe soluții sau cînd nu are soluție. La unele probleme se compară diferitele variante de rezolvare sub raportul volumului de relații utilizat și, ca atare, se pune în evidență redundanța sistemului de informații conținut în enunțul problemei.

În multe probleme, se dau doar relațiile prin tabele, rezolvitorul urmînd să întocmească un text corespunzător și să găsească soluția. Pe baza analizei soluțiilor unei grupe de probleme, se fac observații asupra metodei continue de rezolvare utilizate, avantajele și limitele ei.

Studiul cărții este facilitat de o clasificare și ordonare, care reliefează ideile și metodele principale și îndrumă pe cei ce vor să parcurgă cartea pe sărite sau profesorii care vor să rezolve unele probleme cu elevi de diferite clase. Un indice special dă indicații asupra noțiunilor noi introduse. Prezentarea grafică este ireproșabilă. Traducerea cărții în limba germană este semnată de Dr. Johann Raab.

David Rimer

A. A. KIRILLOV: *Elements of the Theory of Representations*, Springer-Verlag, Berlin, Heidelberg, New York, 1976, 315 p.

Cartea este o traducere a ediției rusești, apărută în anul 1972 în editura Nauka din Moscova. În esență, aici sînt expuse sistematic, atît unele capitole din alte discipline matematice, necesare în teoria generală a reprezentărilor, cît și fundamentele teoriei reprezentărilor și diversele dezvoltări ale acestei teorii, așa cum au fost sistematizate, după ce au fost prezentate în cadrul cursului de teoria reprezentărilor, ținut de autor la Universitatea de Stat din Moscova.

Pentru o expunere cît mai sistematică, materialul a fost împărțit în trei părți. În prima parte (§§ 1—§§ 6) se expun rezultate din alte capitole ale matematicilor care sînt necesare pentru prezentarea teoriei reprezentărilor. Partea a doua (§§ 7—§§ 15) conține conceptele și metodele principale din teoria reprezentărilor. În partea a treia (§§ 16—§§ 19) sînt ilustrate, prin exemple, construcțiile și teoremele generale conținute în partea a doua.