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SPIN STRUCTURES ON HILBERT MANIFOLDS

BY

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1. Introduction. Let H be a separable, real Hilbert space. We denote by $L(H)$ the algebra of bounded linear operators on H, by $O(H)$ the orthogonal operators on H and by I the identity operator on H. Let $GL_P(H)$ be the group of those invertible operators on H which can be written as $I+A$, where A is in a „perturbation class" $P(H)$ [2, p. 46] of $L(H)$. The group $O(H)_P = O(H) \cap GL_P(H)$ is doubly connected and we denote by $SO(H)_P$ the connected component of I.

Let $F(H)$ be the ideal of finite rank operators on H. We denote by $F(H)_p$ the closure of the ideal $F(H)$ in the p-norm defined by $\| X \|_p =$ (trace $(X^*X)^{\frac{p}{2}})^{\frac{1}{p}}$ for $1 \leqslant p < \infty$ and by the usual norm, for $p = \infty$. The ideals $F(H)_p$ are „perturbation classes" for $L(H)$. For $p = 1$ (resp. $p = 2$) we obtain the ideal of nuclear operators (resp. the ideal of Hilbert-Schmid operators) and in this case the groups $O(H)_P$, $SO(H)_P$ will be denoted by $O(H)_1$, $SO(H)_1$ (resp. $O(H)_2$, $SO(H)_2$). It follows, from general principles, that the universal covering of $SO(H)_P$ is a Banach-Lie group and that the covering map is 2-sheeted. An explicit construction of the universal covering group Spin $(H)_1$ of $SO(H)_1$ has been given by P. de la Harpe [3]. Later, R. J. Plymen and R. F. Streater [9] gave an explicit construction of the universal covering group Spin $(H)_2$ of $SO(H)_2$. Both groups Spin $(H)_1$ and Spin $(H)_2$ will be called spinor groups and will be denoted by Spin (H).

We denote by $SO(H)$ both the groups $SO(H)_1$ and $SO(H)_2$ and by ρ : Spin $(H) \to SO(H)$ the corresponding covering maps.

In the following, we define the spin structures using the groups Spin (H) and we give some properties of these structures. Some results about the Riemannian P-structures are given, too.

2. Definitions of the spin structures. All bundles, manifolds and morphisms considered in the following will be assumed of class C^∞. Let M be a connected and paracompact manifold, modelled on a Banach space and let $\xi : E \to M$ be a vector bundle over M with fibre H.

Definition 2.1. *A Riemannian P-structure on the vector bundle ξ is a reduction of its structural group to the group $O(H)_P$.*

Remark 2.1. The existence of the Riemannian P-structures is a direct consequence of the fact that $GL(H)$ is contractible (Kuiper's theorem).

Definition 2.2. *A Riemannian P-structure on the vector bundle ξ is said* *to be orientable if* ξ *admits a further reduction of its structural group to the group* $SO(H)_P$.

Theorem 2.1. *A Riemannian P-structure on the vector bundle* ξ *is orientable if and only if the first Stiefel-Whitney class* $w_1(\xi) \in H^1(M, Z_2)$ *vanishes.*

Proof. The proof of Proposition 6.2 from [6] can be repeated using the homomorphism $O(H)_P \to O(H)_P/SO(H)_P$. We give an alternative proof. The exact sequence

$$(2.1) \qquad 1 \to SO(H)_P \to O(H)_P \xrightarrow{p} Z_2 \to 1$$

induces an exact sequence of the cohomology groups and sets ([5], 3.1 and 2.10.1)

$$(2.2) \qquad 0 \to H^1(M, SO(H)_P) \to H^1(M, O(H)_P) \xrightarrow{p^*} H^1(M, Z_2)$$

We denote by L the principal bundle of linear frames of ξ (for definition see B o u r b a k i [1]), interpreted as an element of $H^1(M, O(H)_P)$. From exactness of the sequence (2.2) it follows that the *Riemannian P-structure of* ξ *is orientable iff* $p^*(L) = 0$. Now we prove that $p^*(L) = w_1(\xi)$. By the naturality property of the characteristic classes, it is sufficient to do so when M is the classifying space BO of the group $O(H)_P$. But $H^1(BO, Z_2) = Z_2$ [6], hence the map p^* is either identically zero, or is the class w_1. The first alternative is not possible, because there exists at least a vector bundle with a nonorientable Riemannian P-structure (see Exemple 4 from [2]). For the theory of the characteristic classes considered here see U. K o s c h o r k e [6].

Corollary 2.1. *A connected and paracompact manifold* N, *modelled on* H, *endowed with a Riemannian P-structure is orientable with respect to this structure iff* $w_1(N) = 0$.

Theorem 2.2. *A connected and paracompact manifold* N, *modelled on* H, *is orientable with respect to all Riemannian P-structures which are compatible with its manifold structure if* $H^1(N, Z_2) = 0$.

Proof. It follows from a result of U. K o s c h o r k e [6, Proposition 6.3].

A reduction of the structural group of the vector bundle ξ to the group $O(H)_1$ (resp. $O(H)_2$) will be called a *Riemannian nuclear structure* (resp. a *Riemannian Hilbert-Schmidt structure*).

Let G be a Banach-Lie group and let $P(M, \pi, G)$ (where $\pi : P \to M$), be a principal bundle over M with group G. Let G' be another Banach-Lie group.

Definition 2.3. *A principal bundle* $P'(M, \pi', G')$ *where* $\pi' : P' \to M$, *is said to be an extension of the principal bundle* $P(M, \pi, G)$, *associated to the homomorphism* $\varphi : G' \to G$, *if there exists a morphism* $\tilde{\varphi} : P' \to P$ *such that* $(\tilde{\varphi}, \varphi)$ *is a morphism of principal bundles.*

We suppose that the vector bundle ξ is endowed with a reduction of its structural group to $SO(H)$ and we denote by $P(M, \pi, SO(H))$ the principal bundle of linear frames of it.

Definition 2.4. *A spin structure on the vector bundle* ξ, *endowed with a reduction of its structural group to* $SO(H)$, *is an extension of principal bundle* $P(M, \pi, SO(H))$, *associated to the covering map* $\rho : \mathrm{Spin}(H) \to SO(H)$.

Such an extension will be denoted by $\Sigma(M, \pi', \mathrm{Spin}(H))$ and will be called a *spin structure on* $P(M, \pi, SO(H))$ or *a spin structure on* M *with respect to* $P(M, \pi, SO(H))$, too.

The morphism $\tilde{\rho} : \Sigma \to P$ is a 2-sheeted covering map and its restriction to fibres are 2-sheeted covering maps. For every a in Spin (H) and u in Σ, we have $\tilde{\rho}(ua) = \tilde{\rho}(u)\rho(a)$ and $\pi(\tilde{\rho}(u)) = \pi'(u)$. It follows that the Definition 2.4 is equivalent to the following definition, given by A. L i c h n e r o w i c z [7] in a different context.

Definition 2.5. *A spin structure on the vector bundle* ξ, *endowed with a reduction of its structural group to* $SO(H)$, *is a principal bundle* $\Sigma(M, \pi', \mathrm{Spin}(H))$ *such that* Σ *is a 2-fold covering of* P, *the restriction of the covering map* $\tilde{\rho} : \Sigma \to P$ *to fibres are 2-sheeted covering maps and* $\tilde{\rho}(ua) = \tilde{\rho}(u)\rho(a)$, $\pi(\tilde{\rho}(u)) = \pi'(u)$ *hold, for every* $a \in \mathrm{Spin}(H)$ *and* $u \in \Sigma$.

Remark 2.2. By a general result (see B o u r b a k i [1]), the principal bundle $P(M, \pi, SO(H))$ is determined (up to an isomorphism) by an open covering $\{U_i\}$ of M and a cocycle $g_{ij} : U_i \cap U_j \to SO(H)$. From Definition 2.4 it follows that, the principal bundle $\Sigma(M, \pi', \mathrm{Spin}(H))$, when it exists, is determined by a cocycle $\tilde{g}_{ij} : U_i \cap U_j \to \mathrm{Spin}(H)$ such that $\rho(\tilde{g}_{ij}) = g_{ij}$.

The following theorem gives another definition of the spin structures.

Theorem 2.3. *Let* $P(M, \pi, SO(H))$ *be the principal bundle of linear frames of* ξ. *The vector bundle* ξ *admits a spin structure if and only if there exists a cohomology class* $\sigma \in H^1(P, Z_2)$ *whose restriction to each fibre is non-zero.*

Proof. From the isomorphism $H^1(P, Z_2) \simeq \mathrm{Hom}(H_1(P), Z_2)$ it follows that, there exists a not trivial homomorphism $\sigma : H_1(P) \to Z_2$. If $\varphi_1 : \pi_1(P) \to H_1(P)$ denotes Hurewicz's homomorphism, $\sigma \circ \varphi_1 : \pi_1(P) \to Z_2$ is an epimorphism, hence $\ker(\sigma \circ \varphi_1)$ is a subgroup of index 2 in $\pi_1(P)$. Consequently, since the manifold P is locally contractible, there exists a covering space Σ of P such that $\tilde{\rho}_*(\pi_1(\Sigma)) = \ker(\sigma \circ \varphi_1)$, where $\tilde{\rho} : \Sigma \to P$ is the covering map. The covering space Σ can be taken as the total space of a principal bundle over M with group Spin (H) which is an extension of $P(M, \pi, SO(H))$, therefore a spin structure of ξ.

Conversely, if $P(M, \pi, SO(H))$ admits a spin structure $\Sigma(M, \tilde{\pi}, \mathrm{Spin}(H))$, the total space Σ is a two-fold covering of P. Let s_0 and s_1 be two points in $\tilde{\rho}^{-1}(u)$, where u is a fixed point in P. Denote by c a loop about u and by $\hat{c}$ its lift to Σ with $\hat{c}(0) = s_0$. The endpoint $\hat{c}(1)$ depends on $[c] \in \pi_1(P, u)$ the homotopy class of c. Define the homomorphism $\tau : \pi_1(P, u) \to Z_2$ by $\tau([c]) = 0$ if $\hat{c}(1) = s_0$ and $\tau([c]) = 1$ if $\hat{c}(1) = s_1$. Since Z_2 is commutative, τ vanishes on the commutator subgroup $[\pi_1(P), \pi_1(P)]$ of $\pi_1(P, u)$, therefore τ induces a homomorphism $\sigma : \pi_1(P, u)/[\pi_1(P, u), \pi_1(P, u)] \to Z_2$. We can identify σ with an element of $H^1(P, Z_2)$ via the isomorphisms $\pi_1(P, u)/[\pi_1(P), \pi_1(P)] \simeq H_1(P)$, $\mathrm{Hom}(H_1(P), Z_2) \simeq H^1(P, Z_2)$.

The exact sequence of groups

$$1 \to Z_2 \to \text{Spin}\,(H) \to SO\,(H) \to 1 \tag{2.3}$$

induces an exact sequence of the cohomology groups and sets

$$0 \to H^1\,(M, Z_2) \to H^1(M, \text{Spin}\,(H)) \to H^1(M, SO(H)) \xrightarrow{v} \ldots H^2(M, Z_2) \tag{2.4}$$

Denote by P the element of $H^1(M, SO(H))$ determined by $P(M, \pi, SO(H))$. Using the Remark 2.2. and the exactness of the sequence (2.4), we obtain

Theorem 2.4. *The vector bundle ξ admits a spin structure if and only if $v\,(P) = 0$.*

When $SO(H) = SO(H)_1$, P. de la H a r p e [3] has proved that $v\,(P) = w_2\,(\xi)$, where $w_2\,(\xi)$ is the second Stiefel-Whitney class of ξ.

The following exact sequence

$$0 \to H^1(M, Z_2) \overset{\pi^*}{\to} H^1(P, Z_2) \overset{i^*}{\to} H^1(SO\,(H), Z_2) \to H^2\,(M, Z_2) \tag{2.5}$$

where i is the natural inclusion of the fibre in the total space, can be obtained from spectral sequence associated to the principal bundle $P\,(M, \pi, SO(H))$ (see J.-P. S e r r e [11] p. 456).

If ξ admits a spin structure $\sigma \in H^1\,(P, Z_2)$, then $\sigma + \pi^*\,(b)$ where $b \in H^1\,(M, Z_2)$ is *the most general spin structure of* ξ. It follows that, *there is a bijection between the set of isomorphism classes of spin structures of* ξ *and* $H^1\,(M, Z_2)$. Consequently, a spin structure of ξ is unique (up to an isomorphism) iff $H^1(M, Z_2) = 0$.

Theorem 2.5. *Let $\xi_1 \oplus \xi_2$ be the Whitney sum of the vector bundles ξ_1 and ξ_2 over M. Given spin structures on two of the three vector bundles ξ_1, ξ_2, $\xi_1 \oplus \xi_2$, there is a uniquely determined spin structure on the third.*

Proof. As in J. M i l n o r [8].

Let M' be another manifold and let $f : M' \to M$ be a morphism of manifolds. Denote by $Pf\,(M', \pi', SO(H))$ the principal bundle induced from $P\,(M, \pi, SO\,(H))$ by the map f. This principal bundle is determined (up to an isomorphism) by the cocycle $g_{ij} \circ f$ associated to the open covering $\{f^{-1}\,(U_i)\}$, where g_{ij} is the cocycle of $P\,(M, \pi, SO(H))$ associated to an open covering $\{U_i\}$. Using the Remark 2.2 we obtain

Theorem 2.6. *Let be $f : M' \to M$. If M admits a spin structure $\Sigma\,(M, \tilde{\pi}, \text{Spin}\,(H))$ with respect to $P\,(M, \pi, SO\,(H))$; then $\Sigma\, f(M', \pi'$ Spin $(H))$ is a spin structure on M' with respect to $Pf\,(M', \pi', SO\,(H))$.*

Suppose now that $f : M' \to M$ is a principal bundle with group G. It follows that, there is an action of G on Pf defined by $(p', u)\, g = (p'g, u)$ for $(p', u) \in Pf$ and $g \in G$.

Theorem 2.7. *Let be $f : M' \to M$ a principal bundle with group G. Then the following conditions are equivalent:*

(1) *There exists a spin structure $\Sigma\,(M, \tilde{\pi}$, Spin $(H))$ on $P(M, \pi, SO(H))$.*

(2) *There exists a spin structure $\Sigma'(M', \tilde{\pi}'$, Spin $(H))$ on $Pf(M', \pi', SO(H))$ with the following properties.*

a) *G acts on Σ' such that Σ'/G is a manifold and the projection $\Sigma' \to \Sigma'/G$ is a submersion,*

b) *The action of G on Σ' commutes with the action of* Spin (H) *on Σ',*

c) *$\tilde{\rho}'\,(wg) = \tilde{\rho}'\,(w)\, g$ holds, for every $g \in G$ and $w \in \Sigma'$, where $\tilde{\rho}' : \Sigma' \to Pf$ is the covering map.*

Proof. (1) $\Rightarrow$ (2). By the Theorem 2.6, $\Sigma f\,(M', \tilde{\pi}'$, Spin $(H))$ is a spin structure on $Pf\,(M', \Pi', SO\,(H))$. Define an action of G on $\Sigma' = \Sigma f$ by $(p', v)\, g = (p'g, v)$, where $(p', v) \in \Sigma'$ and $g \in G$. This action is propre and free. Moreover, the map $g \to (p'g, v)$ is an immersion of G in Σ', since $f : M' \to M$ is a principal bundle. It follows that $\Sigma' \to \Sigma'/G$ is just a principal bundle (see B o u r b a k i [1]), hence the property a) is verified. From $(p', v)\, b = (p', vb)$ for $b \in$ Spin (H), it follows $(p'g, vb) = (p', v)\, gb = (p', v)\, bg$ i.e. the property b).

For $w = (p', v) \in \Sigma$, we have $\tilde{\rho}'\,(wg) = (p'g, \tilde{\rho}(v)) = \tilde{\rho}'\,(w)\, g$, i. e. the property c).

(2) $\Rightarrow$ (1) Define an action of Spin (H) on Σ'/G by $wGb = wbG$, where $w \in \Sigma'$, $b \in$ Spin (H) and wG is the orbit of w, and a surjection $h : \Sigma'/G \to M$ by $h\,(wG) = f\,(\tilde{\pi}'\,(w))$. The local isomorphism $t : f^{-1}(U) \times$ Spin $(H) \to \Sigma'$, where U is an open subset of M, defines a local isomorphism $s : U \times$ Spin $(H) \to \Sigma'/G$ by

$$s(p, b) = t\,(p'\,G, b) = t\,(p', b)\, G, \text{ where } f\,(p') = p. \tag{2.6}$$

The last equality from (2.6) is a consequence of

$$\begin{cases} \pi'(\tilde{\rho}'\,(wg)) = \pi'\,(\tilde{\rho}'\,(w))g \\ f^*(\tilde{\rho}'\,(wg)) = f^*\,\tilde{\rho}'\,(w) \end{cases} \qquad g \in G, w \in \Sigma'. \tag{2.7}$$

where $f^* : Pf \to P$ is the morphism induced by f. But (2.7) is equivalent to property c). It is not difficult to see that $h\,(s\,(p, b)) = p$ and $s\,(p, bb') = s(p, b)\, b'$, where $b' \in$ Spin (H), hence $h : \Sigma'/G \to M$ is a principal bundle with group Spin (H). The morphism $\tilde{\rho} : \Sigma'/G \to P$ defined by $\tilde{\rho}\,(wG) = f^*(\tilde{\rho}'(w))$ satisfies $\pi \circ \tilde{\rho} = h$ and $\tilde{\rho}\,(wGb) = \tilde{\rho}\,(wG)\, \rho\,(b)$, therefore $\Sigma'/G(M, h$, Spin $(H))$ is a spin structure on $P(M, \pi, SO(H))$.

Suppose that $f : M' \to M$ is a principal bundle with group Z_2. Let η denotes the involution of Pf defined by the action of Z_2 on it.

Corollary 2.7. *Let $f : M' \to M$ be a principal bundle with group Z_2. The following conditions are equivalent:*

(1) *There exists a spin structure $\Sigma\,(M, \tilde{\pi}$, Spin $(H))$ on $P(M, \pi, SO(H))$,*

(2) *There exists a spin structure $\Sigma'\,(M', \tilde{\pi}'$, Spin $(H))$ on $Pf\,(M', \pi', SO\,(H))$ endowed with an involution η' which corresponds to identity on* Spin (H) *and which commutes with the involution η.*

Proof. In order to apply the Theorem 2.7 it is sufficient to remark that η' defines an action of Z_2 on Σ', which commutes with the action of Spin (H), such that $\Sigma' \to \Sigma'/Z_2$ is a submersion and $\tilde{\rho}'\,(wZ_2) = \tilde{\rho}'\,(w)\, Z_2$ holds, for every $w \in \Sigma'$.

Remark 2.3. The above corollary has been obtained by I. P o p o v i c i [10] in non-orientable and finite dimensional case.

Definition 2.6. *Let N be a manifold modelled on H, with an oriented riemannian nuclear structure (resp. an oriented riemannian Hilbert-Schmidt structure). A spin structure on N is a spin structure on the tangent bundle TN.*

3. Examples. a) The trivial bundle $M \times SO(H)$ admits a spin structure, unique if M is simply connected.

b) Let $\left\{ x = (x_1, x_2, \ldots) / x_i \in R, \sum_{i=1}^{\infty} x_i^2 < \infty \right\}$ be the Hilbert space l_2. The Hilbert torus $T = l_2 / \sum_{i=1}^{\infty} Ze_i$, where $e_1, e_2, \ldots$ is a base of l_2, is a Hilbert-Lie group modelled on l_2. It admits a canonical analytic atlas such that the derivatives of the coordinate changes is always the identity on l_2. It follows that the corresponding riemannian nuclear structure is orientable and T with this structure, admits a spin structure (with respect to Spin $(l_2)_1$) since $w_2(T) = 0$.

c) Let $M \subset H$, be a smoothly imbedded manifold with an oriented riemannian nuclear structure. From Theorem 2.6, it follows that a spin structure on M determines a spin structure on the normal bundle to M and conversely.

4. Clifford bundles. In this section we limit our considerations to the group $SO(H)_1$. Let $Cl(H)$ be the Clifford algebra of H viewed as a C^*-algebra [3]. A*-automorphism ψ of $Cl(H)$ which satisfies $\psi(H) = H$ is called a Bogoliubov automorphism. Denote by Bog $(Cl(H))$ the group of Bogoliubov automorphisms and by $C : O(H) \to \text{Bog}(Cl(H))$ the canonical isomporhism described in [3]. A Bogoliubov automorphism ψ is said to be *inner* if there exists $u \in Cl(H)$ such that $\psi(v) = uvu^{-1}$ for every $v \in Cl(H)$ and is said to be *inner and even* if u is even. Let $S(M, O(H))$ be a principal bundle. Its associated fibre bundle with fibre $Cl(H)$ (the action of the group $O(H)$ on $Cl(H)$ is given by C) is an algebric bundle, called the *Clifford bundle*. We can obtain the Clifford bundle in another way. For this, let be $\xi : E \to M$ a Riemannian vector bundle. The fibres of ξ are Hilbert spaces. Let E_p be the fibre of ξ in $p \in M$ and let $Cl(E_p)$ be the Clifford algebra of E_p. The set $\bigcup_{p \in M} Cl(E_p)$ and the projection

$Cl(E_p) \to p$ can be taken as the total space and projection of the Clifford bundle. We remark that ξ can be identified with a subbundle of the Clifford bundle. Let Bog $(Cl(H))_i$ be the group of inner Bogoliubov automorphisms. The isomorphism $O(H)_1 \simeq$ Bog $(Cl(H)$ i (see [4]) implies the following

Theorem 4.1. *There exists a Riemannian nuclear structure on the Riemannian vector bundle $\xi : E \to M$ iff the Clifford bundle admits a reduction to* Bog $(Cl(H))_i$.

From the isomophism $SO(H)_1 \simeq$ Bog $(Cl(H))_{ie}$ (see [3]), where Bog $(Cl(H))_{ie}$ is the group of inner and even Bogoliubov automorphism, it follows

Theorem 4.2. *A Riemannian nuclear structure on the Riemannian vector bundle $\xi : E \to M$ is orientable iff the Clifford bundle admits a reduction to* Bog $(Cl(H))_{ie}$.

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