$$K_{ij} = \delta_{ij}, \quad K_{i+3;j} = 0, \quad K_{i;j+3} = 0, \quad K_{i+3;j+3} = I_{ij}^{-1}$$

$$L_{ij} = \frac{\partial t_{ki}(c_{qn}^{+}, \gamma_{rs}^{+})}{\partial c_{pj}} N_k N_p, \quad L_{i;j+3} = \frac{\partial t_{ki}(c_{qn}^{+}, \gamma_{rs}^{+})}{\partial \gamma_{pj}} N_p N_k,$$

$$L_{i+3;j} = \frac{\partial m_{ki}(c_{qn}^{+}, \gamma_{rs}^{+})}{\partial c_{pj}} N_k N_p, \quad L_{i+3;j+3} = \frac{\partial m_{ki}(c_{qn}^{+}, \gamma_{rs}^{+})}{\partial \gamma_{pj}} N_p N_k.$$

If the matrix L satisfies the condition

$$L_{jk} \lambda_j \lambda_k > 0. \tag{2.15}$$

for arbitrary non-vanishing vectors λ, then as in [5] one can prove that the eigenvalues of the operator $P'(\theta)$ are positive.

If the function W satisfies the relation

$$\frac{\partial^2 W(c_{kn}^{+}, \gamma_{kn}^{+})}{\partial c_{ij}\, \partial c_{pq}} \eta_i \eta_p \lambda_j \lambda_q + 2\, \frac{\partial^2 W(c_{kn}^{+}, \gamma_{kn}^{+})}{\partial c_{ij}\, \partial \gamma_{pq}} \eta_i \eta_p \lambda_j \lambda_{q+3} + \frac{\partial^2 W(c_{kn}^{+}, \gamma_{kn}^{+})}{\partial \gamma_{ij}\, \partial \gamma_{pq}} \eta_i \eta_p \lambda_{j+3} \lambda_{q+3} > 0, \tag{2.16}$$

for all non-vanishing vectors $\lambda : (\lambda_k)\ (k = \overline{1,6})$, $\eta = (\eta_i)\ (i = \overline{1,3})$, then the condition (2.15) is satisfied for every direction of propagation. From theorem 1.1 it follows

Theorem 2.2. *At a point of a deformed hyperelastic Cosserat body, for which the internal energy function satifies (2.16) and has continuous third order derivatives, the operator $P'(\theta)$ has characteristic value of odd multiplicity, the shock waves propagate in every directions.*

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ON THE SEMILINEAR THEORY FOR MATERIALS WITH INTERNAL STATE VARIABLES

BY

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Introduction. In the frame of nonlinear theories for materials with internal state variables, the acceleration waves are not, in general, symmetric with respect to the propagation direction and their symmetry is obtained only under additional hypotheses.

The present work considers materials with internal state variables, in the framework of [1], attention being focussed on the conditions under which one can obtain real and symmetric thermomechanically coupled acceleration waves as well as certain consequences of these hypotheses on the constitutive equations. Most of the introductory results are reproduced from [2] (see also [16], ch. V). The semilinear theory and the case of a material with no thermal dilatation (which case, from the point of view of the propagation of acceleration waves, may be called the weakly coupled case), have not been treated in [2]. The content of this work has been partially presented in several lectures held by I. Suliciu at the Summer School in Jablonna, Poland, the 3—8 may 1976.

The main assumptions used in the paper are the following:

a) The quasilinearity assumptions (6), leading to a quasilinear system of partial differential equations for the unknown functions.

b) Symmetry assumptions I and II, for the case when the stress — temperature moduli (or the thermal dilatation coefficients) form a nonsingular matrix (the instantaneously coupled case) and an additional hypothesis that strengthens the symmetry assumption II, when these moduli vanish (the weakly coupled case or the case of a material without thermal dilatation).

c) The assumption on the existence of strong equilibrium states.

d) The strong ellipticity condition, which represents here a natural extension of that used in the purely mechanical case.

In the frame of all these assumptions, in the neighborhood of a strong equilibrium state, one proves there are four real and symmetric possible acceleration waves and at least two of them carry both mechanical and thermal jumps (in the instantaneously coupled case); four real and symmetric acceleration waves are still possible in the weakly coupled case but three of them are mechanical ones and the last is a thermal one; they are uncoupled although the mechanical and thermal effects are not completely uncoupled. In the same neighborhood, heat flux can be approximated by a function that depends on the internal state variables only.

Another assumption concerns the internal state variables: they are chosen to be frame indifferent but at least a certain number of them are not invariant under the isotropy group of the material. An illustration of these results is given by means of the linear and the semilinear isotropic cases.

In the case of the isotropic semilinear theory, the structure of acceleration waves is the same as for the linear case. The heat flux constitutive equation may be nonlinear while the evolution equations for the scalar and tensorial internal state variables do not depend on the temperature gradient.

If one assumes that internal state variables are behaving as the components of a vector and a second-order tensor under the isotropy group then, the isotropic linear theory includes the Maxwell viscoelastic theory where for heat flux one uses Cattaneo constitutive equation. The semilinear theory attached to this case will then include the Sokolovski-Malvern-Perzyna theory for elastic-viscoplastic materials.

Symmetric acceleration waves. 1. Consider a body $\mathcal{B}$. Its motion with respect to a reference configuration $\mathcal{R}$ is described by the function $\mathbf{x} = \chi(\mathbf{X}, t)$ that gives the coordinates $\mathbf{x}$ at the moment t of that point which had coordinates $\mathbf{X}$ in the reference configuration.

Balance of mass, momentum and energy are then

$$(1)\qquad \rho_0 = \rho j,\ \operatorname{Div}\tilde{\mathbf{S}} + \rho_0\mathbf{b} = \rho_0\ddot{\mathbf{x}},$$

$$\rho_0\dot{\Psi} + \rho_0(\dot{\theta}\eta + \theta\dot{\eta}) - \tilde{\mathbf{S}}.\dot{\mathbf{F}} + \operatorname{Div}\tilde{\mathbf{q}} = \rho_0 r,$$

where the notations are the following

$$(2)\qquad \mathbf{F} = \operatorname{Grad}\chi,\ j = \det\mathbf{F},\ \ddot{\mathbf{x}} = \frac{\partial^2\chi}{\partial t^2},$$

$$\tilde{\mathbf{S}} = j\mathbf{T}(\mathbf{F}^{-1})^T,$$

and ρ_0 and ρ are the densities in the reference and actual configurations respectively, $\mathbf{T}$ is the Cauchy stress tensor, θ is temperature, $\Psi = e - \theta\eta$ is the free energy and e and η are the internal energy and entropy respectively, $\mathbf{q}$ is the heat flux, $\tilde{q} = j\mathbf{F}^{-1}\mathbf{q}$ while $\mathbf{b}$ and r are the body forces and the heat supply respectively.

The second law of thermodynamics is expressed by the Clausius-Duhem inequality

$$(3)\qquad -\dot{\Psi} - \eta\dot{\theta} + \frac{1}{\rho_0}\tilde{\mathbf{S}}.\dot{\mathbf{F}} - \frac{1}{\rho_0\theta}\tilde{\mathbf{q}}.\mathbf{g}_0 \geq 0$$

with

$$\mathbf{g}_0 = \mathbf{F}^T\mathbf{g},\ \mathbf{g} = \operatorname{grad}\theta,\ \left(g_i = \frac{\partial\theta}{\partial x_i}\right).$$

Constitutive equations are assumed to have the form

$$(4)\qquad \Psi = \hat{\Psi}(\mathbf{F}, \theta, \mathbf{g}, \alpha),\ \eta = \hat{\eta}(\mathbf{F}, \theta, \mathbf{g}, \alpha),\ \tilde{\mathbf{S}} = \hat{\mathbf{S}}(\mathbf{F}, \theta, \mathbf{g}, \alpha),$$

$$\tilde{\mathbf{q}} = \hat{\mathbf{q}}(\mathbf{F}, \theta, \mathbf{g}, \alpha),\ \dot{\alpha} = \mathbf{f}(\mathbf{F}, \theta, \mathbf{g}, \alpha),$$

where $\alpha = (\alpha_1, \ldots, \alpha_N)$ is the vector of internal state variables and the functions $\hat{\Psi}, \hat{\eta}, \hat{\mathbf{S}}, \hat{\mathbf{q}}, \mathbf{f}$ are as smooth as required.

The restrictions imposed by the Clausius-Duhem inequality on the constitutive equations (4) are the following (C o l e m a n and G u r t i n [1])

$$(5)\qquad \Psi = \hat{\Psi}(\mathbf{F}, \theta, \alpha),\quad \eta = -\frac{\partial\hat{\Psi}}{\partial\theta},\quad \tilde{\mathbf{S}} = \rho_0\frac{\partial\hat{\Psi}}{\partial\mathbf{F}}.$$

We now lay down two additional constitutive assumptions (M i h ă i l e s c u and S u l i c i u [2]), called the quasilinearity assumptions

$$(6)\qquad \frac{\partial\hat{\mathbf{q}}}{\partial\mathbf{g}_0} = 0,\ \mathbf{f} = \mathbf{A}(\mathbf{F}, \theta, \alpha)\,\mathbf{g}_0 + \mathbf{b}(\mathbf{F}, \theta, \alpha).$$

Then, inequality (3) requires that

$$(7)\qquad \hat{\mathbf{q}}(\mathbf{F}, \theta, \alpha) = -\rho_0\theta\mathbf{A}^T\frac{\partial\hat{\Psi}}{\partial\alpha},\quad \frac{\partial\hat{\Psi}}{\partial\alpha}.\mathbf{b} \leqslant 0.$$

The internal state variables α are chosen to be frame indifferent but at least a certain number of these variables are not invariant under the isotropy group of the material.

2. A regular surface $\varphi(\mathbf{X}, t) = 0$ with $\mathbf{X} \in \mathcal{R}$ and $t \in R$ is called an acceleration wave if $\dot{x}$, $\mathbf{F}$, θ and α are continuous but their derivatives with respect to $\mathbf{X}$ and t may have jumps when crossing this surface. One assumes that $\mathbf{b}$ and r have no jumps across the acceleration wave.

In this case, the geometric and kinematic compatibility conditions will be (see for instance [3])

$$\left[\frac{\partial^2\chi_k}{\partial t^2}\right] = U^2a_k,\quad \left[\frac{\partial F_{kl}}{\partial X_j}\right] = a_kn_jn_l,\quad \left[\frac{\partial F_{ij}}{\partial t}\right] = -Ua_in_j,$$

$$(8)\qquad \left[\frac{\partial\theta}{\partial t}\right] = -U\nu,\quad \left[\frac{\partial\theta}{\partial X_j}\right] = \nu n_j,$$

$$\left[\frac{\partial\alpha_i}{\partial t}\right] = -U\gamma_i,\quad \left[\frac{\partial\alpha_i}{\partial X_j}\right] = \gamma_in_j,$$

where

$$(9)\qquad n_i = \frac{\dfrac{\partial\varphi}{\partial X_i}}{|\operatorname{Grad}\varphi|},\quad U = -\frac{\dfrac{\partial\varphi}{\partial t}}{|\operatorname{Grad}\varphi|},$$

and the vector $\mathbf{a} = (a_1\,a_2\,a_3)$ is called the mechanical amplitude of the wave, the scalar ν is the thermal amplitude and the N-dimensional vector $\gamma = (\gamma_1, \ldots, \gamma_N)$

is called the internal state amplitude of the wave. Writing now the dynamic compatibility conditions and using (5), (7) and (8), one obtains ([2])

$$(\mathbf{Q} - U^2\mathbf{I})\,\mathbf{a} + \mathbf{P}\nu + \mathbf{H}\gamma = \mathbf{0}, \tag{10}$$

$$(\rho_0\theta U\mathbf{P} + \mathbf{E})\,\mathbf{a} + (L - Uv)\,\nu + (U\mathbf{G} + \mathbf{K})\,\gamma = 0,\ \mathbf{M}\nu + U\gamma = \mathbf{0},$$

where the following notations were introduced

$$Q_{ik} = \frac{\partial^2\hat{\Psi}}{\partial F_{ij}\,\partial F_{kl}}\,n_j n_l,\quad v = -\rho_0\theta\,\frac{\partial^2\hat{\Psi}}{\partial\theta^2},\quad P_i = \frac{\partial^2\hat{\Psi}}{\partial\theta\,\partial F_{ij}}\,n_j,$$

$$E_i = \frac{\partial\hat{q}_k}{\partial F_{il}}\,n_k n_l,\quad H_{ik} = \frac{\partial^2\hat{\Psi}}{\partial F_{ij}\partial\alpha_k}\,n_j,\quad K_k = \frac{\partial\hat{q}_i}{\partial\alpha_k}\,n_i, \tag{11}$$

$$G_k = \rho_0\theta\,\frac{\partial^2\hat{\Psi}}{\partial\theta\,\partial\alpha_k} - \rho_0\,\frac{\partial\hat{\Psi}}{\partial\alpha_k},\quad L = \frac{\partial\hat{q}_i}{\partial\theta}\,n_i,\quad M_k = A_{kj}n_j.$$

Since, for a fixed state (F, θ, α) and a given direction of propagation $\mathbf{n}$, relations (10) represent an homogeneous linear system for the $N + 4$ unknowns: $\mathbf{a}$, ν and γ, non-trivial solutions will exist if and only if the determinant of the system vanishes. This condition leads to an algebraic equation of degree $N + 7$, for the speed of propagation U:

$$\begin{aligned}
&U^{N-1}\{-vU^8 + wU^7 + (vI_Q + z + \rho_0\theta\mathbf{P}\cdot\mathbf{P})\,U^6 + [-wI_Q + \mathbf{P}\cdot\\
&\cdot(\mathbf{E} - \rho_0\theta\mathbf{D})\,]\,U^5 - (vII_Q + zI_Q + \mathbf{D}\cdot\mathbf{E} + \rho_0\theta I_Q\,\mathbf{P}\cdot\mathbf{P} - \rho_0\theta\mathbf{QP}\cdot\mathbf{P})U^4 +\\
&+ [wII_Q + I_Q\mathbf{P}\cdot(\rho_0\theta\mathbf{D} - \mathbf{E}) + \mathbf{QP}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D})\,]\,U^3 + [vIII_Q +\\
&+ zII_Q + I_Q\mathbf{D}\cdot\mathbf{E} - \mathbf{QD}\cdot\mathbf{E} - \rho_0\theta I_Q\mathbf{QP}\cdot\mathbf{P} + \rho_0\theta II_Q\mathbf{P}\cdot\mathbf{P} +\\
&+ \rho_0\theta\mathbf{Q}^2\mathbf{P}\cdot\mathbf{P}]\,U^2 + [-wIII_Q + II_Q\mathbf{P}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D}) + I_Q\mathbf{QP}\cdot\\
&\cdot(\rho_0\theta\mathbf{D} - \mathbf{E}) + \mathbf{Q}^2\mathbf{P}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D})]\,U - zIII_Q + I_Q\mathbf{QD}\cdot\mathbf{E} -\\
&II_Q\mathbf{D}\cdot\mathbf{E} - (\mathbf{Q}^2\mathbf{D})\cdot\mathbf{E}\} = 0,
\end{aligned} \tag{12}$$

where

$$\mathbf{D} = \mathbf{HM},\ z = -\mathbf{K}\cdot\mathbf{M},\ w = L - \mathbf{G}\cdot\mathbf{M} \tag{13}$$

and I_Q, II_Q, III_Q, are the invariants of $\mathbf{Q}$.

From the physical point of view one expects to obtain symmetric acceleration waves, i.e. one expects equation (12) to have symmetric roots, which does not happen as our conditions are still too general. We therefore lay down two new constitutive assumptions:

Symmetry assumption I. (Suliciu [4], Mihăilescu and Suliciu [2]). For any fixed state $(\mathbf{F}, \theta, \alpha)$ and any fixed direction of propagation $\mathbf{n}$, equation (12) for the speeds of propagation, must admit symmetric roots.

The symmetry assumption I is satisfied if and only if

$$w = 0,\ \mathbf{P}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D}) = 0,\ \mathbf{QP}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D}) = 0,\ \mathbf{Q}^2\mathbf{P}\cdot(\mathbf{E} - \rho_0\theta\mathbf{D}) = 0, \tag{14}$$

for any state $(\mathbf{F}, \theta, \alpha)$ and any n.

According to this hypothesis, equation (12) can be rewritten, if one leaves appart the $N - 1$ zero roots, as an algebraic equation of degree 4 for $Z = U^2$.

Symmetry assumption II (Mihăilescu and Suliciu [2]). For a real acceleration wave that travels in the direction n with the mechanical amplitude $\mathbf{a}$, U is a possible speed of propagation if and only if $-U$ is.

This assumptions is verified in the case of a nonlinear elastic material and it is extended here for materials with internal state variables.

3. A state $(\mathbf{F}^*, \theta^*, \mathbf{g}^* = o, \alpha^*)$ is called an equilibrium state if

$$\mathbf{f}^* = \mathbf{f}(\mathbf{F}^*, \theta^*, 0, \alpha^*) = \mathbf{0}. \tag{15}$$

An equilibrium state that satisfies in addition

$$\frac{\partial\hat{\Psi}^*}{\partial\alpha_k} = \frac{\partial\hat{\Psi}}{\partial\alpha_k}(\mathbf{F}^*, \theta^*, \alpha^*) = \mathbf{0}, \tag{16}$$

is called a strong equilibrium state (see Coleman and Gurtin [1], Bowen [5], Truesdell [6]). In the following we shall use a star as upper index to denote a quantity at a strong equilibrium state.

Relation $(7)_1$ implies, at a strong equilibrium state

$$\begin{aligned}
&\tilde{\mathbf{q}}^* = \tilde{\mathbf{q}}(\mathbf{F}^*, \theta^*, \alpha^*) = o,\ \frac{\partial\tilde{\mathbf{q}}^*}{\partial\theta} + \rho_0\theta^*\mathbf{A}^{*T}\,\frac{\partial^2\hat{\Psi}^*}{\partial\alpha\partial\theta} = o,\ \frac{\partial\tilde{\mathbf{q}}^*}{\partial\mathbf{F}} +\\
&+ \rho_0\theta^*\mathbf{A}^{*T}\,\frac{\partial^2\hat{\Psi}^*}{\partial\alpha\,\partial\mathbf{F}} = o,\ \frac{\partial\tilde{\mathbf{q}}^*}{\partial\alpha} + \rho_0\theta^*\mathbf{A}^{*T}\frac{\partial^2\hat{\Psi}^*}{\partial\alpha^2} = o.
\end{aligned} \tag{17}$$

We now strengthen the above conditions with an elliptic type inequality ([2]):

The functions $\hat{\Psi}$ and f have to be defined such that the inequality

$$\begin{aligned}
&\hat{\Psi}(\mathbf{F}, \theta, \alpha) - \hat{\Psi}(\tilde{\mathbf{F}}, \tilde{\theta}, \tilde{\alpha}) - (\theta - \tilde{\theta})\,\frac{\partial\hat{\Psi}}{\partial\theta}(\mathbf{F}, \theta, \alpha) -\\
&\mathrm{tr}\left\{(\mathbf{F} - \tilde{\mathbf{F}})^T\,\frac{\partial\hat{\Psi}}{\partial F}(\tilde{\mathbf{F}}, \tilde{\theta}, \tilde{\alpha})\right\} - (\alpha - \tilde{\alpha})\,\frac{\partial\hat{\Psi}}{\partial\alpha}(\tilde{\mathbf{F}}, \tilde{\theta}, \tilde{\alpha}) > 0,
\end{aligned} \tag{18}$$

be verified for any $(\mathbf{F}, \theta, \alpha)$ and $(\tilde{\mathbf{F}}, \tilde{\theta}, \tilde{\alpha})$, with

$$\begin{aligned}
&(\mathbf{F}, \theta, \alpha) \neq (\tilde{\mathbf{F}}, \tilde{\theta}, \tilde{\alpha}),\\
&\mathbf{F} = \tilde{\mathbf{F}} + \mathbf{a}\otimes\mathbf{b},\ \theta = \lambda + \tilde{\theta},\ \alpha = \tilde{\alpha} + \mathbf{f}_g(\tilde{\mathbf{F}}, \tilde{\theta}, o, \tilde{\alpha})\,\mathbf{c},
\end{aligned} \tag{19}$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c} \in R^3$ and $\lambda \in R$ satisfy the conditions

$$(\det\tilde{F})\,\det\,(\tilde{F} + a\otimes b) > 0,\ \lambda + \tilde{\theta} > 0. \tag{20}$$

Such types of conditions have been frequently used in the literature for other classes of materials (see for instance T r u e s d e l l and N o l l [7] for the ellipticity condition).

Two immediate consequences of the strict inequality (18), are the following : $\mathbf{Q}$ is a positive definite tensor and

$$v > 0. \tag{21}$$

At a strong equilibrium state, condition (18) leads to (see [2])

$$z^* > 0, \quad \frac{\partial \hat{\mathbf{q}}^*}{\partial \theta} = 0, \quad \mathbf{A}^{*T} \frac{\partial^2 \hat{\mathbf{H}}^*}{\partial \alpha \partial \theta} = 0, \tag{22}$$
$$\mathbf{E}^* = -\rho_0 \theta^* \mathbf{D}^*, \quad \mathbf{P}^* . \mathbf{D}^* = 0 .$$

4. We state now one of the main results obtained in [2] :

If one assumes that, for any strong equilibrium state and any propagation direction,

$$\mathbf{P}^* \neq 0 ,$$

then, in the neighborhood of a strong equilibrium state, under the symmetry assumptions I and II and the strong ellipticity hypothesis, all acceleration waves are real and symmetric and at least two of them carry thermal jumps.

In particular, under the above conditions there results that, at a strong equilibrium state, the thermal amplitude has the same symmetry property as the mechanical amplitude.

Let us now consider the case, that was not considered in [2], when

$$\mathbf{P}^* = 0 , \tag{23}$$

for any propagation direction $\mathbf{n}$, at a given strong equilibrium state.

If $\mathbf{P}^*$ vanishes at a strong equilibrium state, then $(\partial^2 \hat{\Psi}^*)/\partial\theta\partial\mathbf{F} = 0$; in the frame of a linearized theory in the neighborhood of the strong equilibrium state, this condition implies the fact that temperature variation has no influence on stress. In other words, the condition $\mathbf{P}^* = 0$ describes an instantaneously un coupled phenomenon at the given strong equilibrium state.

Let us examine the structure of the acceleration waves at a strong equilibrium state, when (23) is verified. One observes from (14) that symmetry assumption I is not completely satisfied as w can remain different form zero ; hence, this hypothesis is still necessary.

The system (10) becomes

$$(v^* U^{*2} - z^*)\nu = U^* \mathbf{E}^* . \mathbf{a}, \quad U^{*2}\mathbf{a} - \mathbf{Q}^*\mathbf{a} = -\frac{\nu}{U^*}\mathbf{D}^*, \tag{24}$$

while the equation (12) giving the speeds of propagation, will be

$$\begin{gathered} v^* Z^4 - (v^* I_{Q^*} + z^*) Z^3 + [v^* II_{Q^*} + z^* I_{Q^*} - \rho_0\theta^* \mathbf{D}^* \cdot \mathbf{D}^*] Z^2 - \\ [v^* III_{Q^*} + z^* II_{Q^*} - \rho_0\theta^* \mathbf{D}^* . \mathbf{D}^* + \rho_0\theta^* \mathbf{Q}^* \mathbf{D}^* . \mathbf{D}^*] Z + z^* III_{Q^*} + \\ + \rho_0\theta^* I_{Q^*} \mathbf{Q}^* \mathbf{D}^* . \mathbf{D}^* - \rho_0\theta^* II_{Q^*} \mathbf{D}^* . \mathbf{D}^* - \rho_0\theta^* \mathbf{Q}^{*2} \mathbf{D}^* . \mathbf{D}^* = 0, \end{gathered} \tag{25}$$

where $Z = U^2$. In the proof of the above stated result (for $\mathbf{P}^* \neq 0$) one finds, without using the asumption $\mathbf{P}^* \neq 0$, that equation (12) for $Z = U^2$ has real roots. Hence, it only remains to see what conditions will ensure their positiveness too.

Since $v^* > 0$, $z^* > 0$, $I_{Q^*} > 0$, at least one root is positive. For such a root, the symmetry assumption II used in (24), leads to

$$(\nu + \tilde{\nu})\mathbf{D}^* = 0, \quad (v^* U_4^{*2} - z^*)(\nu + \tilde{\nu}) = 0. \tag{26}$$

If $\mathbf{D}^* = \mathbf{0}$ for a given direction of propagation $\mathbf{n}$, hence $\mathbf{E}^* = \mathbf{0}$ (from (22)), the roots of equation (25) will be : $Z_i = \lambda_i^*$, $i = 1, 2, 3$, where λ_i^* are the eigenvalues of $\mathbf{Q}^*$ and $Z_4 = z^*/v^*$ and therefore all acceleration waves are real.

If $Z_4 = z^*/v^*$ is different from all three eigenvalues λ_i^*, the solutions of the system (24) are : $(a_i, 0)$ for $Z_i = \lambda_i^*$, $i = 1, 2, 3$, and $(0, \nu)$ for Z_4. In other words, one obtains uncoupled acceleration waves, three of them being mechanical waves of speeds $U_i^* = \pm\sqrt{\lambda_i^*}$ and amplitudes $(a_i, 0)$, $i = 1, 2, 3$ while the last one is a thermal wave of speed $\pm\sqrt{z^*/v^*}$ and amplitude $(0, \nu)$.

If $\mathbf{P}^* = 0$, the quasilinearity assumptions together with the symmetry assumptions and the strong ellipticity condition do not generally lead to the conclusion that $\mathbf{D}^* = 0$ for any direction of propagation $\mathbf{n}$; on the other hand, if $\mathbf{D}^* \neq 0$, it does not generally follow that equation (25) has only positive roots. This time, the symmetry assumption II is no more sufficient to ensure the positiveness of the roots since it refers to the mechanical amplitude only while the condition $\mathbf{P}^* = 0$, as we already pointed out before, describes an instantaneously uncoupled phenomenon ; therefore the symmetry of the thermal amplitudes will no more follow. But, if we strengthen the symmetry assumption II by requiring that thermal amplitude be symmetric too (i.e. $\nu = \tilde{\nu}$) then $\mathbf{D}^*$ must vanish for any propagation direction and one reaches the above result. This analysis shows that symmetry assumption II is properly chosen. Moreover, it represents another argument that the whole assumption frame is a natural one for the wave propagation problems in materials with internal state variables.

Let us remark that $\mathbf{P}^* = o$ does not imply a complete uncoupling of the mechanical and thermal effects as coupling is still performed by means of the evolution equations for the internal state variables ; however, acceleration waves propagate in an uncoupled way.

Heat flux in the neighborhood of a strong equilibrium state. If one applies the principle of frame indifference to the heat flux constitutive equation, one obtains at a strong equilibrium state ([2])

$$\frac{\partial \hat{\mathbf{q}}^*}{\partial \mathbf{F}} = 0 . \tag{27}$$

Thus, we can state another important result of [2] :

In a neighborhood of a strong equilibrium state, heat flux can be approximated by a function that depends on the internal state variables only.

Conversely, if at a strong equilibrium state, the partial derivatives of $\hat{q}$ with respect to $\mathbf{F}$ and θ are vanishing, then, for that strong equilibrium state, the symmetry assumptions I and II are satisfied.

The linear theory. Let us consider, as an illustration to the above results, the isotropic linear theory for small deformations.

We did assume from the beginning that at least some of the internal state variables are not invariant under the isotropy group of the material. In [2], the isotropic linear case is treated under the assumption that $\alpha = (\alpha_1, \dots, \alpha_N)$ contains two groups of components:

$$\alpha = (\beta, \alpha),$$

where $\beta = (\alpha_1, \alpha_2, \alpha_3)$ is behaving as $\tilde{q}$ and $\alpha = (\alpha_4, \dots, \alpha_N)$ are invariant under the isotropy group. One will consider here that the vector of internal state variables contains three groups of components with respect to their behaviour under the isotropy group: 1) a group $\bar{\alpha} = (\alpha_{10}, \dots, \alpha_N)$ containing scalar variables, 2) a group $\beta = (\alpha_1, \alpha_2, \alpha_3)$ that behaves as a three-dimensional vector and 3) a group $\tilde{\alpha} = (\alpha_4, \dots, \alpha_9)$ of six components that behaves as a symmetric second-order tensor, i.e.

$$\alpha = (\beta, \tilde{\alpha}, \alpha). \tag{28}$$

Let $(\mathbf{F} = I,\ \theta = \theta_0,\ \mathbf{g} = 0,\ \alpha = 0)$ be a strong equilibrium state with the property

$$\hat{\Psi}^* = 0,\ \eta^* = 0,\ \tilde{\mathbf{S}}^* = 0. \tag{29}$$

One assumes that

$$(\varepsilon_{ij}\varepsilon_{ij})^{1/2} \ll 1,\ |\vartheta| \ll 1,\ |\alpha| \ll 1, \tag{30}$$

where $\vartheta = (\theta - \theta_0)/\theta_0$ and $\varepsilon = 1/2\,(\mathbf{F} + \mathbf{F}^T - 2\mathbf{I})$ is the symmetric small deformations tensor.

The principle of frame indifference requires that $\hat{\Psi}(\mathbf{F}, \theta, \alpha) = \tilde{\Psi}(\mathbf{C}, \theta, \alpha)$, where $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ and, according to the linearity hypothesis, $\mathbf{C}$ is approximated by its linear part, that is $\mathbf{I} + 2\varepsilon$. On the other hand, isotropy will require $\tilde{\Psi}(\mathbf{C}, \theta, \alpha)$ to be an isotropic function of two tensors: $\mathbf{C}$ and $\tilde{\alpha}$ and a vector β. Then, the hypotheses (30) together with the representation theorem for isotropic scalar functions (see for instance Wang [20], [21], [22]), lead to the following expression for the free energy, in the neighborhood of the considered strong equilibrium state:

$$\begin{aligned}\rho_0\check{\Psi}(\varepsilon, \vartheta, \beta, \tilde{\alpha}, \alpha) &= \frac{\lambda}{2}(\operatorname{tr}\varepsilon)^2 + \mu\operatorname{tr}(\varepsilon^2) - \varkappa\vartheta(\operatorname{tr}\varepsilon) + \Lambda_i\alpha_i\operatorname{tr}\varepsilon + \\ &+ \Lambda'\operatorname{tr}(\varepsilon\tilde{\alpha}) + \Lambda''(\operatorname{tr}\varepsilon)(\operatorname{tr}\tilde{\alpha}) + \Delta'(\operatorname{tr}\tilde{\alpha})^2 + \Delta''\operatorname{tr}(\tilde{\alpha}^2) + \\ &+ \Gamma\vartheta\operatorname{tr}\tilde{\alpha} + \Gamma_i\vartheta\alpha_i + \Delta_i\alpha_i\operatorname{tr}\tilde{\alpha} - \frac{1}{2}\tau\theta_0\vartheta^2 + \zeta\beta\cdot\beta + \Omega_{ij}\bar{\alpha}_i\bar{\alpha}_j.\end{aligned} \tag{31}$$

In the expression (31), λ and μ are Lamé constants, $\varkappa/\theta_0$ is the stress-temperature modulus, v/α_0 is the heat capacity and $\Lambda', \Lambda'', \Delta', \Delta'', \Gamma, \zeta, \Lambda_i, \Gamma_i, \Delta_i$ and Ω_{ij} are defined as follows

$$\begin{aligned}&2\zeta\delta_{kl} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\beta_k\partial\beta_l},\quad \Gamma\delta_{ij} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\tilde{\alpha}_{ij}\partial\theta},\quad 2\Delta'\delta_{ij}\delta_{kl} + 2\Delta''\delta_{ik}\delta_{jl} = \\ &= \rho_0\frac{\partial^2\psi^*}{\partial\tilde{\alpha}_{ij}\partial\tilde{\alpha}_{hl}},\quad \frac{1}{2}\Lambda'(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) + \Lambda''\delta_{ij}\delta_{kl} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\tilde{\alpha}_{ij}\partial F_{kl}}, \\ &\Gamma_k = \rho_0\theta_0\frac{\partial^2\hat{\psi}^*}{\partial\bar{\alpha}_k\partial\theta},\quad \Lambda_k\sigma_{ij} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\bar{\alpha}_k\partial F_{ij}},\quad \Delta_k\delta_{ij} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\bar{\alpha}_k\partial\tilde{\alpha}_{ij}}, \\ &\Omega_{kl} = \rho_0\frac{\partial^2\hat{\psi}^*}{\partial\bar{\alpha}_k\partial\bar{\alpha}_l}.\end{aligned} \tag{32}$$

Relations $(5)_{1,2}$ will determine the stress and entropy

$$\tilde{S}_{ij} = \lambda(\operatorname{tr}\varepsilon)\delta_{ij} + 2\mu\varepsilon_{ij} - \varkappa\vartheta\delta_{ij} + \Lambda_k\bar{\alpha}_k\delta_{ij} + \Lambda'\tilde{\alpha}_{ij} + \Lambda''(\operatorname{tr}\tilde{\alpha})\delta_{ij} \tag{33}$$

$$\rho_0\theta_0\eta = \varkappa(\operatorname{tr}\varepsilon) - \Gamma\operatorname{tr}\tilde{\alpha} + v\theta_0\vartheta - \Gamma_i\bar{\alpha}_i, \tag{34}$$

while the internal energy is given by

$$\begin{aligned}\rho_0 e &= \frac{\lambda}{2}(\operatorname{tr}\varepsilon)^2 + \mu\operatorname{tr}(\varepsilon^2) + \Lambda_i\bar{\alpha}_i\operatorname{tr}\varepsilon + \Lambda'\operatorname{tr}(\varepsilon\tilde{\alpha}) + \Lambda''(\operatorname{tr}\varepsilon)(\operatorname{tr}\tilde{\alpha}) + \\ &+ \Delta'(\operatorname{tr}\tilde{\alpha})^2 + \Delta''\operatorname{tr}(\tilde{\alpha}^2) + \Delta_i\bar{\alpha}_i\operatorname{tr}\tilde{\alpha} + \zeta\beta\cdot\beta + \Omega_{ij}\bar{\alpha}_i\bar{\alpha}_j + \\ &+ \frac{1}{2}v\theta_0\vartheta(2 + \vartheta) + \varkappa\operatorname{tr}\varepsilon - \Lambda\operatorname{tr}\tilde{\alpha} - \Gamma_i\bar{\alpha}_i.\end{aligned} \tag{35}$$

A previous result stated that in the neighborhood of a strong equilibrium state, the heat flux can be considered as depending on the internal state variables only. Since $\tilde{q}$ is an isotropic vector function of a vector β and a symmetric tensor $\tilde{\alpha}$, in the present linear frame one gets

$$\tilde{q} = -\gamma\beta,\ \gamma = \text{const.} \tag{36}$$

According to relation (36), $\tilde{q}$ would obviously be identical zero (i.e. the material would be a nonconductor) if among the internal state variables there were only scalars and tensors.

Let us now consider the evolution equations $(4)_5$, written as

$$\dot{\bar{\alpha}} = \bar{f}(\varepsilon, \vartheta, g_0, \beta, \tilde{\alpha}, \bar{\alpha}),\ \dot{\beta} = \hat{f}(\varepsilon, \vartheta, g_0, \beta, \tilde{\alpha}, \bar{\alpha}),\ \dot{\tilde{\alpha}} = \tilde{f}(\varepsilon, \vartheta, g_0, \beta, \tilde{\alpha}, \bar{\alpha}), \tag{37}$$

where $\bar{f}$, $\hat{f}$ and $\tilde{f}$ have to be isotropic linear scalar, vector and tensor functions respectively, of two vectors and two symmetric second-order tensors. Then (see [20]),

$$\dot{\bar{\alpha}}_j = \bar{A}_j^1 \operatorname{tr} \varepsilon + \bar{A}_j^2 \operatorname{tr} \tilde{\alpha} + \bar{A}_j^3 \vartheta + \bar{A}_{jk}^4 \bar{\alpha}_k, \quad \dot{\beta} = \mathbf{A}\mathbf{g}_0 + \delta\beta$$
$$(38) \qquad \dot{\tilde{\alpha}} = \tilde{A}_1 \varepsilon + \tilde{A}_2 \tilde{\alpha} + (\tilde{A}_3 \vartheta + \tilde{A}_4 \operatorname{tr} \varepsilon + \tilde{A}_5 \operatorname{tr} \tilde{\alpha} + \tilde{A}_{6k} \bar{\alpha}_k) \,.\, I,$$

where A, δ, $\tilde{A}_i$ $(i = 1, \ldots, 5)$, $\tilde{A}_{6k}$ $(k = 10, \ldots, N)$, $\bar{A}_j^p$ $(p = 1, 2, 3, j = 10, \ldots, N)$ and $\bar{A}_{jk}^4$ $(j, k = 10, \ldots, N)$ are scalar constants.

Let us remark that the temperature gradient $\mathbf{g}_0$ is no more present but in the evolution equation of β, which contains neither the strain nor the temperature and is completely decoupled from the evolution equations of $\bar{\alpha}$ and $\tilde{\alpha}$. Thus, if the vector α of internal state variables is considered to contain only, two groups of components, i.e. $\alpha = (\beta, \tilde{\alpha})$, the two corresponding evolution equations will be completely separated, as the evolution equation for β does not depend on $\tilde{\alpha}$ and the evolution equation for $\tilde{\alpha}$ does not depend on β.

This result is obtained here as a consequence of isotropy and linearity (compare to K o s i n s k i [19], see also the semilinear theory section).

One can easily see that (36) and (37)$_2$ lead to

$$(39) \qquad -\frac{1}{\gamma}\dot{\mathbf{q}} = \mathbf{A}\mathbf{g}_0 - \frac{\delta}{\gamma}\mathbf{q}\,,$$

which is nothing else than Cattaneo's heat conduction equation. Hence in the frame of an isotropic linear theory, heat flux is an internal state variable and its evolution equation is given by Cattaneo's equation. For the one-dimensional case, Suliciu [4] has reached similar conclusions by using another hypothesis instead of that requiring the existence of a strong equilibrium state.

Let us observe that the restriction $(22)_2$ as well as the condition

$$\frac{\partial^2 \hat{\psi}^*}{\partial F_{ij} \partial \alpha_r} A_{rk}^* = 0, \quad i, j, k = 1, 2, 3,$$

(see [2], relation (5.8)) are satisfied in the linear case.

The linear constitutive equations (38) are generalizing most of the linear rheological models (see for instance R e i n e r [23], ch. B).

We will describe the way one may obtain what in rheology is known as the linear standard model. Let us suppose $\Lambda_k = 0$ in (33) and $\tilde{A}_{6k} = 0$ in $(38)_3$. We also assume that relation (33) is invertible with respect to $\tilde{\alpha}$, that is

$$\tilde{\alpha} = \mathbf{L}(\tilde{\mathbf{S}}, \varepsilon, \vartheta)\,,$$

where $\mathbf{L}$ is a linear function of its arguments. If (33) is differentiated with respect to t and $\dot{\tilde{\alpha}}$ and $\tilde{\alpha}$ are replaced by the expressions $(38)_3$ and $\mathbf{L}(\tilde{\mathbf{S}}, \varepsilon, \vartheta)$ respectively, one obtains the linear standard model.

We present in the following the restrictions imposed by the Clausius-Duhem inequality as well as by inequality (18) on the numerical coefficients (in this case) that enter the constitutive equations of the linear theory.

The Clausius-Duhem inequality requires that relations (7) be satisfied. Condition $(7)_1$ leads to

$$(40) \qquad \gamma = 2\,\theta_0 A\zeta\,,$$

while $(7)_2$ leads to the following inequalities

$$(41) \qquad \begin{gathered} \Gamma_i \bar{A}_i^3 + 3\Gamma\tilde{A}_3 \leqslant 0, \; \zeta\delta \leqslant 0, \; 3\,[\Lambda_i \bar{A}_i^1 + \tilde{A}_1 (\Lambda' + \Lambda'')] \leqslant 2\Lambda'\tilde{A}_1 \leqslant 0, \\ 3\,[\Delta_i \bar{A}_i^2 + 2\tilde{A}_2 (\Delta' + \Delta'')] \leqslant 4\Delta''\tilde{A}_2 \leqslant 0, \; 2\Omega_{kl}\bar{A}_{lk}^4 + 3\Delta_k \tilde{A}_{6k} \leqslant 0 (k!). \end{gathered}$$

Inequality (18) requires that

$$(41_1) \qquad \mu > 0, \; \lambda + 2\mu > 0, \; \vartheta > 0, \; \varepsilon > 0,$$

where the last restriction, according to $(41)_2$, leads to $\delta \leqslant 0$.

The semilinear theory. The semilinear theory is a theory that leads to a semilinear system of partal differential equations for the unknown functions. One of the characteristics of such a theory is the property of having constant speeds of propagation for acceleration and shock waves.

Such a theory is experimentally justified in the domain of small deformations (i.e. $\varepsilon_{ij}\varepsilon_{ij} \ll 1$) for certain materials as metals for instance. Thus, if a stress pulse is superposed over a constant state of strain and temperature, in the plastic domain, it will propagate with the elastic wave speed. This kind of behaviour has been shown to be present for several metals, for tension and compression as well (see B e l l [9], S t e r n g l a s s and S t u a r t [10], A l t e r and C u r t i s [11], B e l l and S t e i n [12], F o k u o k a and M a s u i [13]).

These experimental results have been used by S u l i c i u, M a l v e r n and C r i s t e s c u [14], S u l i c i u [15] (see also C r i s t e s c u and S u l i c i u [16]) in order to formulate a constitutive assumption containing two parts : the speed of acceleration waves at a given equilibrium state is path-independent and, the time a state with constant strain and temperature needs (from the moment it has been achieved) to reach an equilibrium state, is finite ; this time has been called the relaxation time. They have studied the consequences of these constitutive assumptions, for the one-dimensional case and quasilinear rate type constitutive equations. In connection with the existence of an „absolute plateau" in plasticity, K u k u d j a n o v [17] has underlined, by assimptotical methods, a large class of semilinear rate type constitutive equations that satisfy the finite relaxation time hypothesis (see also S u l i c i u [15]).

A similar study can be performed for materials with internal state variables. In the one-dimensional case of a semilinear theory in this frame S z m i t [18] has shown that, in the thermo-mechanically coupled case, the finite relaxation time hypothesis is verified even if one uses a linear evolution equation for the thermal internal variable. Therefore, one can describe thermoplasticity by means of Cattaneo constitutive equation for the heat flux hence, from this point of view it is unnecessary to consider a semilinear constitutive equation for the heat flux.

In order to obtain a semilinear theory for isotropic materials, it is sufficient to maintain the assumptions of the first section together with (31) and with the following additional constitutive assumptions

$$\alpha = (\beta, \tilde{\alpha}, \bar{\alpha}), \ \dot{\tilde{\alpha}}_{ij} = \tilde{A}_{ij,k} g_{0k} + \tilde{b}_{ij}(\varepsilon, \theta, \alpha), \ \dot{\beta}_i = A_{ik} g_{0k} +$$

$$+ b_i(\varepsilon, \theta, \alpha), \ \dot{\bar{\alpha}}_m = \bar{A}_{mk} g_{0k} + \bar{b}_m(\varepsilon, \theta, \alpha), \ \tilde{A}_{ij,k} = \text{const.}, \tag{42}$$

$$A_{ik} = \text{const.}, \ \bar{A}_{mk} = \text{const.}, \ i, j, k = 1, 2, 3, \ m = 10, \dots, N.$$

The functions $\bar{b}_m$, b_i and $\tilde{b}_{ij}$, $i, j = 1, 2, 3$, $m = 10, \dots, N$ are assumed to be continuous of their arguments in a neighborhood $\mathcal{U}^*$ of the equilibrium state $(\varepsilon^*, \theta^*, g = 0, \alpha^*)$, which also contains the strong equilibrium state (o, θ^0, o, o) and

$$\bar{b}^*_m = \bar{b}_m(\varepsilon^*, \theta^*, \alpha^*) = 0, \ b^*_i = b_i(\varepsilon^*, \theta^*, \alpha^*) = 0 \ \tilde{b}_{ij} = \tilde{b}_{ij}(\varepsilon^*, \theta^*, \alpha^*) = 0.$$
$$i, j = 1, 2, 3, \ m = 10, \dots, N. \tag{43}$$

Let us now consider the following process in the body $\mathcal{B}$, with the reference configuration $\mathcal{R}$

$$\varepsilon(Y, t) = \varepsilon^*, \ \theta^*(Y, t) = \theta^* + g_0(Y - X), \ Y \in \mathcal{R}, \ t \geq 0 \tag{44}$$

(where $X \in \mathcal{R}$ is fixed), that starts with

$$\alpha(0) = \alpha^*. \tag{45}$$

Then, at $X \in \mathcal{R}$ and $t = 0$, one obtains

$$\dot{\tilde{\alpha}}_{ij}(0) = \tilde{A}_{ij,k} g_{0k}, \ \dot{\beta}_i(0) = A_{ik} g_{0k}, \ \dot{\bar{\alpha}}_i(0) = \bar{A}_{ik} g_{0k}, \tag{46}$$

that is, $(\tilde{A}_{ij,k} g_{0k})$ $i, j = 1, 2, 3$, $(A_{ik} g_{0k})$ $i = 1, 2, 3$ and $\bar{A}_{mk} g_{0k}$, $m = 10, \dots, N$ have to be isotropic tensor, vector and scalar functions respectively of a vector variable. Hence

$$A_{ik} = A_{ik}, \ \tilde{A}_{ij,k} = 0, \ \bar{A}_{mk} = 0, \ i, j, k = 1, 2, 3,$$
$$m = 10, \dots, N, \ A = \text{const.} \tag{47}$$

and therefore

$$\dot{\tilde{\alpha}}_{ij} = \tilde{b}_{ij}(\varepsilon, \theta, \alpha), \ \dot{\beta}_i = A g_{0i} + b_i(\varepsilon, \theta, \alpha), \ \dot{\bar{\alpha}}_m = \bar{b}_m(\varepsilon, \theta, \alpha)$$
$$i, j = 1, 2, 3, \ m = 10, \dots, N \tag{48}$$

which are similar relations to (38). The form of these evolution equations is close to that of the evolution equations used by K o s i n s k i [19]. The main difference consistes in the fact that, considering only the case $\alpha = (\tilde{\alpha}, \beta)$,, K o s i n s k i postulates two evolution equations for β and $\tilde{\alpha}$, independent from each other which, generally, does not follow from (48). For instance, if one assumes that in the neighborhood of a strong equilibrium state $(\varepsilon^*, \theta^*, \beta^*, \tilde{\alpha}^*)$, one can approximate $b_i(\varepsilon, \theta, \alpha)$ $i = 1, 2, 3$, by linear functions of $\varepsilon - \varepsilon^*$, $\theta - \theta^*$, $\tilde{\alpha} - \tilde{\alpha}^*$, $\beta - \beta^*$, then due to isotropy

$$b_i = \delta(\beta_i - \beta^*_i), \quad i = 1, 2, 3, \tag{49}$$

with $\delta = \text{const.}$ and the evolution equation $(48)_2$ becomes independent of $(48)_3$ but the latter one still contains β.

The semilinear theory described by (48) and (49) may lead to a finite relaxation time, in the coupled case, if b_i verifies certain conditions of the same nature as those discussed by S u l i c i u [15].

Let us now remark there exists a connection between materials with internal state variables and rate type materials (for a more detailed discussion see C r i s t e s c u and S u l i c i u [16]).

In order to describe a purely mechanical behaviour, let us consider the semilinear rate type constitutive equations

$$\dot{S}_{ij} = C_{ijkl}\,\dot{\varepsilon}_{kl} + b_{ij}(\varepsilon, S), \tag{50}$$

where C_{ijkl}, $i, j, k, l = 1, 2, 3$ is a non-singular constant matrix. For any given strain history on a time interval $[0, t_1]$, $t_1 > 0$, one can write (using Lagrange method of variation of parameters)

$$S_{ij} = C_{ijkl}(\varepsilon_{kl} - \tilde{\varepsilon}_{kl}), \tag{51}$$

where $\tilde{\varepsilon}$ remains to be determined such that (51) be a solution of equation (50), that is

$$\dot{\tilde{\varepsilon}}_{ij} = -C^{-1}_{ijkl}\, b_{kl}(\varepsilon, C(\tilde{\varepsilon} - \varepsilon)) \ \ \tilde{\varepsilon}_{ij}(0) = -C^{-1}_{ijkl}\, S^0_{kl}. \tag{52}$$

$\tilde{\varepsilon}$ is usually interpreted as the viscoplastic part of the strain and $\varepsilon^e_{kl} = \varepsilon_{kl} - \tilde{\varepsilon}_{kl}$ as its elastic part. Therefore, there is an internal tensor state variable $\tilde{\varepsilon}$ that corresponds to the stress tensor S.

In this way one obtains an interpretation of the internal state variables in terms of the rate type constitutive equations. Such an interpretation has been used by S u l i c i u in [4] for the one-dimensional case and, it explains why the assumption (28) has been laid down.

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RECENZII

ARPAD SZABÓ: *The Beginnings of Greek Mathematics*. Akadémiai Kiadó, Budapest, 1978, 360 p.

Este o carte cu totul remarcabilă de istoria matematicii scrisă de un autor cu o foarte solidă pregătire ca matematician, filolog și filozof. Cartea analizează o perioadă istorică relativ restrînsă (secolele 6—4 î.e.n.) asigurîndu-și astfel unitatea. Buna pregătire de filolog a autorului a permis o analiză extrem de atentă a izvoarelor istorice; prin analiza detaliată a înțelesului fiecărui cuvînt o serie de texte adesea citate capătă un sens nou, profund și corect. Remarcăm, de asemenea, analiza aprofundată ce o face autorul relației matematică-muzică (pentru perioada menționată).

Cea mai importantă parte a cărții (a treia) analizează evoluția conceptului de demonstrație matematică, a delimitării matematicii ca știință. O serie de concluzii formulate în paragraful 3.30 semnalează cîteva opinii inedite ale autorului, în opoziție cu teoriile anterioare, dovedite eronate. De exemplu apare clar, că în afara construcțiilor efective ce pun în evidență unele entități matematice, există în matematica greacă a secolului 4 î.e.n. și demonstrații neconstructive de existență.

Dan Brânzei

VLADIMIR G. BOLTIANSKII: *Hilbert's Third Problem*. Translated by Richard A. Silverman and introduced by Albert B. J. Novikoff. Scripta Series in Mathematics. V. H. Winston & Sons, Washington, D. C., 1978. X+228 p. Distributed by Halsted Press, John Wiley & Sons, Inc., New York, London.

Dintre cele 23 de probleme enunțate de Hilbert în 1900, problema a treia este singura avînd un enunț elementar. În același timp, este prima căreia i s-a dat un răspuns (răspunsul negativ dat de Max Dehn chiar în 1900). În capitolul 1 al cărții, autorul prezintă axiomatica legată de noțiunile de arie și volum. În capitolul 2 se demonstrează teorema lui F. Bolyai și P. Gerwien: două poligoane au aceeași arie dacă și numai dacă sînt echidecompozabile. Urmează extinderi ale problemei la cazul neeuclidian și la cazul euclidian relativ la un grup de mișcări. Problema echidecompozabilității poliedrelor este discutată în capitolul 3. Se dă soluția negativă a lui Dehn. În continuare, se demonstrează teorema lui Hadwiger și teorema Dehn-Sydler. După studiul echidecompozabilității în raport cu un grup de mișcări ale spațiului, se definesc invarianții Dehn-Hadwiger și se demonstrează teorema lui Jessen. Cartea se încheie cu o anexă asupra conceptului de lungime.

Jack Weinstein

IMRE RUSZA: *Die Begriffswelt der Mathematik*. Volk und Wissen, Volkseigener Verlag Berlin, 1976, 471 p.

Cartea se adresează unui public foarte larg fără pregătire matematică superioară. Se realizează o introducere corectă și accesibilă (dar nu pedantă și tehnică) în teme fundamentale, ca: formarea noțiunii de număr, mulțimi finite și infinite (cap. 1), dezvoltarea noțiunii de număr, numere iraționale și nume reale și prima inițiere în analiză (cap. 2), funcții reale, limită, derivată, integrală (cap. 3), geometrie euclidiană, geometrii neeuclidiene (cap. 4), mulțimi infinite, cardinale transfinite, antinomii, axiomatizări ale teoriei mulțimilor (cap. 5), logică matematică, teoria demonstrației (cap. 6). Titlurile metaforice ale capitolelor și subcapitolelor stimulează atenția cititorului, iar exemplele judicios alese, claritatea expunerii și stilul curgător fac lectura antrenantă. Un indice de noțiuni și unul de nume completează lucrarea.

David Rimer