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SPLINE FUNCTIONS IN H-LOCALLY CONVEX SPACES

BY

VASILE POSTOLICĂ

It is known that the abstract theory of spline functions was developed on Hilbert spaces and contains characterizations and properties of approximation and optimal interpolation by these functions ([1], [2], [5]).

In this paper we introduce the notion of spline function in H-locally convex spaces, spaces introduced and studied in [4] and defined as locally convex spaces with the semi-norms satisfying the parallelogram law, and we establish the basic properties of approximation and interpolation of spline functions. [Theorem 2 and Theorem 3]. Thus, we generalize the classic properties of approximation and interpolation of spline functions and, taking particular H-locally convex spaces, we obtain usual spaces of minimizing spline functions.

Let $(X, P=\{p_\alpha\}_{\alpha\in A})$, $(Y, Q=\{q_\alpha\}_{\alpha\in A})$ be two real (or complex) H-locally convex spaces, $U: X\to Y$ a linear continuous operator and define

$$M=\{x\in X : (x, y)_\alpha = <Ux, Uy>_\alpha,\ \forall y\in X,\ \forall\alpha\in A\},$$

where $(.,.)_\alpha$ is the scalar semi-product which induces the semi-norm $p_\alpha(.)$ and $<.,.>_\alpha$ is the scalar semi-product which induces the semi-norm $q_\alpha(.)$.

Theorem 1. *M is a closed linear subspace of X.*

Proof. For every $y\in X$ and $\alpha\in A$, the mapping $f_{\alpha,y}(\cdot)=(.,y)_\alpha - <U, Uy>_\alpha : X\to C$ is a linear continuous functional with the kernel $\ker f_{\alpha,y} = \{x\in X : (x, y)_\alpha = <Ux, Uy>_\alpha\}$. We have $M=\bigcap\limits_{y\in X,\,\alpha\in A} \operatorname{Ker} f_{\alpha,y}$ and because the kernel of a linear continuous functional on a linear topological space is a closed linear subspace, it follows that M is a closed linear subspace and the theorem is proved. We call the linear space $M^\perp=\{x\in X : x\perp M\}$ ($x\perp M$ means that x is orthogonal to each $y\in M$ in the H-locally convex space sense, i.e., $(x, y)_\alpha=0$ for every $y\in M$ and $\alpha\in A$) *the space of spline functions with respect to U*.

We remark that

$$M^\perp=\{x\in X : <Ux, U\zeta>_\alpha=0,\ \forall\zeta\in M,\ \forall\alpha\in A\}. \tag{1}$$

Indeed, $\forall x\in M^\perp \Rightarrow 0=(\zeta, x)_\alpha = <U\zeta, Ux>_\alpha$, $\forall\zeta\in M$, $\forall\alpha\in A$ which implies that $M^\perp\subseteq\{x\in X: <Ux, U\zeta>_\alpha=0,\ \forall\zeta\in M,\ \forall\alpha\in A\}$. On the other hand, if $<Uy, U\zeta>_\alpha=0$, $\forall\zeta\in M$, $\forall\alpha\in A$, then $(\zeta, y)_\alpha=<U\zeta, Uy>_\alpha=0$, $\forall\zeta\in M$, $\forall\alpha\in A$, hence $y\in M^\perp$. Therefore $\{x\in X: <Ux, U\zeta>_\alpha=0,\ \forall\zeta\in M,\ \forall\alpha\in A\}\subseteq M^\perp$ and taking into account the converse inclusion we obtain (1).

Let us consider the direct sum $X' = M \oplus M^{\perp}$. For every $x \in X'$ we denote the projection of x onto $M^{\perp}$ by s_x and call it *the spline U-approximation of x*; we also call $x - s_x \in M$ *the error in the approximation of x by* s_x.

Lemma. *For every* $x \in X'$ *we have*

(i) $$<Us_x, Ux - Us_x>_{\alpha} = 0, \quad \forall \alpha \in A.$$

(ii) $$p_{\alpha}^2(x) - p_{\alpha}^2(s_x) = p_{\alpha}^2(x - s_x), \quad \forall \alpha \in A.$$

(iii) $$q_{\alpha}^2(Ux) - q_{\alpha}^2(Us_x) = q_{\alpha}^2(U(x - s_x)), \quad \forall \alpha \in A.$$

(iv) $$p_{\alpha}^2(x - s_x) = q_{\alpha}^2(U(x - s_x)), \quad \forall \alpha \in A.$$

Proof.

(i) $$\left.\begin{array}{r} s_x \in M^{\perp} \\ x - s_x \in M \end{array}\right\} \Rightarrow <Us_x, U(x - s_x)>_{\alpha} = 0, \ \forall \alpha \in A, \text{ i.e.,}$$

$$<Us_x, Ux - Us_x>_{\alpha} = 0, \quad \forall \alpha \in A.$$

(ii) $x = (x - s_x) + s_x$ with $x - s_x \in M$ and $s_x \in M^{\perp}$, hence

$$p_{\alpha}^2(x) = p_{\alpha}^2(x - s_x) + p_{\alpha}^2(s_x), \quad \forall \alpha \in A.$$

(iii) From $Ux = (Ux - Us_x) + Us_x$, combined with (i), we obtain $q_{\alpha}^2(Ux) = q_{\alpha}^2(Ux - Us_x) + q_{\alpha}^2(Us_x) + q_{\alpha}^2(Us_x)$, $\forall \alpha \in A$.

(iv) $$x - s_x \in M \Rightarrow p_{\alpha}^2(x - s_x) = q_{\alpha}^2(U(x - s_x)), \quad \forall \alpha \in A.$$

Theorem 2. *For every* $\sigma, s \in M^{\perp}$ *we have*

(2) $$q_{\alpha}(U(\sigma - s)) = \inf \{q_{\alpha}(U(\eta - s)) : \eta \in X \text{ and } \eta - \sigma \in M\}, \quad \forall \alpha \in A.$$

Proof.

$$\left.\begin{array}{r} \sigma - s \in M^{\perp} \\ \eta - \sigma \in M \end{array}\right\} \Rightarrow <U(\eta - \sigma), U(\sigma - s)>_{\alpha} = 0, \ \forall \alpha \in A, \text{ hence}$$

$q_{\alpha}^2(U(\eta - s)) = <U(\eta - s), U(\eta - s)>_{\alpha} = <U(\eta - \sigma) + U(\sigma - s), U(\eta - \sigma) + U(\sigma - s)>_{\alpha} = q_{\alpha}^2(U(\eta - \sigma)) + q_{\alpha}^2(U(\sigma - s))$, $\forall \alpha \in A$. Therefore $q_{\alpha}(U(\eta - s)) \geq q_{\alpha}(U(\sigma - s))$, $\forall \alpha \in A$, $\forall \eta \in X$ with $\eta - \sigma \in M$, with equality if and only if $q_{\alpha}(U(\eta - \sigma)) = 0$, $\forall \alpha \in A$.

Corollary. *If* $s = 0$ *in* (2) *we obtain*

(3) $$q_{\alpha}(U\sigma) = \inf \{q_{\alpha}(U\eta) : \eta \in X \text{ and } \eta - \sigma \in M\}, \quad \forall \alpha \in A,$$

hence for every $x \in X'$ we have

(4) $$q_{\alpha}(Us_x) = \inf \{q_{\alpha}(Uy) : y - s_x \in M\} = \inf \{q_{\alpha}(Uy) : y - x \in M\}, \quad \forall \alpha \in A.$$

Remark. Theorem 2 is a result of global optimal interpolation.

Theorem 3. *For every* $x \in X'$ *we have*

(5) $$q_{\alpha}(U(s_x - x)) = \inf \{q_{\alpha}(U(\eta - x)) : \eta \in M^{\perp}\}, \quad \forall \alpha \in A.$$

Proof.

$$\forall \eta \in M^{\perp}, \ \forall x \in X' \Rightarrow \eta - x = \eta - s_x + s_x - x.$$

$$\left.\begin{array}{r} \eta - s_x \in M^{\perp} \\ s_x - x \in M \end{array}\right\} \Rightarrow <U(\eta - s_x), U(s_x - x)>_{\alpha} = 0, \ \forall \alpha \in A.$$

or

$<Us_x - Ux, U\eta - Us_x>_{\alpha} = 0$, $\forall_{\alpha} \in A$. Therefore $q_{\alpha}^2(U(\eta - x)) = <U(\eta - x), U(\eta - x)>_{\alpha} = <U\eta - Us_x + Us_x - Ux, U\eta - Us_x + Us_x - Ux>_{\alpha} = q_{\alpha}^2(U(\eta - s_x)) + q_{\alpha}^2(U(s_x - x))$, $\forall \alpha \in A$. Then $q_{\alpha}^2(U(\eta - x)) \geq q_{\alpha}(U(s_x - x))$, $\forall_{\alpha} \in A$, $\forall \eta \in M^{\perp}$, with equality if and only if $q_{\alpha}(U(\eta - s_x)) = 0$, $\forall \alpha \in A$.

Theorem 4. *For every* $x \in X'$ *we have*

(6) $$p_{\alpha}(x - s_x) = \inf_{y \in M^{\perp}} p_{\alpha}(x - y), \quad \forall \alpha \in A$$

and

(7) $$\inf_{y \in M^{\perp}} \sup_{\alpha \in A} p_{\alpha}(x - y) = \sup_{\alpha \in A} p_{\alpha}(x - s_x).$$

Proof.

$$\forall y \in M^{\perp} \Rightarrow p_{\alpha}^2(x - y) = (x - s_x + s_x - y, x - s_x + s_x - y)_{\alpha} = p_{\alpha}^2(x - s_x) + p_{\alpha}^2(s_x - y), \quad \forall \alpha \in A,$$

because $x - s_x \in M$ and $s_x - y \in M^{\perp}$. Thus (6) is proved. From the last equality, we obtain $p_{\alpha}(x - s_x) \leq p_{\alpha}(x - y)$, $\forall \alpha \in A$, $\forall y \in M^{\perp}$ which implies

$$\sup_{\alpha \in A} p_{\alpha}(x - s_x) \leq \sup_{\alpha \in A} p_{\alpha}(x - y), \quad \forall y \in M^{\perp}$$

and (7) is justified.

Remark. In (6), we remark that for every $x \in X'$, the spline U-approximation s_x is the best approximation for x with respect to $M^{\perp}$, independent on $\alpha \in A$.

Example 1. Let $H^m(-\infty, +\infty) = H^m(R) = \{f \in C^{m-1}(R) : f^{(m-1)}$ locally absolutely continuous and $f^{(m)} \in L^2_{loc}(R)\}$, $m \geq 1$. We consider $Y = L^2_{loc}(R)$ with the H-locally convex topology induced by the scalar semi-products $<x, y>_k = \int^{k} x(t)y(t)dt$ $k = 0, 1, 2, \ldots$ and $X = H^m(R)$ with the H-locally convex topology generated by the scalar semi-products

$$(x, y)_k = \sum_{n=0}^{m-1} [x^{(h)}(k)y^{(h)}(k) + X^{(h)}(-k)y^{(h)}(-k)] + \int_{-k}^{k} x^{(m)}(t)y^{(m)}(t)\, dt,$$

$$k = 0, 1, 2, \ldots$$

Let $U : X \to Y$ be the derivation operator of order m. We have

$$M = \{x \in H^m(R) : x^{(h)}(\nu) = 0, \ \forall h = \overline{0, m-1}, \ \forall \nu \in Z\}$$

and

$$M\perp=\left\{s\in H^m(R):\int_{-k}^{k}s^{(m)}(t)x^{(m)}(t)dt=0,\quad \forall x\in M,\ \ \forall k=0,1,2,\ldots\right\}.$$

Let $(\nu,\nu+1)$ $(\nu\in Z)$ be an interval and $g\in C^\infty((\nu,\nu+1)$ with compact support contained in $(\nu,\nu+1)$. There exists $k=0,1,2,\ldots$ such that $(\nu,\nu+1)\subset$ $\subset[-k\ \ k]$ and, for every $s\in M\perp$, we have

$$0=\int_{-k}^{k}s^{(m)}(t)g^{(m)}(t)dt=\int_{\nu}^{\nu+1}s^{(m)}(t)g^{(m)}(t)\,dt=(-1)^{(m)}\int_{\nu}^{\nu+1}s(t).\ \ g^{(2m)}(t)\,dt.$$

Therefore (see [3], p. 92) $s\in C^{2m}(\nu,\nu+1)$ and $s^{(2m)}(t)=0$, $\forall t\in(\nu,\nu+1)$ Hence the restriction of s to $(\nu,\nu+1)$ is a polynomial of degree $2m-1$ at most. It follows that every function $s\in M\perp$ is a piecewise order $2m$ (degree $2m-1$) polynomial having knots at the integer points of the real axis. Since every piecewise order $2m$ (degree $2m-1$) polynomial function of $H^m(R)$ having the knots at the integer points is in $M\perp$, it follows that $M\perp=$ $=\{s\in H^m(R):s/(\nu,\nu+1)$ is a polynomial of degree $\leqslant 2m-1$, $\forall\nu\in Z\}$.

Let now $y=(y_\nu)$, $y'=(y'_\nu)$, $y''=(y''_\nu),\ldots,y^{(m-1)}=(y^{(m-1)}_\nu)$ be m sequences of real numbers. There exists a unique spline $S\in M\perp$ satisfying $S^{(h)}(\nu)=y^{(h)}_\nu$, $\forall h=\overline{0,m-1}$, $\forall\nu\in Z$. Indeed, every function $S\in M\perp$ coincides on every interval $(\nu,\nu+1)$ $(\nu\in Z)$ with a polynomial of degree $\leqslant 2m-1$, hence it is completely determined (on $(\nu,\nu+1)$) by the conditions $S^{(h)}(\nu)=$ $=y^{(h)}_\nu$ and $S^{(h)}(\nu+1)=Y^{(h)}_{\nu+1}$, $\forall h=\overline{0,m-1}$. Since the homogeneous system corresponding to the above conditions is $S^{(h)}(\nu)=S^{(h)}(\nu+1)=0$, $\forall h=$ $=\overline{0,m-1}$ and has only the null solution (because, in the contrary case, it results that ν and $\nu+1$ are roots of order of multiplicity at least m for a polynomial of degree $\leqslant 2m-1$, which is a contradiction), it follows the stated conclusion.

Therefore, for every function $f\in H^m(R)$, there exists a unique function denoted by $S_f\in M\perp$ such that $S^{(h)}_{(f)}(\nu)=f^{(h)}_{(\nu)}$ $\forall h=\overline{0,m-1}$, $\forall\nu\in Z$; hence M and $M\perp$ realize an orthogonal decomposition of the space $H^m(R)$.

By virtue of Theorem 2 we have

$$\text{(9)}\qquad \forall S,s\in M\perp\Rightarrow\int_{-k}^{k}[S^{(m)}(t)-s^{(m)}(t)]^2dt=\inf\left\{\int_{-k}^{k}[g^{(m)}(t)-s^{(m)}(t)]^2dt:g\in H^m(R)\right.$$

$$\text{and}\qquad \left.g^{(h)}(\nu)=S^{(h)}(\nu),\ \ \forall h=\overline{0,m-1},\ \ \forall\nu\in Z\right\},\ \ \forall k=0,1,2,\ldots$$

which is equivalent with

$$\text{(10)}\qquad \int_{-k}^{k}[S^{(m)}(t)-s^{(m)}(t)]^2dt\leqslant\int_{-k}^{k}[g^{(m)}(t)-s^{(m)}(t)]^2dt,\ \ \forall g\in H^m(R)\ \ \text{with}$$

$$g^{(h)}(\nu)=S^{(h)}(\nu),\ \ \forall h=\overline{0,m-1},\ \ \forall\nu\in Z,\ \ \forall k=0,1,2,\ldots.$$

with equality (see the last part of the proof of Theorem 2) if and only if $\int_{-k}^{k}[g^{(m)}(t)-S^{(m)}(t)]^2dt=0$, $\forall k=0,1,2,\ldots$ i.e. if and only if $\int_{\nu}^{\nu+1}[(g-S)^{(m)}(t)]^2dt=0$, $\forall\nu\in Z$. Therefore we have equality in (10) if and only if $g-S$ is a polynomial of degree $\leqslant m-1$ in every interval $(\nu,\nu+1)$. Taking $s=0$ in (9),

$$\text{(11)}\qquad \forall S\in M\perp\Rightarrow\int_{-k}^{k}[S^{(m)}(t)]^2dt=\inf\left\{\int_{-k}^{k}[g^{(m)}(t)]^2dt:g\in H^m(R)\ \text{and}\right.$$

$$\left.g^{(h)}(\nu)=S^{(h)}(\nu)\ \ \forall h=\overline{0,m-1},\ \ \forall\nu\in Z\right\},\ \ \forall k=0,1,2,\ldots$$

If for every function $f\in H^m(R)$ we denote its orthogonal projection on $M\perp$ by S_f and we take $S=S_f$ in (11) we obtain

$$\text{(12)}\qquad \forall f\in H^m(R)\Rightarrow\int_{-k}^{k}[S_f^{(m)}(t)]^2dt=\inf\left\{\int_{-k}^{k}[g^{(m)}(t)]^2dt:g\in H^{(m)}R\ \text{and}\right.$$

$$\left.g^{(h)}(\nu)=f^{(h)}(\nu),\ \ \forall h=\overline{0,m-1},\ \ \forall\nu\in Z\right\},\ \ \forall k=0,1,2\ldots$$

Applying Theorem 3 we have

$$\text{(13)}\qquad \forall f\in H^m(R)\Rightarrow\int_{-k}^{k}[S_f^{(m)}(t)-f^{(m)}(t)]^2\,dt\leqslant\int_{-k}^{k}[s^{(m)}(t)-f^{(m)}(t)]^2\,dt,$$

$\forall s\in M\perp$, $\forall k=0,1,2,\ldots$ with equality if and only if $\int_{-k}^{k}[(s-S_f)^{(m)}(t)]^2\,dt=0$, $\forall k=0,1,2,\ldots$, i.e., if and only if the restriction of $s-S_f$ to every interval $(\nu,\nu+1)$ is a polynomial of degree $m-1$ at most.

Example 2. Let $X=F_m=\{f\in C^{m-1}(R):f^{(m-1)}$ locally absolutely continuous and $f^{(m)}\in L^2(-\infty,+\infty)\}$ (see [6], p. 25) with the H-locally convex topology induced by the scalar semi-products $(x,y)_\nu=x(\nu)y(\nu)+$ $+\int_R x^{(m)}(t)\,y^{(m)}(t)\,dt$, $\nu\in Z$, $Y=L^2(-\infty,+\infty)$ with the topology generated by the inner products $<x,y>_\nu=\int_R x(t)\,y(t)\,dt$ $(\nu\in Z)$ and $U:X\to Y$ be the derivation operator of order m.

We have $M=\{x\in F_m:\ x(\nu)=0,\forall\nu\in Z\}$, $M\perp=\left\{x\in F_m:\int_R x^{(m)}(t)\zeta^{(m)}(t)dt=\right.$ $\left.=0,\ \forall\zeta\in M\right\}$.

In a similar manner as in Example 1, it may be proved that $M\perp$ coincides with the class of piecewise polynomial functions of order $2m$ (degree $2m-1$) having their knots at the integer points. By virtue of Theorem 7 of [6] it follows that for every $f\in F_m$, there exists a unique function $S_f\in M\perp$ satisfying the relations $S_f(\nu)=f(\nu)$ for all $\nu\in Z$, hence M and $M^{\perp}$ realize an orthogonal decomposition of the space F_m.

Applying Theorem 2 we obtain

$$(14)\qquad \forall S,s\in M^{\perp}\Rightarrow\int_R[S^{(m)}(t)-s^{(m)}(t)]^2\,dt\leqslant\int_R[g^{(m)}(t)-s^{(m)}(t)]^2\,dt,$$

$\forall g\in F_m$ with $g(\nu)=S(\nu)$, $\forall\nu\in Z$, with equality if and only if $\int_R[g^{(m)}-S^{(m)}(t)]^2\,dt=0$, i.e., if and only if $g-S$ is a polynomial of degree $\leqslant m-1$. Since $(g-S)(\nu)=0$, $\forall\nu\in Z$, it follows that we have equality in (14) if and only if $g=S$. If for every function $f\in H^m(R)$, we denote its orthogonal projection on $M^{\perp}$ by S_f and we take $S=S_f$ and $S=0$ in (14) we obtain

$$(15)\qquad \int_R[S_f^{(m)}(t)]^2\,dt=\inf\left\{\int_R[g^{(m)}(t)]^2\,dt : g\in F_m \text{ and } g(\nu)=f(\nu),\ \forall\nu\in Z\right\}.$$

By virtue of Theorem 3 it follows

$$(16)\qquad \int_R[S_f^{(m)}(t)-f^{(m)}(t)]^2dt\leqslant\int_R[s^{(m)}(t)-f^{(m)}(t)]^2\,dt,\quad \forall s\in M^{\perp}$$

with equality if and only if $\int_R[S_f^{(m)}(t)-s^{(m)}(t)]^2\,dt=0$, i.e., if and only if $s-S_f$ is a polynomial of degree $\leqslant m-1$. Thus we have obtained the parts (*ii*) and (*iii*) of the Theorem 7 from [6].

Remark. The above examples show that the abstract construction can be used to solve some frequent problems of interpolation and approximation, having the possibility to choose the spaces and the scalar semi-products. We also remark that the construction leads to finding orthogonal decompositions for H-locally convex spaces which, in general, do not exist.

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Faculty of Mathematics
University of Iaşi
Iaşi, R.S. Romania

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TIME-PERIODIC SOLUTIONS TO A CLASS OF FIRST ORDER HYPERBOLIC SYSTEMS

BY

MILAN ŠTĚDRÝ

1. Introduction. We shall investigate the existence of functions $u=(u_1,u_2,\dots,u_n)$ and $v=(v_1,v_2,\dots,v_n)$ defined on $(t,x)\in R^2$ which satisfy the system of equations,

$$(1.1)\qquad u_{it}-v_{ix}-\varepsilon f_i^1(t,x,u(t,x),v(t,x))=0,\quad (t,x)\in R^2,$$

$$(1.2)\qquad v_{it}-u_{ix}-\varepsilon f_i^2(t,x,u(t,x),v(t,x))=0,\quad (t,x)\in R^2,\ i=1,2,\dots,n.$$

the conditions of π-antiperiodicity in x,

$$(1.3)\qquad u_i(t,x+\pi)=-u_i(t,x),\quad (t,x)\in R^2,$$

$$(1.4)\qquad v_i(t,x+\pi)=-v_i(t,x),\quad (t,x)\in R^2,\ i=1,2,\dots,n,$$

and the conditions of 2π-periodicity in t,

$$(1.5)\qquad u_i(t+2\pi,x)=u_i(t,x),\quad (t,x)\in R^2,$$

$$(1.6)\qquad v_i(t+2\pi,x)=v_i(t,x),\quad (t,x)\in R^2,\ i=1,2,\dots,n.$$

In Section 3, using two auxiliary lemmas, whose proofs will be postponed and accomplished in Sections 4 and 5, we prove that for $|\varepsilon|$ small, there is a classical solution of the problem (1.1)–(1.6) provided that the functions f_i^k satisfy certain assumptions ((3.1) and (3.2) of Section 3). The crucial assumption is the second one, which requires the monotone behaviour of f_i^1 in u_i and f_i^2 in v_i to dominate the possible dependence of f_i^1 and f_i^2 on other variables.

In Section 6, imposing some additional assumptions on f_i, we show that the arguments used in the previous sections can be used to prove the existence of u and v, which besides (1.1)–(1.6) satisfy,

$$(1.7)\qquad u_i(t,-x)=-u_i(t,x),\quad (t,x)\in R^2,$$

$$(1.8)\qquad v_i(t,-x)=v_i(t,x),\quad (t,x)\in R^2,\qquad i=1,2,\dots,n.$$

Since for functions u_i satisfying (1.7) and (1.3) we have

$$(1.9)\qquad u_i(t,0)=u_i(t,\pi)=0,$$

it is also proved that there exists a classical solution to (1.1), (1.2), and (1.9) which is 2π-periodic in t.