Beweis. Weil $< e, Le > \leqslant f^*$, für jedes $e \in C^2(D, R^n)$ mit $|e| = 1$, gilt auch $< e, Le > \leqslant -\alpha^2, \alpha \in R, \alpha \neq 0$ und man verwendet Satz 1 von[3].

Bemerkungen : 1) Satz 2 ist eine Verallgemeinerung des Satzes 1 aus [2] für den Fall von Systemen mit zwei partiellen Differentialgleichungen.

2) Die Voraussetzungen des Satzes 2 sind stärker als die des Satzes 1 aus [3], aber sie werden nur den Koeffizienten des Systems (1) auferlegt, während der Einheitsvektor e ausgeschaltet wird.

3) Satz 2 ist eine Verbesserung des Satzes 1.1. aus [1] in Falle der stark elliptischen Systeme von Differentialgleichungen, wenn die Koeffizienten der Glieder die Ableitungen zweiter Ordnung enthalten nichtdiagonale, symmetrische Matrizen A_{ij} sind.

2. In diesem Abschnitt zeigen wir, daß das Grenzproblem (Dirichlet)

$$\left\{\begin{array}{l} Lu = B, \\ u \in C^2(D, R^n), \\ u|_\Gamma = b, \end{array}\right. \tag{15}$$

wo $b \in C(\Gamma, R^n)$ ein Functionvektor ist, eine einzige Lösung hat.

Satz 3. *In den Bedingungen des Satzes 2 ist die Lösung des Grenzproblems* (15) *einzig.*

Beweis. Wir zeigen, daß die homogene Grenzaufgabe

$$\left\{\begin{array}{l} Lu = 0 \\ u \in C^2(D, R^n) \\ u|_\Gamma = 0 \end{array}\right. \tag{16}$$

nur die Lösung $u \equiv 0$ hat.

In den Bedingungen in denen wir arbeiten, kann man das Prinzip des Maximums (Satz 2) für die Lösung des Systems (16) anwenden. (11) zeigt, daß $|u| \leqslant 0$, also $|u| \equiv 0$ oder $u \equiv 0$.

Es seien u_1 und u_2 zwei Lösungen für das Grenzproblem (15). Dann ist die Differenz $u = u_1 - u_2$ eine Lösung der homogenen Grenzaufgabe (16), also $u \equiv 0$, oder $u_1 \equiv u_2$.

Somit ist die Eindentigkeit festgesetzt und der Satz bewiesen.

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APPROXIMATE STABILITY OF GENERALIZED DYNAMICAL SYSTEMS

BY

C. IGNAT

The notion of stability of sets with respect to a preorder relation has been introduced by Bushaw in [1] in uniform spaces. This work has suggested us a generalization of the same notion in topological spaces [5]. In the present paper we will propose another generalization in spaces endowed with a preorder relation and an approximation structure, defined by us in [6].

We recall the definition of an approximation structure [6].

Let X and A be two sets and $\mathcal{R}(A, X)$ the collection of all relations $\alpha \subset A \times X$.

Definition 1. *An approximation relation of X directed by A is a subset $a \subset X \times \mathcal{R}(A, X)$ such that*

$$\forall x \in X \quad \exists \alpha \in \mathcal{R}(A, X) : (x, \alpha) \in a. \tag{1}$$

We shall say that $\{X, a, A\}$ is an approximation structure space.

We denote

$$r(\alpha, a) = \{z \in X : (a, z) \in \alpha\}, \ \alpha \in \mathcal{R}(A, X), \ a \in A; \tag{2}$$

$$a^*[\gamma] = \{z \in X : (z, \gamma) \in a\}, \ \gamma \in \mathcal{R}(A, X); \tag{3}$$

$$a[x] = \{\alpha \in \mathcal{R}(A, X) : (x, \alpha) \in a\}, \ x \in X; \tag{4}$$

$$\mathcal{S}[\alpha] = \{y \in X : \exists a \in A, \ (a, y) \in \alpha\}, \ \alpha \in \mathcal{R}(A, X). \tag{5}$$

1. We introduce now the notion of approximative generalized metric (a.g.m.).

Definition 2. *Let X, G, A be three nonempty sets, $\mathcal{K} \subset \mathcal{R}(A, G)$ and let ρ be a mapping of $X \times X$ in the set of functions G^A; denote*

$$\lambda[a] = \{\rho \in G : (a, \rho) \in \lambda\}, \ \lambda \in \mathcal{K}, \ a \in A,$$

and

$$\mathcal{D}(\lambda) = \{a \in A : \lambda[a] \neq \varnothing\} \ \lambda \in \mathcal{K}. \tag{6}$$

The relation

$$B(x, \varkappa) = \{(a, y) \in \mathcal{D}(\varkappa) \times X : \rho(x, y) \in \varkappa[a]\} \tag{7}$$

is said to be an approximation ball centred in $x \in X$ with radius $\varkappa$.

The quintuple $\{X, G, A, \mathcal{K}, \rho\}$ is said to be an approximative generalised metric (a.g.m.) on X, directed by A.

Let $\{X, G, A, \mathcal{K}, \rho\}$ be an a.g.m. on X.

The approximation relation a_ρ on X defined by

$$(8) \qquad (x, \alpha) \in a_\rho \Leftrightarrow \exists\, \varkappa \in \mathcal{K} : \forall a \in \mathcal{D}(\varkappa) \;\; r(\alpha, a) = \{y \in X : \rho(x, y) [a] \in \varkappa [a], \;\; \forall a \notin \mathcal{D}(\varkappa) \;\; r(\alpha, a) = \varnothing\}$$

is said to be canonicaly induced by the a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$.

Proposition 1. *Let X be a set endowed with an approximation relation $\{a, A\}$. Then, there exists an a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ on X such that the approximation relation a_ρ defined by (6) coincides with a.*

Proof. Let us denote by $\varphi_M(x)$ the characteristic function of $M \subset X$, $\psi(M; x) = 1 - \varphi_M(x) = \varphi_{M^c}(x)$ and $S = \{0,1\}$.

For every $(b, \beta) \in A \times \mathcal{R}(A, X)$ consider the map

$$\rho_b^\beta : X \times X \to S \times S$$

defined by

$$(9) \qquad \rho_b^\beta(x, y) = (\psi(a^*[\beta]; x), \psi(r(\beta, b); y)).$$

Let us consider the a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ with: $G = (S \times S)^{\mathcal{R}(A,X)}$ and ρ defined by:

$$(10) \qquad pr_\beta\{\rho(x, y)[b]\} = \rho_b^\beta(x, y) : x, y \in X, b \in A, \beta \in \mathcal{R}(A, X).$$

Define the equivalence relation on X:

$$(11) \qquad x \underset{a}{\sim} y \Leftrightarrow a[x] = a[y].$$

and denote $\tilde{x} = \{z \in X : z \underset{a}{\sim} x\}$.

For $(\tilde{x}, \beta)$, $\beta \in a[x]$ consider the relation on A:

$$(12) \qquad \varkappa[(\tilde{x}, \beta)] = \{(b, p) \in \mathcal{D}(\beta) \times G : pr_\alpha(p) = (i_\alpha^{b,\beta}, j_\alpha^{b,\beta}) \;\; \alpha \in \mathcal{R}(A, X)$$

$$i_\alpha^{b,\beta} \in \begin{cases} \{0\} & \text{if } a^*[\beta] \subset a^*[\alpha], \\ \{1\} & \text{if } a^*[\alpha] \cap a^*[\beta] = \varnothing, \\ \{0,1\} & \text{if } a^*[\alpha] \cap a^*[\beta] \neq \varnothing.\; a^*[\beta] \setminus a^*[\alpha] \neq \varnothing, \end{cases}$$

$$j_\alpha^{b,\beta} \in \begin{cases} \{0\} & \text{if } r(\beta, b) \subset r(\alpha, b), \\ \{1\} & \text{if } r(\alpha, b) \cap r(\beta, b) = \varnothing, \\ \{0,1\} & \text{if } r(\alpha, b) \cap r(\beta, b) \neq \varnothing, r(\beta, b) \setminus r(\alpha, b) \neq \varnothing. \end{cases}$$

and

$$(13) \qquad \mathcal{K} = \{\varkappa[(\tilde{x}, \beta)] : \tilde{x} \in X/\tilde{a}, \beta \in a[x]\}.$$

Let $(z, \gamma) \in a$. For $a \in \mathcal{D}[\varkappa[(\tilde{z}, \gamma)]$ one has $r(\gamma, a) = \{y : \rho(x, y) \in \varkappa[(\tilde{z}, \gamma)][a]\}$, and $\mathcal{D}(\gamma) = \mathcal{D}(\varkappa[\tilde{z}, \gamma)]]$, i.e. $(z, \gamma) \in a_\rho$. Therefore $a \subset a_\rho$.

If $(u, \delta) \in a_\rho$, by (8) there exists a relation $\varkappa[(x, a)]$ such that

$$\forall a \in \mathcal{D}(\varkappa[(\tilde{z}, \alpha)]) = \mathcal{D}(\alpha); \; r(\delta, a) = \{y : \rho(u, y)[a] \in \varkappa[(\tilde{x}, \alpha)][a]\},$$

$$\forall a' \notin \mathcal{D}(\varkappa[(\tilde{x}, \alpha)]) : \quad r(\delta, a') = \varnothing.$$

It follows that

$$\mathcal{D}(\delta) = \mathcal{D}(\alpha),$$

$$u \in [\bigcap_\beta \{a^*[\beta] : a^*[\alpha] \subset a^*[\beta]\}] \cap [\bigcap_\gamma \{Ca^*[\gamma] : a^*[\alpha] \cap a^*[\gamma] = \varnothing\}] = a^*[\alpha],$$

$$r(\delta, a) = [\bigcap_\beta \{r(\beta, a) : r(\alpha, a) \subset r(\beta, a)\}] \cap [\bigcap_\gamma \{Cr(\gamma, a) : r(\alpha, a) \cap r(\gamma, a) = \varnothing\}] = \begin{cases} r(\alpha, a) & \text{if } a \in \mathcal{D}(\alpha). \\ \varnothing & \text{if } a \notin \mathcal{D}(\alpha). \end{cases}$$

Then $\delta = \alpha$ and therefore $(u, \delta) \in a$.

2. We shall establish conditions such that an a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ induces by (8) an approximation structure associated with a Hammer closure map [2], [3].

Let $\{X, G, A, \mathcal{K}, \rho\}$ be an a.g.m. on X. We introduce the equivalence relation on $X \times \mathcal{K}$ defined by

$$(14) \qquad (x, \lambda) \underset{\rho}{\sim} (x', \lambda') \Leftrightarrow B(x, \lambda) = B(x', \lambda'),$$

and denote the equivalence classes by $\overline{(x, \lambda)}^\rho$. For $(x, \varkappa) \in X \times \mathcal{K}$, set

$$(15) \qquad \mathcal{E}(x, \varkappa) = \{y \in X : \exists\, \varkappa' \in \mathcal{K} : (x, \varkappa) \underset{\rho}{\sim} (y, \varkappa')\}.$$

We shall say that the a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ is strictly monotone if

$$(16) \qquad B(x, \varkappa) \subset B(y, \varkappa') \text{ and } \mathcal{D}(\varkappa) = \mathcal{D}(\varkappa') \Rightarrow \mathcal{E}(x, \varkappa) \subset \mathcal{E}(y, \varkappa').$$

Recall [6] that an approximation relation a on X is said to be monotone if

$$(\text{SAM}) \qquad \mathcal{S}(\alpha) \subset \mathcal{S}(\beta) \Rightarrow a^*[\alpha] \subset a^*[\beta], \; \alpha, \beta \in \mathcal{R}(A, x).$$

It is easy to prove

Proposition 2. *Let $\{X, G, A, \mathcal{K}, \rho\}$ be an a.g.m. which satisfies (16). Then the approximation relation a induced by (8) is monotone. An a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ is said to be expansive if*

$$(17) \qquad \forall \varkappa \in \mathcal{K} : (\exists\, a \in \mathcal{D}(\varkappa), \;\; \exists\, z \in X, \;\; (a, x) \in B(z, \varkappa)) \Rightarrow (x, \varkappa) \underset{\rho}{\sim} (z, \varkappa).$$

Proposition 3. *An expansive a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ generates by (8) an approximation relation a on X which is expansive* [6] i.e.:

$$(\text{SAE}) \qquad x \in \mathcal{S}[\alpha] \Rightarrow (x, \alpha) \in a \;\; x \in X, \; \alpha \in \mathcal{R}(A, X).$$

Since a monotone and expansive approximation relation a on X generates a closure map $\mathcal{J} : \mathcal{P}(X) \to \mathcal{P}(X)$ by

(18) $\mathcal{J}_a(M) = \{y \in X : (y, x) \in a\ \mathcal{S}(x) = M\} \subset X\ M \subset X$,

it follows that an a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ which satisfies (16) and (17) generates by a_ρ in (8) and (18) a Hammer closure map.

3. The stability of sets in spaces with filltral approximation structure. We shall say that an approximation structure a on X is filltral if it is canonically induced by an a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ with card $(\mathcal{K}) \geqslant 2$ and

(19) $\forall\, \varkappa', \varkappa'' \in K \Rightarrow \exists\, \varkappa \in \mathcal{K},\ \varkappa \subset \varkappa' \cap \varkappa''$.

Let σ be a binary relation on X reflexive and transitive and denote

$$\sigma(u) = \{z \in X : (u, z) \in \sigma\},\ u \in X,$$
$$\sigma(M) = \bigcup_x \{\sigma(x) : x \in M\},\ M \subset X.$$

Let $\{X, G, A, \mathcal{K}, \rho\}$ be a filltral a.g.m. on X. We shall say that

(20) $\mathcal{A}(M ; \varkappa) = \{(a, y) \in A \times X : a \in \mathcal{D}(\varkappa)\ \exists\, x \in M,$
$\rho(x, y)[a] \in \varkappa[a]\}$

is the approximation of order $\varkappa$ for $M \subset X$.

Definition 3. *The set $S \subset X$ is said to be stable with respect to a preorder σ and a filtral a.g.m. $\{X, G, A, \mathcal{K}, \rho\}$ ((σ, ρ)-stable) if:*

(21) $\forall\, \varkappa \in \mathcal{K}\ \exists\, \varkappa' \in \mathcal{K}\ \forall\, a \in \mathcal{D}(\varkappa) : \sigma[\mathcal{A}(S ; \varkappa')[a]] \subset \mathcal{A}[S, \varkappa][a]$.

Liapunov functions [1]. Let $\{\mathcal{H}, \leqslant\}$ be an ordered set, with $h_0 \leqslant h$ $\forall\, h \in \mathcal{H}$ and such that $\mathcal{H}^+ = \mathcal{H}/\{h_0\} \neq \emptyset$. Let U be an uniform approximation of $S \subset X$, i.e. $U \subset A \times X$, $\exists\, \varkappa \in \mathcal{K}$, $A(S ; \varkappa) \subset U$ and $\forall\, a \in A$ $\sigma[U[a]] = U[a]$.

Definition 4. *The mapping $\mathcal{L} : U \to \mathcal{H}$ is called a Liapunov function for S if:*

(L1) $\forall\, (a, x) \in U\ (x, y) \in \sigma \Rightarrow \mathcal{L}(a, y) < \mathcal{L}(a, x)$;

(L2) $\forall\, h \in \mathcal{H}^+\ \exists\, \varkappa \in \mathcal{K}\ \forall\, (a, x) \in A(S ; \varkappa) \cap U \Rightarrow h \not< \mathcal{L}(a, x)$;

(L3) $\forall\, \varkappa \in \mathcal{K}\ \exists\, h \in \mathcal{H}^+\ \forall\, (a, x) \in U / A(S ; \varkappa) \Rightarrow h < \mathcal{L}(a, x)$.

This is a generalization of the definition for Liapunov function due to B u s h a w [1]. Let $\mathcal{T}$ be a collection of subsets of a set $Y (\mathcal{T} \subset \mathcal{P}(Y))$. We shall say that

(22) $\mathcal{E}(\mathcal{T}) = \{\psi \subset \mathcal{T}\ M \in \psi\ N \in \mathcal{T},\ M \subset N \Rightarrow N \in \psi\}$,

ordered by

(23) $\psi < \chi \Leftrightarrow \chi \subset \psi$

is the scale of $\mathcal{T}$.

We remark that the scale of a collection $\mathcal{T}$ has as first element $\mathcal{T}$ itself.

We consider the scale of the system $\mathcal{K} \subset \mathcal{P}(A \times G)$.

We shall prove a generalization of the results of B u s h a w [1] and [5].

Proposition 4. *A set $S \subset X$ is (σ, ρ)-stable if and only if there exists a Liapunov function for S with range in the scale $\mathcal{E}(\mathcal{K})$.*

Proof. Necessity. Let $S \subset X$ be stable.

Obviously the relation $U = A \times X$ is an uniform approximation of S invariant with respect to $\sigma : \sigma[U[a]] = U[a]$ $\forall\, a \in A$.

Consider the map

(24) $\mathcal{L}(a, x) = \{\varkappa \in \mathcal{K} : \sigma(x) \in A[S ; \varkappa][a]\}$.

Let $\varkappa_1 \in \mathcal{L}(a, x)$ and $\varkappa_2 \in \mathcal{K}, \varkappa_1 \subset \varkappa_2$. Since

$$\sigma(x) \subset A(S ; \varkappa_1)[a] \subset A(S, \varkappa_2)[a],$$

it follows $\varkappa_2 \in \mathcal{L}(a, x)$ that is $\mathcal{L}(a, x) \in \mathcal{E}(\mathcal{K})$. Let $(a, x_0) \in A \times X$ and $\varkappa \in \mathcal{K}$ be such that $\sigma(x_0) \subset A(S, \varkappa)[a]$. Let x' be any element of $\sigma(x_0)$. From $\sigma(x') \subset \sigma(x_0)$ it follows $\sigma(x') \subset A(S, \varkappa)[a]$ i.e. $\varkappa \in \mathcal{L}(a, x_0) \Rightarrow$ $\Rightarrow \varkappa \in \mathcal{L}(a, x')$. Thus the condition (L1) is fulfilled.

Let $\Gamma \in \mathcal{E}(\mathcal{K})$ and $x \in \mathcal{K} \setminus \Gamma \neq \emptyset$. Since S is stable there exists an element $\varkappa' \in \mathcal{K}$ such that

$$\forall\, a \in \mathcal{D}(\varkappa)\ \sigma[A(S ; \varkappa')[a]] \subset A(S ; \varkappa)[a].$$

Suppose that there exists $(a, x) \in A(S, \varkappa')$ such that $\mathcal{L}(a, x) > \Gamma$. Since $(a, x) \in A(S, \varkappa')$ it follows $\sigma(x) \subset \sigma(A(S ; \varkappa')) \subset A(S, \varkappa)[a]$ that is $\varkappa \subset \mathcal{L}.(a, x)$. But $\mathcal{L}(a, x) \subset \Gamma$ which is a contradiction.

Therefore, for $\Gamma \in \mathcal{E}^+(\mathcal{K})$ there exists $\varkappa' \in \mathcal{K}$ such that $(a, x) \in A(s, \varkappa') \Rightarrow \Gamma \not< \mathcal{L}(a, x)$ i.e. (L2).

Let x_0 be an element of $\mathcal{K}$ and set

(25) $\Delta = \bigcup_{(a, x)} \{\mathcal{L}(a, x) : (a, x) \in C\, A(S ; \varkappa_0)\}$.

Because $\mathcal{L}(a, x) \in \mathcal{E}(\mathcal{K})$ obviously $\Delta \in \mathcal{E}(\mathcal{K})$. Let us prove that $\Delta = \mathcal{K}$. Indeed, if $\Delta = \mathcal{K}$ then $x_0 \in \Delta$ and there it exists (a, x) such that $x_0 \in$ $\in \mathcal{L}(a, x)$. From (24) we have $\sigma(x) \subset A(S ; \varkappa_0)[a]$ i.e. $(a, x) \in A(S, \varkappa_0)$ which contradicts (25). Then $\Delta \in \mathcal{E}^+(\mathcal{K})$ so that (L3) is fulfilled.

Sufficiency. Suppose that there exists a Liapunov function for the set S, $\mathcal{L} : U \to \mathcal{E}(\mathcal{K})$ where U is an invariant with respect to σ uniform approximation of S.

Assume for the moment that:

(*) $\exists\, \varkappa_0 \in \mathcal{K}\ U \setminus A(S, \varkappa_0) \neq \emptyset$

and set

(26) $\mathcal{M} = \bigcup_{(a, x)} \{\mathcal{L}(a, x) : (a, x) \in U \setminus A(S ; \varkappa_0)\}$.

Obviously $\mathcal{M} \in \mathcal{E}(\mathcal{K})$.

From (L3) it follows that there exists $\Sigma \in \mathcal{E}^+(\mathcal{K})$ such that

$$\forall\, (a, x) \in U \setminus A(S ; \varkappa_0) \Rightarrow \Sigma < \mathcal{L}(a, x),$$

i.e. $\mathcal{L}(a, x) \subset \sum$ therefore $M \subset \Sigma$. Then $\mathcal{M} \in \mathcal{E}^{+}(\mathcal{X})$. By (L2) for $\mathcal{M} \in \mathcal{E}^{+}(\mathcal{X})$ there exists $\varkappa' \in \mathcal{X}$ such that

$$(a, x) \in U \cap A(S; \varkappa') \Rightarrow M \not\subset \mathcal{L}(a, x).$$

Suppose

$$(**) \qquad A(S; \varkappa') \subset U.$$

Because $\forall a\, \sigma[U[a]] = U[a]$, it follows

$$\sigma[A(S, \varkappa')[a]] \subset U[a], \ \forall a \in A.$$

Let (a, x_1) be an element of $A(S; \varkappa')$; therefore:

$$\mathcal{M} \not\subset \mathcal{L}(a, x_1) \quad \text{or} \quad \mathcal{L}(a, x_1) \notin \mathcal{M}.$$

For $x_2 \in \sigma(x_1)$ we have by (L1) $\mathcal{L}(a, x_2) < \mathcal{L}(a, x_1)$ i.e.

$$(27) \qquad \forall x \in \sigma[A(S; \varkappa')[a]] \ \mathcal{L}(a, x) \notin \mathcal{M}.$$

From (26) and (27) we get

$$\sigma(A(S, \varkappa')[a]) \subset A(\mathcal{S}; \varkappa_0)[a] \ \forall a \in \mathcal{D}(\varkappa_0).$$

If for $\varkappa'$ the condition (**) is false, then since U is an uniform approximation of S there is a $\varkappa_1 \in \mathcal{X}$ such that $A(S, \varkappa_1) \subset U$. Since K is filtral there exists $\varkappa'' \subset \varkappa' \cap \varkappa_1$.

By the same argument replacing $\varkappa'$ by $\varkappa''$ we deduce that: for every $\varkappa_0$ satisfies (*) there is a $\varkappa \in K$

$$(28) \qquad \sigma[A(S; \varkappa)[a]] \subset A(S; \varkappa_0)[a] \ \forall a \in \mathcal{D}(\varkappa_0).$$

We shall now prove that assumption (*) is unnecessary. Let $\lambda \in \mathcal{X}$ for which (*) is not satisfied. If for $\varkappa_1 \in \mathcal{X}$ which exists by the definition of uniform approximation U of S, the condition (*) is fulfilled, then by the same argument for $\varkappa''' \subset \varkappa_1 \cap \lambda$ instead of $\varkappa'$, we derive (28).

If for every $\varkappa \in \mathcal{X} \not\subset A(S, \varkappa) = \varnothing$ then there exists $\varkappa_2 \in K$ such that $\mathcal{A}(S, \varkappa_2) = U$ since U is an uniform approximation of S and

$$\forall \varkappa \in \mathcal{X}: \sigma(A(S; \varkappa_2)[a]) = \sigma(U[a]) = U[a] \subseteq A(S; \varkappa)[a].$$

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APPROXIMATION OF AN INTEGRO-DIFFERENTIAL EQUATION IN A HILBERT SPACE

BY

V. ARNĂUTU

1. Notations and General Assumptions. We consider the nonlinear Volterra equation

$$(1.1) \qquad \frac{du}{dt}(t) + Bu(t) + \int_0^t a(t-s)\, Au(s)\, ds = f(t), \qquad t \in [0, T],$$

with initial value condition

$$(1.2) \qquad u(0) = u_0$$

in a Hilbert space H.

We list the notations and general assumptions which will be in effect throughout the paper.

(a) H will denote a real Hilbert space with inner product $(.,.)$ and norm $|\,.\,|$. V is a real reflexive Banach space dense in H, V' is the dual space of V and then the relation

$$V \subset H \subset V'$$

means that the injection of V in H is continuous, H is identified with its own dual and $(.,.)$ will denote also the duality between V' and V. $\|.\|$ will denote the norm of V.

(b) $B = \partial\varphi$ where $\varphi : H \to (-\infty, +\infty]$ is convex, lower semicontinuous, $D(\varphi) \subset V$ and, for any $M > 0$, the level set

$$(1.3) \qquad \{u \in H \mid \varphi(u) + |u|^2 \leq M\} \text{ is compact in } V.$$

$\partial\varphi$ denotes the subdifferential of φ (recall that $\partial\varphi$ is a maximal monotone operator from H into itself).

(c) $A : V \to V'$ is single valued, everywhere defined and maximal monotone from V into V'. Moreover, there exists a convex and lower semicontinuous function $\psi : V \to (-\infty, +\infty]$ such that $A = \partial\psi$. Further, assume that