Theorem 3.5. *If the system (S) has the minimal stability property and there exists a solution (P, Q, R) of $\mathcal{L}(S)$ with $P^*J = JP$, then $P^* \geq 0$.*

Proof. By Definition 3.2 and Lemma 2.4 we have

$$<P^*\tilde{x}(T), \tilde{x}(T)>_{V^*} - <P^*\tilde{x}(0), \tilde{x}(0)>_{V^*} \leq F(T, x_0, u) \leq 0.$$

Hence

$$<P^*\tilde{x}(0), \tilde{x}(0)>_{V^*} \geqslant <P^*\tilde{x}(T), \tilde{x}(T)>_{V^*} \to 0 \text{ if } T \to \infty$$

and because $\pi^* : H \to V^*$ is surjective we obtain $P^* \geq 0$.

Theorem 3.6. *If there exists a solution (P, Q, R) for $\mathcal{L}(S)$ such that $P^* \geqslant 0$ then (S) is an hyperstable system.*

Proof. Is obvious from the Lemma 2.4.

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EXISTENCE FOR NONLINEAR DIFFERENTIAL SYSTEMS OF HYPERBOLIC TYPE

BY

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1. Introduction. Our objective is to investigate the existence and uniqueness of solutions to the nonlinear hyperbolic differential system

$$(1.1)\qquad \begin{aligned} &\alpha(x)\frac{\partial u}{\partial t} - \frac{\partial v}{\partial x} + A(x, u) = f(x, t), \\ &\beta(x)\frac{\partial v}{\partial t} - \frac{\partial u}{\partial x} + B(x, v) = g(x, t), \\ &\qquad \text{for } 0 < x < 1,\ 0 < t < T, \end{aligned}$$

with the initial data

$$(1.2)\qquad u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad 0 \leqslant x \leqslant 1,$$

and boundary conditions

$$(1.3)\qquad (u(0, t)\ -u(1, t)) \in L\ (v(0, t), v(1, t)),\ 0 \leqslant t \leqslant T,$$

where A and B are some functions from $]0, 1[\times R$ into R, $(R =]-\infty, \infty[)$ and L is a nonlinear (possibly) multivalued mapping from $R^2 = R \times R$ into itself.

Such problems are very much discussed in the literature. They arise in the theory of electrical transmission lines and in hydraulics. Existence, uniqueness and asymptotic behavior of solutions for problems of this type have been studied by many authors (see e.g. [2], [3], [5—11], [14] and Răsvan's book [13] for further references). Our starting point is the paper of V. Barbu [2], where the mapping L in the boundary conditions (1.3) was supposed to be a subdifferential. In the present paper we shall conditions, assume only that

(H_1) *$L : R^2 \to R^2$ is an arbitrary maximal monotone operator.*

Here are now some other basic hypotheses which will be used in what follows.

(H_2) The functions $r \to A(x, r)$ and $r \to B(x, r)$ are continuous and monotone increasing on R, a.e. $x \in]0,1[$. Furthermore, the functions $x \to A(x, r)$ and $x \to B(x, r)$ belong to $L^2(0,1)$, for each fixed $r \in R$.

(H_3) The functions α and β belong to $L^2(0,1)$ and in addition there exists some constant $\delta>0$ such that $\alpha(x)\geq\delta$, $\beta(x)\geq\delta$, a.e. $x\in]0,1[$.

We are now able to formulate the main result of this Note.

Theorem 1. *Suppose that* (H_1), (H_2) *and* (H_3) *hold and in addition*

(1.4) $$f, g\in W^{1,1}(0,T\,;\,L^2(0,1)),$$

(1.5) $$u_0, v_0\in H^1(0,1),$$

(1.6) $$(u_0(0),\ -u_0(1))\in L(v_0(0),\ v_0(1))\ (\textit{consistence condition}).$$

Then, the problem (1.1), (1.2), (1.3) *has a unique solution* $(u(x,t),\ v(x,t))$ *which satisfies*

(1.7) $$u, v\in W^{1,\infty}(0,T\,;\,L^2(0,1)\cap L^\infty(]0,1[\times]0,T[),$$

(1.8) $$\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial x}\in L^\infty(0,T\,;\,L^2(0,1)).$$

For the sake of brevity we refer the reader to the books [1] and [4] for all the notations and concepts used in this Note and for background material related to monotone operators, convex functions and nonlinear evolution equations developed on general Banach spaces. It should only be emphasized that throughout we write „function" to denote an element (i.e. an equivalence class of functions) of some L^p-space, $1\leq p\leq\infty$.

2. Proof of Theorem 1. Let X denote the product space $L^2(0,1)\times\times L^2(0,1)$ endowed with the usual scalar product and the corresponding norm. Define the operator $\mathcal{A}: X\to X$ by

(2.1) $$\mathcal{A}\begin{pmatrix}u\\ v\end{pmatrix}=\begin{pmatrix}-\dfrac{dv}{dx}\\[2mm] -\dfrac{du}{dx}\end{pmatrix},\ \text{for }(u,v)\in D(\mathcal{A}),$$

where

(2.2) $$D(\mathcal{A})=\{(u,v)\in H^1(0,1)\,;\,(u(0),\ -u(1))\in L(v(0),\ v(1))\}.$$

We begin the proof of Theorem 1 by proving the following auxiliary result.

Lemma 1. *The operator* $\mathcal{A}$ *defined by* (2.1) *and* (2.2) *is maximal monotone.*

Proof. An elementary verification shows that $\mathcal{A}$ is monotone. It remains to prove the maximality of $\mathcal{A}$. In other words, it remains to prove that for each pair $(p,q)\in X$ there exists a pair $(u,v)\in D(\mathcal{A})$ such that (u,v) satisfies the following boundary value problem

(2.3) $$u-\frac{dv}{dx}=p\,,\quad v-\frac{du}{dx}=q,$$

(2.4) $$(u(0),\ -u(1))\in L(v(0),\ v(1)).$$

We look for a solution to (2.3), (2.4) of the form $u=u_1+u_2$, $v=v_1+v_2$, where (u_1, v_1) satisfies the problem

(2.5) $$u_1-\frac{dv_1}{dx}=p,\ v_1-\frac{du_1}{dx}=q$$

(2.6) $$v_1(0)=v_1(1)=0,$$

while (u_2, v_2) is a solution to

(2.7) $$u_2-\frac{dv_2}{dx}=0,\ v_2-\frac{du_2}{dx}=0,$$

(2.8) $$(u_2(0),\ -u_2(1))\in L(v_2(0),\ v_2(1))-(u_1(0),\ -u_1(1)).$$

According to [2] the first problem (2.5), (2.6) has a unique solution $(u_1, v_1)\in\in H^1(0,1)\times H_0^1(0,1)$ because the boundary conditions (2.6) can be expressed in the form (2.4) where L is actually the subdifferential of the function $\varphi: R^2\to]-\infty,\infty]$, defined by

$$\varphi(x,y)=0\ \text{if}\ x=y=0,$$
$$=+\infty\ \text{otherwise.}$$

On the other hand, the second problem (2.7), (2.8) is equivalent with the following

(2.9) $$\frac{d^2v_2}{dx^2}=v_2\,,\qquad 0<x<1,$$

(2.10) $$\left(\frac{dv_2}{dx}(0),\ -\frac{dv_2}{dx}(1)\right)\in\overline{L}(v_2(0),\ v_2(1))$$

(2.11) $$u_2=\frac{dv_2}{dx},\qquad 0<x<1,$$

where $\overline{L}(x,y)=L(x,y)-(u_1(0),\ -u_1(1))$. Since $v_2=\tilde{x}e^x+\tilde{y}e^{-x}$ satisfies (2.9) for every constants $\tilde{x},\tilde{y}\in R$, to solve (2.9) (2.10) it suffices to find some pair $(\tilde{x},\tilde{y})\in R^2$ which satisfies the equation

(2.12) $$\left(\tilde{x}-\tilde{y},-e\tilde{x}+\frac{1}{e}\tilde{y}\right)\in\overline{L}\ \left(\tilde{x}+\tilde{y},\ e\tilde{x}+\frac{1}{e}\tilde{y}\right).$$

Let $z=\tilde{x}+\tilde{y}$, $w=e\tilde{x}+\frac{1}{e}\tilde{y}$, $a=2e/(1+e^2)$, $b=(e^2-1)/(e^2+1)$ and $F(z,w)=(z-aw,\ w-az)$. After an elementary calculation (2.12) becomes

(2.13) $$0\in F(z,w)+b\overline{L}(z,w).$$

It is easy to see that F is linear, continuous and strongly positive with the constant $1-a$. See also [12], p. 204. Therefore (2.13) has a unique solution (see e.g. [1], Corollary 2.1, p. 48). This completes the proof of Lemma 1.

Now let us define the operator $\mathcal{B}: X \to X$ by

$$\mathcal{B}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\dfrac{dv}{dx} + A(.,u) \\ -\dfrac{du}{dx} + B(.,v) \end{pmatrix} \tag{2.14}$$

By (H_2) clearly $D(\mathcal{B}) = D(\mathcal{A})$ and besides $\mathcal{B}$ is monotone. We continue the proof of Theorem 1 by proving another auxiliary result.

Lemma 2. *The operator $\mathcal{B}$ is maximal monotone.*

Proof. Consider the boundary value problems

$$\begin{aligned} &u_\lambda - \frac{dv_\lambda}{dx} + A_\lambda(x, u_\lambda) = p, \\ &v_\lambda - \frac{du_\lambda}{dx} + B_\lambda(x, v_\lambda) = q, \end{aligned} \tag{2.15}$$

$$(u_\lambda(0), -u_\lambda(1)) \in L(v_\lambda(0), v_\lambda(1)), \tag{2.16}$$

where (p, q) is an arbitrary element of X and $A_\lambda(x,.)$, $B_\lambda(x,.)$ are the Yosida approximations of $A(x,.)$, $B(x,.)$ respectively, i.e.

$$A_\lambda(x,r) = A(x, (I + \lambda A(x,.))^{-1} r),$$

$$B_\lambda(x,r) = B(x, (I + \lambda B(x,.))^{-1} r).$$

Note that $|A_\lambda(x,r)| \leqslant (2/\lambda)\,|r| + |A(x,0)|$ and the function $x \to A_\lambda(x,r)$ is measurable, for each $r \in R$ and $\lambda > 0$. The same properties hold for B_λ.

It follows that the operator

$$\mathcal{C}_\lambda\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A_\lambda(.,u) \\ B_\lambda(.,v) \end{pmatrix}$$

is everywhere defined and continuous on X. By Lemma 1 and a well-known perturbation result due to Rockafellar $\mathcal{A} + \mathcal{C}_\lambda$ is maximal monotone, i.e. the problem (2.15), (2.16) has a unique solution $(u_\lambda, v_\lambda) \in D(\mathcal{A})$ for each $\lambda > 0$. Furthermore (see [1], p. 81)

$$\{u_\lambda\},\ \{v_\lambda\} \text{ are bounded in } L^2(0,1). \tag{2.17}$$

Following an idea of [3], let us define the function $L_\lambda :]0,1[\, \times R \times R \to R$ by

$$L_\lambda(x,r,s) = \varphi_\lambda(x,r) + (u_\lambda - p)\, r + \psi^*_\lambda(x, s + q - v_\lambda), \tag{2.18}$$

where

$$\varphi_\lambda(x,r) = \int_0^r A_\lambda(x,\tau)\, d\tau, \quad \psi_\lambda(x,r) = \int_0^r B_\lambda(x,\tau)\, d\tau, \tag{2.19}$$

and $\psi^*_\lambda(x,.)$ is the conjugate function associated to $\psi_\lambda(x,.)$ (see [1], p. 52).

Note that (2.15) can be reformulated as

$$\begin{aligned} &\frac{dv_\lambda}{dx} = \partial\varphi_\lambda(x, u_\lambda) + u_\lambda - p, \\ &v_\lambda = \partial\psi^*_\lambda\left(x, \frac{du_\lambda}{dx} + q - v_\lambda\right), \end{aligned} \tag{2.20}$$

where $\partial\varphi_\lambda(x,.)$ and $\partial\psi^*_\lambda(x,.)$ stand for the subdifferentials of $\varphi_\lambda(x,.)$ and $\psi^*_\lambda(x,.)$ respectively.

From (2.18) and (2.20) one gets

$$\left(\frac{dv_\lambda}{dx},\ v_\lambda\right) = \partial L_\lambda\left(x, u_\lambda, \frac{du_\lambda}{dx}\right). \tag{2.21}$$

Here $\partial L_\lambda(x,.,.)$ is the subdifferential associated to $L_\lambda(x,.,.)$. Obviously

$$\begin{aligned} L_\lambda\left(x, u_\lambda, \frac{du_\lambda}{dx}\right) &\leqslant L_\lambda(x, 0, u^* - q + v_\lambda) + \frac{dv_\lambda}{dx} u_\lambda + \\ &+ v_\lambda\left(\frac{du_\lambda}{dx} - u^* + q - v_\lambda\right) \leqslant \psi^*_\lambda(x, u^*) + \frac{d}{dx}(u_\lambda v_\lambda) + v_\lambda(q - u^*), \end{aligned} \tag{2.22}$$

for all $u^* \in R$ and a.e. $x \in]0,1[$.

If in particular $u^* = \partial\psi_\lambda(x,0)$, then $\psi^*_\lambda(x,u^*) = 0$.

Since

$$\frac{d}{dx}(u_\lambda v_\lambda) = \frac{d}{dx}(u_\lambda - u_0)(v_\lambda - v_0) + \frac{du_0}{dx}(v_\lambda - v_0) + \frac{dv_0}{dx}(u_\lambda - u_0) + \frac{dv_\lambda}{dx} u_0 + \frac{du_\lambda}{dx} v_0,$$

where (u_0, v_0) is a fixed element of $D(\mathcal{A})$, it follows that

$$\int_0^1 \frac{d}{dx}(u_\lambda v_\lambda)\, dx \leqslant \text{const}\left[1 + \int_0^1 \left(\left|\frac{du_\lambda}{dx}\right| + \left|\frac{dv_\lambda}{dx}\right|\right) dx\right]. \tag{2.23}$$

On the other hand, using the definition of the conjugate function we obtain

$$\begin{aligned} L_\lambda\left(x, u_\lambda, \frac{du_\lambda}{dx}\right) &\geqslant u_\lambda A_\lambda(x,0) + (u_\lambda - p)\, u_\lambda + \\ &+ \rho\left|\frac{du_\lambda}{dx} + q - v_\lambda\right| - \psi_\lambda(x, \rho w_\lambda), \end{aligned} \tag{2.24}$$

where $\rho>0$ is arbitrary and $w_\lambda=\operatorname{sign}\left(\frac{du_\lambda}{dx}+q-v_\lambda\right)$. By (2.22), (2.23) and (2.24) it follows the inequality

$$\rho\int_0^1\left|\frac{du_\lambda}{dx}+q-v_\lambda\right|dx\leq\rho\int_0^1|B(x,\rho w_\lambda)|\,dx+$$
$$+\text{const}\left[1+\int_0^1\left(\left|\frac{du_\lambda}{dx}\right|+\left|\frac{dv_\lambda}{dx}\right|\right)dx\right].$$

This together with the simmetric one for dv_λ/dx leads to

(2.25) $$\left\{\frac{du_\lambda}{dx}\right\},\ \left\{\frac{dv_\lambda}{dx}\right\}\text{ are bounded in } L^1(0.1).$$

Clearly, we have

$$u_\lambda(x)=\int_0^1\left(\xi\frac{du_\lambda}{d\xi}+u_\lambda\right)d\xi-\int_x^1\frac{du_\lambda(\xi)}{d\xi}\,d\xi$$

as well as the similar equality for v_λ.

By (2.17) and (2.25) we get

(2.26) $$\{u_\lambda\},\ \{v_\lambda\}\text{ are bounded in } C[0,1].$$

Taking into account (H_2), (2.15) and (2.26) we infer that

(2.27) $$\{A_\lambda(\cdot,u_\lambda)\},\ \{B_\lambda(\cdot,v_\lambda)\}\text{ are bounded in } L^2(0,1),$$

(2.28) $$\left\{\frac{du_\lambda}{dx}\right\},\ \left\{\frac{dv_\lambda}{dx}\right\}\text{ are bounded in } L^2(0,1).$$

Note that (2.27) relies on the following well-known inequalities

$$|A_\lambda(x,r)|\leq|A(x,r)|,\ |B_\lambda(x,r)|\leq|B(x,r)|.$$

By (2.26) and (2.28) according to Arzelà's criterion we have (on some subsequences)

(2.29) $$u_\lambda\to u,\ v_\lambda\to v\text{ in } C[0,1],\text{ as }\lambda\to 0.$$

Finally, by (2.27) and (2.28) it follows that (on some subsequences) when $\lambda\to 0$

(2.30) $$\frac{du_\lambda}{dx}\to\frac{du}{dx},\ \frac{dv_\lambda}{dx}\to\frac{dv}{dx}\text{ weakly in } L^2(0,1),$$

(2.31) $$A_\lambda(\cdot,u_\lambda)\to A(\cdot,u),\ B_\lambda(\cdot,v_\lambda)\to B(\cdot,v)\text{ weakly in } L^2(0,1).$$

The reasoning for (2.31) can be found in [3]. Passing to the limit in (2.15), (2.16) one obtains the maximality of $\mathcal{B}$ and thereby Lemma 2 is completely proved.

To continue the proof of Theorem 1 suppose firstly that $\alpha(x)=\beta(x)=1$, a.e. $x\in]0,1[$. The problem (1.1), (1.2), (1.3) can be expressed as the initial value problem

(2.32) $$\frac{d}{dt}\binom{u}{v}+\mathcal{B}\binom{u}{v}=\binom{f}{g}\text{ in } X,$$

(2.33) $$\binom{u}{v}(0)=\binom{u_0}{v_0}\in D(\mathcal{B}).$$

We derive by the general existence theory that (1.1), (1.2), (1.3) has a unique solution (u,v) such that $u,v\in W^{1,\infty}(0,T;L^2(0,1))$. Repeating the reasoning in the proof of Lemma 2 we deduce starting from (1.1) that u, $v\in L^\infty(]0,1[\times]0,T[)$ and $\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial x}\in L^\infty(0,T;L^2(0,1))$. If α and β are not identically equal to 1 on $]0,1[$ we divide by α and β the equations in (1.1) and work in the weighted space $X_{\alpha\beta}=L^2(0,1;\alpha(x)\,dx)\times L^2(0,1;\beta(x)\,dx)$ which is equivalent algebraically and topologically to X defined above. Since $\mathcal{B}$ is maximal monotone it is only a simple exercise to prove that the operator

$$\mathcal{B}_{\alpha\beta}=\begin{pmatrix}-\frac{1}{\alpha}\frac{dv}{dx}+\frac{1}{\alpha}A(\cdot,u)\\ -\frac{1}{\beta}\frac{du}{dx}+\frac{1}{\beta}B(\cdot,v)\end{pmatrix}$$

is maximal monotone too on $X_{\alpha\beta}$. The proof of Theorem 1 is now complete.

Remarks. 1°. Theorem 1 remains valid for systems of type (1.1) with $2n$ equations, $n=2,3,\ldots$ Other outstanding generalizations of Theorem 1 correspond to the cases:

1) $\mathcal{B}$ is an ω-maximal monotone operator (i.e. $\mathcal{B}+\omega I$, $\omega>0$ is maximal monotone).

2) the time-dependent case, i.e. α, β, A and B depend of t (see [2]).

We leave to the reader to formulate and prove these results.

2°. All the results related to the behavior on solutions stated in [11] remain true for (1.1), (1.2), (1.3) subject to (H_1), (H_2), and (H_3).

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AN INTEGRODIFFERENTIAL EQUATION WITH A DISCONTINUOUS NONLINEARITY

BY

SERGIU AIZICOVICI

1. Let Ω be a bounded domain in R^N with smooth boundary Γ. Let $a:[0,\infty)\to R$, $f:R\to R$, $h:\Omega\times R\to R$ and $u_0:\Omega\to R$ be given functions. In this paper we will be concerned with the existence of solutions to the initial boundary value problem

$$(\mathrm{P})\quad \begin{cases} u_t(t,x)+\int_0^t a(t-s)\,(-\Delta_x u(s,x)+f(u(s,x)))\,ds=h(t,x), & t\geqslant 0,\ x\in\Omega,\\ u(t,x)=0, & t\geqslant 0,\ x\in\Gamma,\\ u(0,x)=u_0(x), & x\in\Omega. \end{cases}$$

This problem has recently been considered by the author [1], under the key assumption that $f(u)$ is continuous and has the same sign as u. In the present paper we only assume that

$$(\mathrm{F}-1)\qquad f\in L^{\infty}_{loc}(R)\,,$$

$$(\mathrm{F}-2)\qquad \operatorname*{ess\,sup}_{-\infty<t<-s_0} f(t)\leqslant \operatorname*{ess\,inf}_{s_0<t<+\infty} f(t).$$

for some $s_0\geqslant 0$.

Our approach rests heavily on the technique of [1]. It also involves some arguments previously used by Kiffe [4] and Rauch [5] in the study of discontinuous scalar Volterra integral equations and discontinuous elliptic equations, respectively.

2. Suppose that $f: R\to R$ satisfies (F—1). Then, for any $\varepsilon>0$ and $s\in R$, one can define, as in [5]:

$$\underline{f}_\varepsilon(s)=\operatorname*{ess\,inf}_{|t-s|<\varepsilon} f(t)\,,\qquad \overline{f}_\varepsilon(s)=\operatorname*{ess\,sup}_{|t-s|<\varepsilon} f(t),$$

and

$$\underline{f}(s)=\lim_{\varepsilon\to 0+}\underline{f}_\varepsilon(s),\qquad \overline{f}(s)=\lim_{\varepsilon\to 0+}\overline{f}_\varepsilon(s)\,.$$

We are now in position to precise what we mean by a solution of (P).