Proposition 6. *Let $f: M \to R$ be a convex function on a Riemannian manifold. Let $\tilde{D}$ be a distribution on the tangent bundle TM such that $\tilde{X} \in \tilde{D}$ implies the conditions:*

$$\sum_{i,j,m} R^m_{ikj} \frac{\partial f}{\partial x^m} X^i Y^j = 0, \ \forall k,$$

for the expression (8) of the vector field $\tilde{X}$.

Then the vertical lift f^v of the function f is a $\tilde{D}$-convex function.

Particularly, if M is a locally Euclidian space, f^v is a convex function on TM.

REFERENCES

1. Kiyotaka Ii — *Minimal submanifolds and convex functions*, Tôhoku Math. J. 24 (1972), 571—579.
2. Udriște C. — *Funcții convexe pe varietăți Riemanniene*, Studii și cercetări matematice, 28, 6 (1976), 735—745.
3. Udriște C. — *Continuity of convex functions on Riemannian manifolds*, Bulletin Mathématique, Tom 21 (69), 1—2, 1977, 215—218.
4. Oproiu V. — *Some remarkable structures and connections defined on the tangent bundle*, Rendiconti di Matematica (3), Vol. 6, Serie VI, 1973, 503—540.

Received 27.XI.1978

Department of Mathematics
Polytechnic Institute of Iași
Iași, R. S. România

Analele științifice ale Universității „Al. I. Cuza" din Iasi
Tomul XXV, s. I a, 1979, f. 2

A METHOD FOR FINDING THE STRONGLY CONNECTED COMPONENTS OF A DIGRAPH

BY

DĂNUȚ MARCU

In [2], [3], [4] and [5] there are four algorithms for computing the strongly connected components of a digraph. Our paper gives a new method similar to that included in [2]. Let $D = <\mathcal{N}, \mathcal{A}>$ be a *digraph* [1] with $N = \{n_1, n_2, \ldots, n_N\}$ the set of *nodes* and $\mathfrak{A}$ the set of *arcs*. If certain elements of $\mathfrak{A}$ can be placed in a sequence of the form $<n_{i_1}, n_{i_2}>$, $<n_{i_2}, n_{i_3}>, \ldots, <n_{i_{s-1}}, n_{i_s}>$, then, the set $\{<n_{i_1}, n_{i_2}>, <n_{i_2}, n_{i_3}>, \ldots, <n_{i_{s-1}}, n_{i_s}>\}$ is a *path* from n_{i_1} to n_{i_s} in D, a path denoted by $\mathfrak{C}[n_{i_1}, n_{i_s}]$.

For a digraph we consider the *path matrix*

$$P = (p_{ij}), \ i, \ j = 1, 2, \ldots, N, \text{ where:}$$

$$p_{ij} = \begin{cases} 1, & \text{if there exists a path } \mathfrak{C}[n_i, n_j], \\ 0, & \text{otherwise.} \end{cases}$$

In [6] we have suggested a very simple algorithm for computing the path matrix of a digraph from his *adjacency matrix* $A = (a_{ij})$ $i, j = 1, 2, \ldots, N$ defined as follows:

$$a_{ij} = \begin{cases} 1, & \text{if } <n_i, n_j> \in \mathcal{A}, \\ 0, & \text{if } <n_i, n_j> \notin \mathcal{A}. \end{cases}$$

Two nodes n and m are *mutually strongly connected* $(n \sim m)$ if and only if there exist the paths $\mathfrak{C}[n, m]$ and $\mathfrak{C}[m, n]$. Obviously, „$\sim$„ is an equivalence and induces a partition on $\mathcal{N}$ in the classes: $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_k$, called *strongly connected components*. Our problem is to find an algorithm for computing $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_k$. Having this purpose, we consider the matrix $P^* = (p^*_{ij})$ $i, j = 1, 2, \ldots, N$, where:

$$p^*_{ij} = \begin{cases} p_{ij}, & \text{if } i \neq j, \\ 1, & \text{if } i = j. \end{cases}$$

Theorem 1. (see [2], p. 28, exercice 9). *If $\mathcal{C}_0$ is the strongly connected component that contains the node n_i, then $[(P^*)^2]_{ii} = |\mathcal{C}_0|$.*

Proof. Let $i, j \in \{1, 2, \ldots, N\}$, $i \neq j$. The element $[(P^*)^2]_{ij}$ is obtained by:

$$[(P^*)^2]_{ij} = \sum_{k=1}^{N} p^*_{ik} \cdot p^*_{kj}.$$

Obviously, the product $p^*_{ik} \cdot p^*_{kj}$ is equal to 0 or to 1. It is equal to 1 if there exists a path $\mathfrak{C}[n_i, n_j]$ passing through n_k, and it is equal to 0 otherwise. Hence, $[(P^*)^2]_{ij}$ represents the number of nodes belonging to at least a path from n_i to n_j, that implies that $[(P^*)^2]_{ii}$ is equal to the number of nodes belonging to at least a circuit including n_i. Because the nodes belonging to a circuit including n_i generate $\mathcal{C}_0$, we have: $[(P^*)^2]_{ii} = | \mathcal{C}_0 |$. (Q.E.D.).

Theorem 2. *Let $\mathcal{C}_0$ be the strongly connected component that contains the node n_i. The node n_j belongs to $\mathcal{C}_0$ if and only if $[(P^*)^2]_{ij} = [(P^*)^2]_{ii}$.*

Proof. Suppose $[(P^*)^2]_{ij} = [(P^*)^2]_{ii}$. In this case the number of nodes belonging to at least a path $\mathfrak{C}[n_i, n_j]$ is equal to the number of nodes of $\mathcal{C}_0$ (see theorem 1). But every node of $\mathcal{C}_0$ belongs to at least a path from n_i to n_j. Hence, $n_j \in \mathcal{C}_0$. (Q.E.D.).

Reciprocally, suppose that $n_j \in \mathcal{C}_0$. In this case, obviously, we have:

$$(1) \qquad [(P^*)^2]_{ij} \leq [(P^*)^2]_{ii}.$$

Because every node of $\mathcal{C}_0$ belongs to at least a path from n_i to n_j, we obtain according to theorem 1:

$$(2) \qquad [(P^*)^2]_{ij} \geq [(P^*)^2]_{ii}.$$

From (1) and (2) it results: $[(P^*)^2]_{ij} = [(P^*)^2]_{ii}$. (Q.E.D.). Theorem 2 suggests a simple algorithm for computing the strongly connected components of D.

Algorithm. 1. $k := 0$, $\mathcal{C}_0 := \varnothing$. 2. $k := k + 1$.

3. If there exists a node $n_{i(k)}$, $i(k) \in \{1, 2, \ldots, N\}$ so that $n_{i(k)} \notin \bigcup_{\alpha=0}^{k-1} \mathcal{C}_\alpha$, then we construct the set $\mathcal{C}_k := \{n_{i(k)}\} \cup \{n_j \mid n_j \notin \bigcup_{\alpha=0}^{k-1} \mathcal{C}_\alpha$ and $[(P^*)^2]_{i(k)j} = [(P^*)^2]_{i(k)i(k)}$, $j = 1, 2, \ldots, N\}$, go to 2; else STOP.

REFERENCES

1. Berztiss A. T. — *Data Structures*. New York, 1971,
2. Christofides N. — *Graph Theory. An Algorithmic Approach*. Academic Press, 1975.
3. Foulkes J. D. — *Directed Graphs and Assembly Schedules*. 10-th Symp. in Applied Math., Amer. Soc. Math. 1960.
4. Muller Y. — *Applications Pratiques des Graphes à la Recherche d'un Optimum*. Paris, 1972.
5. Marcu D. — *Un algoritm de determinare a componentelor tari conexe ale unui graf orientat finit*. Studii şi cercetări matematice, 28, 1 (1976).
6. Marcu D. — *Determinarea arcelor esenţiale într-un graf orientat finit*. Studii şi cercetări de calcul economic şi cibernetică economică, 2 (1975).

Received 12.IX.1977

Faculty of Mathematics
University of Bucharest
R. S. România

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXV, s. I a, 1979, f. 2

THE EQUATIONS OF RADIATIVE RELATIVISTIC MAGNETOHYDRODYNAMICS. INVARIANCE GROUPS FOR ONE-DIMENSIONAL FLOW

BY

GH. PROCOPIUC

1. Introduction. In [1] we have established the equations of radiation field and its interaction with continuous medium in motion. We now derive, in Section 2, the equations which describe the motion of the medium in the presence of the radiation field and of the electromagnetic field, that is the equations of radiative magnetohydrodynamics, making use of the particle, momentum and energy balances. We also allow the medium velocity to be great enough that relativistic effects may become important. However, we also give the nonrelativistic limit of these equations. We use the way followed in [2], but we also take into account the electromagnetic field. We give, in Section 3, the covariant form of the equations of radiative relativistic magnetohydrodynamics, and in Section 4, some considerations about the energy equation and thermodynamics. In Section 5, the equations for a compressible ideal perfectly conducting gas are written.

Following the method given in [1], in Section 6, determine the invariance groups for the system of the hydrodynamics (HD), of the magnetohydrodynamics (MHD), of the radiation hydrodynamics (RHD), and of the radiative magnetohydrodynamics (RMHD), which describe the one-dimensional flow of an ideal perfectly conducting gas. In Section 7, we analyse some group properties of the previous systems.

2. The equations of radiative relativistic magnetohydrodynamics. We consider a compressible ideal fluid, moving with velocity $\mathbf{u}(\mathbf{r}, t)$ with respect to a reference frame R. We introduce a second reference frame R_0, namely the frame moving with the fluid. In this frame, we denote by $\rho(\mathbf{r}, t)$ the mass density of fluid, $p^{(m)}(\mathbf{r}, t)$ the fluid pressure, and $E^{(m)}(\mathbf{r}, t)$ the fluid energy density in excess of the rest energy. If c is the vacuum speed of light, we define

$$\lambda = (1 - u^2/c^2)^{-1/2};$$

then in the reference frame R for which the fluid velocity is $\mathbf{u}$, the mass density of fluid becomes $\lambda\rho$, and the total fluid energy density