Proof: Since $u^0 = \Phi^0(x^0)$, the rank of the matrix M at P^0 is 4. Then in the equations: $b^r \xi^\alpha_r (x^0, u^0) = 0$, $b^r \eta_r (x^0, u^0) = 0$, 4 of the b's are expressible linearly and homogeneously in terms of the remaining 12. Hence, there are 12 linearly independent transformations of order greater than zero at P^0. Each of such transformations leaves the point P^0 invariant. Consequently, all these transformations generate the subgroup of stability of P^0.

In order to find the operators of this subgroup, we observe that the minor of M formed with first four rows is different from zero, that is: $\det \| \xi^\alpha_p (x^0, u^0), \eta_p (x^0, u^0) \| \neq 0$ $(p = \overline{1,4})$. Then, it comes out that the subgroup of stability of P^0 is generated by the operators:

$$U^0 = -(u - u^0)\partial_u, I^0 = (t - t^0) T + (x_j - x^0_j) X_j,$$

$$L^0_i = (x_i - x^0_i) T + c^2_*(t - t^0) X_i, \quad R^0_i = \xi_{ijk} (x_k - x^0_k) X_j, \tag{8.16}$$

$$P^0_0 = [-c^2_*(t^2 - t^{02}) + x_j x_j - x^0_j x^0_j] T + 2c^2_*(t - t^0) I + 2c^2_* t^0 I^0,$$

$$P^0_i = -\frac{1}{2} [-c^2_*(t^2 - t^{02}) + x_j x_j - x^0_j x^0_j] X_i + (x_i - x^0_i) I + x^0_i I^0.$$

Remark 3. The points $P^0(x^0, u^0)$ for which $u^0 \neq \Phi^0(x^0)$ are ordinary points. The subgroup of stability of such a point is of order 11. it is generated by the operators (8.16) without the operator U^0 and where I is replaced by $I + U$.

Remark 4. The group G^0 is transitive.

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Received 28.IX.1977

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Analele științifice ale Universității „Al. I. Cuza" Iași
Tomul XXIV, f. 2, s. I a, 1978

SHOCK WAVES IN NONLINEAR MICROPOLAR ELASTICITY

BY

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§ 1. **Introduction.** Propagation of shock waves in the classical nonlinear theory of elastic bodies was considered in [6].

In this paper we investigate the propagation of the shock waves in the nonlinear theory of hyperelastic Cosserat bodies. The amplitude of the wave must be an eigenvector of a nonlinear operator P, while the speed of propagation is the corresponding eigenvalue. We prove that the eigenvalues of the operator P are positive and the corresponding eigenvectors are real.

In the following H will be a covex subset of a real Banach space E and B a nonlinear mapping from H into E. The zero element of the Banach space will be denoted by θ.

A number μ_0 is said to be a bifurcation point for B, if for every $\varepsilon > 0$, $\delta > 0$ there exists a characteristic value μ and an eigenvector v ($v = \mu B(v)$) such that $|\mu - \mu_0| < \varepsilon$, $\|v\| < \delta$.

The set of eigenvectors is said to form a continuous branch passing through an eigenvector v_0 if every bounded open set in E containing v_0 possesses an eigenvector on its boundary.

We quote also a theorem which will be of interest for us.

Theorem 1.1. *If B is a compact and continuous operator and $B(\theta) = \theta$ then every characteristic value of odd multiplicity of $B(\theta)$ is a bifurcation point for B, corresponding to a continuous branch of eigenvectors passing through* θ ([3], p. 199).

§2. **Shock Waves.** We consider an anisotropic homogeneous Cosserat medium. The motion of a continuum with microstructures is represented by the mappings

$$y_i = y_i(x_k, t), \quad R_{ij} = R_{ij}(x_k, t), \tag{2.1}$$

where x_k and y_k are rectangular coordinates of the place occupied by the material point in the reference configuration and in the configuration at time t, respectively; R_{ij} is a proper orthogonal tensor which determines the position of directors at time t (see [2]).

We consider as strain tensors

$$c_{ij} = y_{k,i} R_{kj}, \quad \gamma_{ij} = \frac{1}{2} \varepsilon_{jmn} R_{kn} R_{km,i}, \tag{2.2}$$

and the angular velocity vector

(2.3) $$\nu_i = \frac{1}{2}\,\varepsilon_{ijk}\, R_{nk}\, \dot{R}_{nj}\,.$$

The deformation tensor γ_{ij} and the angular velocity vector ν_i may be represented in the form (cf. [4])

(2.4) $$\gamma_{ij} = H_{jk}\varphi_{k,i}\,, \qquad \nu_i = H_{ij}\,\dot{\varphi}_j\,,$$

where

$$H_{jm} = \frac{2\,\delta_{jm} + \varepsilon_{jmn}\,\varphi_n}{2\,(1 + 1/4\,\varphi_k\,\varphi_k)}$$

and the functions φ_i are the components of the microrotation vector.

A propagating surface $\Sigma\,(t)$ is said to be a shock wave if the functions y_i, φ_i are continuous everywhere, while their first order derivatives have jump discontinuities across $\Sigma\,(t)$ but are continuous functions everywhere else. The jump of a functions across $\Sigma\,(t)$ is denoted by $[z] = z^- - z^+$, where z^+ is the value of z immediately ahead of the wavefront and z^- is the value of z immediately behind it. The conditions of compatilibity for shock waves are ([1], Sect, 189)

(2.5) $$[z_{i,j}] = s_i\, N_j\,, \qquad [\dot{z}_i] = -\,G s_i\,, \qquad s_i = [z_{i,j}\, N_j],$$

where G is the speed of propagation of the wave, N_i are the components of the unit normal of $\Sigma\,(t)$.

We assume that at time $t = 0$, the wave has been propagating into a region which is at rest in a deformed configuration. Let the tensors $y^0_{i,j}$, R^0_{ij} characterize the deformed configuration.

As in [1] (p. 545) one shows that the momentum balance law and the moment of momentum balance law lead to the following jump conditions across the wavefront:

(2.6) $$[\tilde{t}_{ij}\, N_i] = -\,\rho G\,[\dot{y}_j], \qquad [\tilde{m}_{ij}\, N_i] = -\,\rho G\,[\sigma_j],$$

where $\tilde{t}_{ij}$, $\tilde{m}_{ij}$ are the components of the stress tensor and couple stress tensor at the point y_j measured per unit area in the reference configuration.

We have the relations [2]

(2.7) $$\tilde{t}_{ij} = R_{jk}\, t_{ik}\,, \qquad \tilde{m}_{ik} = R_{kj}\, m_{ij}\,, \qquad \sigma_i = R_{ij}\, I_{jk}\, \nu_k\,,$$
$$t_{ij} = \frac{\partial}{\partial c_{ij}}\, W\,(c_{kj}\,, \gamma_{kn}), \qquad m_{ij} = \frac{\partial}{\partial \gamma_{ij}}\, W\,(c_{kn}\,, \gamma_{jk}),$$

where I_{jk} are the components of the material tensor of inertia density, W is strain energy per unit volume in the reference configuration.

From (2.2) and (2.4), we obtain

(2.8) $$[c_{kj}] = a_j\, N_k\,, \qquad [\gamma_{kj}] = \tilde{a}_j\, N_k\,, \qquad [\nu_j] = -\,G\tilde{a}_j\,,$$

where

(2.9) $$a_j = R^0_{ij}\,[y_{i,k}\, N_k], \qquad \tilde{a}_i = H^0_{ij}\,[\varphi_{j,k}\, N_k].$$

Taking into account the relations (2.7), (2.8) and (2.9) the jump conditions (2.6) become:

(2.10) $$[t_{jk}\, N_j] = \rho G^2\, a_k\,, \qquad I^{-1}_{jk}\,[m_{ik}\, N_i] = \rho G^2\, \tilde{a}_j\,,$$

The vector $A = (A_k)$, $(k = \overline{1,6})$ defined by the relations $A_i = a_i$, $A_{i+3} = \tilde{a}_i$ $(i = \overline{1,3})$ will be called *the amplitude vector* of the shock wave.

We may write the relations (2.10) in the form

(2.11) $$P\,(A) = \rho\, G^2 A\,;$$

in the above relation P is a nonlinear mapping $P : E \to E$, where E is 6 — dimensional real Euclidian space.

The operator P is determined for each value of the unit normal of the wave by the strain energy of the material and by the deformation tensors c_{ij}, γ_{jk}. From (2.11) it follows:

Theorem 2.1. *The amplitude vector of a shock wave travelling in the direction of N is an eigenvector of the operator P corresponding to the eigenvalue $\rho\, G^2$.*

From (2.10) it follows that the operator P vanishes at zero.

The Lagrange formula implies the existence of c^*_{ij}, γ^*_{ij} such that

(2.12) $$[t_{ij}] = \frac{\partial t_{ij}\,(c^*_{qn}\,, \gamma^*_{rs})}{\partial c_{kp}}\, a_p\, N_k + \frac{\partial t_{ij}\,(c^*_{qn}\,, \gamma^*_{rs})}{\partial \gamma_{kp}}\, \tilde{a}_k\, N_p\,,$$
$$[m_{ij}] = \frac{\partial m_{ij}\,(c^*_{qn}\,, \gamma^*_{rs})}{\partial c_{kp}}\, a_p\, N_k + \frac{\partial m_{ij}\,(c^*_{qn}\,, \gamma^*_{rs})}{\partial \gamma_{kp}}\, \tilde{a}_p\, N_k\,.$$

Assuming that the second order derivatives of the function W are bounded and continuous, then from (2.12) it follows that the operator P is continuous and compact.

If the function W has third order continuous derivatives with respect to c_{ij} and γ_{ij}, we have

(2.13) $$[t_{ij}] = \frac{\partial t_{ij}\,(c^+_{qn}\,, \gamma^+_{rs})}{\partial c_{kp}}\, a_p\, N_k + \frac{\partial t_{ij}\,(c^+_{qn}\,, \gamma^+_{rs})}{\partial \gamma_{kp}}\, \tilde{a}_p\, N_k + T_{ij}\,,$$
$$[m_{ij}] = \frac{\partial m_{ij}\,(c^+_{qn}\,, \gamma^+_{rs})}{\partial c_{kp}}\, a_p\, N_k + \frac{\partial m_{ij}\,(c^+_{qn}\,, \gamma^+_{rs})}{\partial \gamma_{kp}}\, \tilde{a}_p\, N_k + M_{ij}\,,$$
$$\lim_{\|A\| \to 0} \frac{|\,T_{ij}\,|}{\|\,A\,\|} = 0, \qquad \lim_{\|A\| \to 0} \frac{|\,M_{ij}\,|}{\|\,A\,\|} = 0,$$

where

$$\|\,A\,\| = \sqrt{\sum_{k=1}^{6} A_k^2}\,.$$

From the relation (2.10) and (2.13) it results

(2.14) $$P'(0) = KL,$$

where the matrix $K = \|\,K_{ij}\,\|_{6\times 6}$ $L = \|\,L_{ij}\,\|_{6\times 6}$ are defined by

$$K_{ij} = \delta_{ij}\,, \quad K_{i+3;j} = 0, \quad K_{i;j+3} = 0, \quad K_{i+3;j+3} = I_{ij}^{-1}$$

$$L_{ij} = \frac{\partial t_{ki}\,(c_{qn}^{+}\,,\gamma_{rs}^{+})}{\partial c_{pj}}\, N_k\, N_p\,, \quad L_{i;j+3} = \frac{\partial t_{ki}\,(c_{qn}^{+}\,,\gamma_{rs}^{+})}{\partial \gamma_{pj}}\, N_p\, N_k\,,$$

$$L_{i+3;j} = \frac{\partial m_{ki}\,(c_{qn}^{+}\,,\gamma_{rs}^{+})}{\partial c_{pj}}\, N_k\, N_p, \quad L_{i+3;j+3} = \frac{\partial m_{ki}\,(c_{qn}^{+}\,,\gamma_{rs}^{+})}{\partial \gamma_{pj}}\, N_p\, N_k\,.$$

If the matrix L satisfies the condition

$$L_{jk}\,\lambda_j\,\lambda_k > 0. \tag{2.15}$$

for arbitrary non-vanishing vectors λ, then as in [5] one can prove that the eigenvalues of the operator $P'(\theta)$ are positive.

If the function W satisfies the relation

$$\frac{\partial^2 W\,(c_{kn}^{+}\,,\gamma_{kn}^{+})}{\partial c_{ij}\,\partial \gamma_{pq}}\,\eta_i\,\eta_p\,\lambda_j\,\lambda_q + 2\,\frac{\partial^2 W\,(c_{kn}^{+}\,,\gamma_{kn}^{+})}{\partial c_{ij}\,\partial \gamma_{pq}}\,\eta_i\,\eta_p\,\lambda_j\,\lambda_{q+3} + \\ + \frac{\partial^2 W\,(c_{kn}^{+}\,,\gamma_{kn}^{+})}{\partial \gamma_{ij}\,\partial \gamma_{pq}}\,\eta_i\,\eta_p\,\lambda_{j+3}\,\lambda_{q+3} > 0, \tag{2.16}$$

for all non-vanishing vectors $\lambda : (\lambda_k)\,(k = \overline{1,6}),\quad \eta = (\eta_i)\,(i = \overline{1,3})$, then the condition (2.15) is satisfied for every direction of propagation. From theorem 1.1 it follows

Theorem 2.2. *At a point of a deformed hyperelastic Cosserat body, for which the internal energy function satifies (2.16) and has continuous third order derivatives, the operator $P'(\theta)$ has characteristic value of odd multiplicity, the shock waves propagate in every directions.*

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Received 22.11.1978

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Analele științifice ale Universității „Al. I. Cuza" Iași
Tomul XXIV, f. 2, s. I a, 1978

ON THE SEMILINEAR THEORY FOR MATERIALS WITH INTERNAL STATE VARIABLES

BY

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Introduction. In the frame of nonlinear theories for materials with internal state variables, the acceleration waves are not, in general, symmetric with respect to the propagation direction and their symmetry is obtained only under additional hypotheses.

The present work considers materials with internal state variables, in the framework of [1], attention being focussed on the conditions under which one can obtain real and symmetric thermomechanically coupled acceleration waves as well as certain consequences of these hypotheses on the constitutive equations. Most of the introductory results are reproduced from [2] (see also [16], ch. V). The semilinear theory and the case of a material with no thermal dilatation (which case, from the point of view of the propagation of acceleration waves, may be called the weakly coupled case), have not been treated in [2]. The content of this work has been partially presented in several lectures held by I. Suliciu at the Summer School in Jablonna, Poland, the 3—8 may 1976.

The main assumptions used in the paper are the following:

a) The quasilinearity assumptions (6), leading to a quasilinear system of partial differential equations for the unknown functions.

b) Symmetry assumptions I and II, for the case when the stress — temperature moduli (or the thermal dilatation coefficients) form a nonsingular matrix (the instantaneously coupled case) and an additional hypothesis that strengthens the symmetry assumption II, when these moduli vanish (the weakly coupled case or the case of a material without thermal dilatation).

c) The assumption on the existence of strong equilibrium states.

d) The strong ellipticity condition, which represents here a natural extension of that used in the purely mechanical case.

In the frame of all these assumptions, in the neighborhood of a strong equilibrium state, one proves there are four real and symmetric possible acceleration waves and at least two of them carry both mechanical and thermal jumps (in the instantaneously coupled case); four real and symmetric acceleration waves are still possible in the weakly coupled case but three of them are mechanical ones and the last is a thermal one; they are uncoupled although the mechanical and thermal effects are not completely