tisfying the relations $S_f(\nu) = f(\nu)$ for all $\nu \in Z$, hence M and $M^\perp$ realize an orthogonal decomposition of the space F_m.

Applying Theorem 2 we obtain

$$\forall S, s \in M^\perp \Rightarrow \int_R [S^{(m)}(t) - s^{(m)}(t)]^2 \, dt \leqslant \int_R [g^{(m)}(t) - s^{(m)}(t)]^2 \, dt, \tag{14}$$

$\forall g \in F_m$ with $g(\nu) = S(\nu)$, $\forall \nu \in Z$, with equality if and only if $\int_R [g^{(m)} - S^{(m)}(t)]^2 \, dt = 0$, i.e., if and only if $g - S$ is a polynomial of degree $\leqslant m-1$. Since $(g - S)(\nu) = 0$, $\forall \nu \in Z$, it follows that we have equality in (14) if and only if $g = S$. If for every function $f \in H^m(R)$, we denote its orthogonal projection on $M^\perp$ by S_f and we take $S = S_f$ and $S = 0$ in (14) we obtain

$$\int_R [S_f^{(m)}(t)]^2 \, dt = \inf \left\{ \int_R [g^{(m)}(t)]^2 \, dt : g \in F_m \text{ and } g(\nu) = f(\nu), \ \forall \nu \in Z \right\}. \tag{15}$$

By virtue of Theorem 3 it follows

$$\int_R [S_f^{(m)}(t) - f^{(m)}(t)]^2 dt \leqslant \int_R [s^{(m)}(t) - f^{(m)}(t)]^2 \, dt, \quad \forall s \in M^\perp \tag{16}$$

with equality if and only if $\int_R [S_f^{(m)}(t) - s^{(m)}(t)]^2 \, dt = 0$, i.e., if and only if $s - S_f$ is a polynomial of degree $\leqslant m-1$. Thus we have obtained the parts *(ii)* and *(iii)* of the Theorem 7 from [6].

Remark. The above examples show that the abstract construction can be used to solve some frequent problems of interpolation and approximation, having the possibility to choose the spaces and the scalar semi-products. We also remark that the construction leads to finding orthogonal decompositions for H-locally convex spaces which, in general, do not exist.

REFERENCES

1. Jerome J.W. and Schumaker L.L.S. — *On Lg-splines*. Journ. Approx. theory, 2, 1969, 29–49.
2. Laurent P.J. — *Approximation et optimisation*. Hermann, Paris 1972.
3. Micula, Gh. — *Funcţii spline şi aplicaţii*. Ed. tehnică, Bucureşti, 1978.
4. Precupanu, T. — *Espaces linéaires à semi-normes hilbertiennes*. An. şt. Univ. Iaşi, Tom XV, 1969, 83–93.
5. Sard, A. — *Approximation based on nonscalar observations*. Journ. Approx. Theory 8, 1973, 315–334.
6. Schoenberg I.T. — *Spline interpolation and the higher derivatives*. Proc. Nat. Acad. Sci. U.S.A. 51, 1964, 24–28.

Received 4. VI. 1980

Faculty of Mathematics
University of Iaşi
Iaşi, R.S. Romania

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXVII, s. I a, 1981 f. 2

TIME-PERIODIC SOLUTIONS TO A CLASS OF FIRST ORDER HYPERBOLIC SYSTEMS

BY

MILAN ŠTĚDRÝ

1. Introduction. We shall investigate the existence of functions $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ defined on $(t, x) \in R^2$ which satisfy the system of equations,

$$u_{it} - v_{ix} - \varepsilon f_i^1(t, x, u(t, x), v(t, x)) = 0, \quad (t, x) \in R^2, \tag{1.1}$$

$$v_{it} - u_{ix} - \varepsilon f_i^2(t, x, u(t, x), v(t, x)) = 0, \quad (t, x) \in R^2, \ i = 1, 2, \ldots, n. \tag{1.2}$$

the conditions of π-antiperiodicity in x,

$$u_i(t, x + \pi) = -u_i(t, x), \quad (t, x) \in R^2, \tag{1.3}$$

$$v_i(t, x + \pi) = -v_i(t, x), \quad (t, x) \in R^2, \ i = 1, 2, \ldots, n, \tag{1.4}$$

and the conditions of 2π-periodicity in t,

$$u_i(t + 2\pi, x) = u_i(t, x), \quad (t, x) \in R^2, \tag{1.5}$$

$$v_i(t + 2\pi, x) = v_i(t, x), \quad (t, x) \in R^2, \ i = 1, 2, \ldots, n. \tag{1.6}$$

In Section 3, using two auxiliary lemmas, whose proofs will be postponed and accomplished in Sections 4 and 5, we prove that for $|\varepsilon|$ small, there is a classical solution of the problem (1.1)–(1.6) provided that the functions f_i^k satisfy certain assumptions ((3.1) and (3.2) of Section 3). The crucial assumption is the second one, which requires the monotone behaviour of f_i^1 in u_i and f_i^2 in v_i to dominate the possible dependence of f_i^1 and f_i^2 on other variables.

In Section 6, imposing some additional assumptions on f_i, we show that the arguments used in the previous sections can be used to prove the existence of u and v, which besides (1.1)–(1.6) satisfy,

$$u_i(t, -x) = -u_i(t, x), \quad (t, x) \in R^2, \tag{1.7}$$

$$v_i(t, -x) = v_i(t, x), \quad (t, x) \in R^2, \quad i = 1, 2, \ldots, n. \tag{1.8}$$

Since for functions u_i satisfying (1.7) and (1.3) we have

$$u_i(t, 0) = u_i(t, \pi) = 0, \tag{1.9}$$

it is also proved that there exists a classical solution to (1.1), (1.2), and (1.9) which is 2π-periodic in t.

This paper was initiated as an attempt to give some answer to a question raised by Prof. Barbu: Under which conditions there exists a periodic solution to a system of equations which he studied in [1]? The result stated in Section 6 answers this question when the system has the form (1.1) – (1.2) and the function L used in Barbu's work to impose boundary conditions equals zero identically.

Admitting all dependent variables in every equation, the system (1.1), (1.2), compared with that in [1], is slightly more general. In turn, it is simultaneously well more restrictive since it deals only with „small" and „strongly monotone" nonlinear perturbations.

The method applied in this paper is taken from [2], where it was used for proving the existence of time-periodic solutions to a weakly nonlinear wave equation.

2. Definitions, notations. $\mathcal{C}^1([0, 2\pi])$ denotes the space of real functions $w(t, x)$ defined for $(t, x) \in [0, 2\pi] \times R$, with continuous first derivatives on $[0, 2\pi] \times R$ and π-antiperiodic in x. The space $\mathcal{C}^1([0, 2\pi])$ is a Banach space if equipped with the norm

$$\| w \|_{\mathcal{C}^1} = \max \{ \| D_t^k D_x^l w \|_{\mathcal{C}} \,;\; k + l \leqq 1 \},$$

where

$$\| w \|_{\mathcal{C}} = \max \{ | w(t, x) | \,;\; 0 \leqq t \leqq 2\pi,\; x \in R \}.$$

$\mathcal{C}^1(R^2)$ denotes the space of real functions $w(t, x)$ defined for $(t, x) \in R^2$, with continuous first derivatives and 2π-periodic in t and π-antiperiodic in x. The function $w \to \| w \mid [0, 2\pi] \times R \|_{\mathcal{C}^1}$ is a norm on $\mathcal{C}^1(R^2)$.

Further C^j denotes the space of real, π-antiperiodic functions on R whose derivatives up to order j are continuous. The space C^j is a Banach space if equipped with the norm

$$\| s \|_{C^j} = \max \{ \| s \|_C, \| s' \|_C, \dots, \| s^{(j)} \|_C \},$$

where

$$\| s \|_C = \max \{ | s(x) | \,;\; x \in R \}.$$

For every space defined above, we denote the Cartesian product of n such spaces by a subscript n. We obtain the spaces

$$\mathcal{C}_n^1([0, 2\pi]),\; \mathcal{C}_n^1(R^2), \text{ and } C_n^j.$$

The norm assigned to an element from any of these spaces is equal to the maximum of the norms of its components, i.e. if, for example, $w \in \mathcal{C}_n^1([0, 2\pi])$, the norm $\| w \|_{\mathcal{C}_n^1}$ is equal to $\max(\| w_i \|_{\mathcal{C}^1} \,;\; i = 1, 2, \dots, n)$.

Finally, the space R^n will be equipped with the norm

$$| p | = \max \{ | p_j | \,;\; j = 1, 2, \dots, n \}.$$

3. Result. Throughout the paper, we shall suppose:

(3.1) On the set $(t, x, u, v) \in R^{2n+2}$, the functions $f_i^\varkappa$, $\varkappa = 1, 2$, $i = 1, 2, \dots, n$ are of class C^2 and satisfy $f_i^\varkappa(t + 2\pi, x, u, v) = f_i^\varkappa(t, x, u, v)$, $f_i^\varkappa(t, x + \pi; -u, -v) = -f_i^\varkappa(t, x, u, v)$.

To state a further assumption on $f_i^\varkappa$, we will need a number of constants $\alpha_{ij}^\varkappa$, $\beta_{ij}^\varkappa$ defined as maxima or minima of some functions taken over the set

$$M(R_0) = \{(t, x, u, v) \,;\; (t, x) \in R^2,\; u, v \in R^n,\; | u | \leqq 2R_0,\; | v | \leqq 2R_0\},$$

where the constant R_0 is a positive number which will be subject to the subsequent condition (3.2). We set

$$\alpha_{ii}^1 = \min \{D_{u_i}^1 f_i^1\}, \qquad \beta_{ij}^1 = \max \{ | D_{v_j}^1 f_i^1 | \},$$

$$\alpha_{ij}^1 = \max \{ | D_{u_j}^1 f_i^1 | \},\; j \neq i \qquad \beta_{ij}^2 = \max \{ | D_{u_j}^1 f_i^2 | \},$$

$$\alpha_{ii}^2 = \min \{D_{v_i}^1 f_i^2\}, \qquad \alpha_{ij}^2 = \max \{ | D_{v_j}^1 f_i^2 | \},\; j \neq i.$$

Denoting $B_0^0 = \max \{ | f_i^\varkappa(t, x, 0, 0) | \,;\; (t, x) \in R^2,\; \chi = 1, 2,\; i = 1, \dots, n\}$ we are able to formulate the last assumption:

(3.2) For every $i = 1, \dots, n$, the numbers B_0^0, R_0, $\alpha_{ij}^\varkappa$, $\beta_{ij}^\varkappa$ satisfy

$$-2B_0^0 + R_0 \sum_{\varkappa=1}^{2} \Big(\alpha_{ii}^\varkappa - \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_{ij}^\varkappa - \sum_{j=1}^{n} \beta_{ij}^\varkappa\Big) > 0.$$

The following theorem is the main result of the paper.

Theorem 3.1. *Let the assumptions (3.1) and (3.2) be satisfied. Then there exist a unique pair of functions, $a^0, b^0 \in C_n^1$, $\| a^0 \|_{C_n^1} \leqq R_0$, $\| b^0 \|_{C_n^1} \leqq R_0$, and two continuous mappings $u = u(\varepsilon)$, $v = v(\varepsilon)$ of an interval $[-\varepsilon_0, \varepsilon_0]$, $\varepsilon_0 > 0$, into $\mathcal{C}_n^1(R^2)$ such that, for every ε, the functions $u = u(\varepsilon)$, $v = v(\varepsilon)$ satisfy the equations (1.1) – (1.6) and*

$$u(0)\,(t, x) = a^0(x + t) + b^0(x - t), \qquad v(0)\,(t, x) = a^0(x + t) - b^0(x - t).$$

The proof will be carried out by reducing the problem to a system of integral equations, which will be solved by applying the implicit function theorem. For $u, v \in \mathcal{C}_n^1([0, 2\pi])$, $a, b \in C_n^1$, and $\varepsilon \in R$, we set, $i = 1, \dots, n$,

$$F_i^\varkappa(u, v)\,(t, x) = f_i^\varkappa(t, x, u(t, x), v(t, x)),$$

$$G_{1i}(u, v, a, b, \varepsilon)\,(t, x) = -u_i(t, x) + a_i(x + t) + b_i(x - t) +$$

$$+ \frac{\varepsilon}{2} \int_0^t (F_i^1(u, v) + F_i^2(u, v))\,(\tau, x + t - \tau)\, d\tau +$$

$$\frac{\varepsilon}{2} \int_0^t (F_i^1(u, v) - F_1^2(u, v))\,(\tau, x - t + \tau)\, d\tau, \quad 0 \leqq t \leqq 2\pi,\; x \in R,$$

$$G_{2i}(u, v, a, b, \varepsilon)\,(t, x) = -v_i(t, x) + a_i(x + t) - b_i(x - t) +$$

$$+ \frac{\varepsilon}{2} \int_0^t (F_i^1(u, v) + F_i^2(u, v))\,(\tau, x + t - \tau)\, d\tau -$$

$$-\frac{\varepsilon}{2}\int_0^t (F_i^1(u,v)-F_i^2(u,v))\,(\tau, x-t+\tau)\,d\tau, \qquad 0\le t\le 2\pi,\ x\in R.$$

$$G_{3i}(u,v)\,(x)=\int_0^{2\pi}(F_i^1(u,v)+F_i^2(u,v))\,(t, x-t)\,dt,\ \ x\in R,$$

$$G_{4i}(u,v)\,(x)=\int_0^{2\pi}(F_i^1(u,v)-F_i^2(u,v))\,(t, x+t)\,dt,\ \ x\in R.$$

For simplicity, we denote further

$$G_\sigma=(G_{\sigma 1}, G_{\sigma 2},\dots, G_{\sigma n}),\ \ \sigma=1,2,3,4,$$

and, finally,

$$G=(G_1, G_2, G_3, G_4).$$

It is easily observed that

$$G(u,v,a,b,\varepsilon)\in(\mathcal{C}_n^1([0,2\pi]))^2\times(C_n^1)^2$$

for every $(u,v,a,b,\varepsilon)\in(\mathcal{C}_n^1([0,2\pi]))^2\times(C_n^1)^2\times R$. It can be proved by simple but lengthy calculations that G has a continuous Fréchet derivative.

Let us suppose that, for some ε, the functions $u, v\in\mathcal{C}_n^1([0,2\pi])$, and $a, b\in C_n^1$ satisfy

$$G(u,v,a,b,\varepsilon)=0. \tag{3.3}$$

If the functions u and v are extended onto $(t,x)\in R^2$ as functions 2π-periodic in t, then we easily find that these extended functions belong to $\mathcal{C}_n^1(R^2)$ and that these extended functions, say again, u, v, and the number ε satisfy (1.1) – (1.6). Thus we are led to solving (3.3).

We begin with the definition of a mapping $K:\ C_n^1\times C_n^1\to C_n^1\times C_n^1$ which will be used later. For $(a,b)\in C_n^1\times C_n^1$, we set

$$K(a,b)=(G_3(Z^+(a,b),\ Z^-(a,b)),\ G_4(Z^+(a,b),\ Z^-(a,b))),$$

where

$$Z^+(a,b)=a(x+t)+b(x-t),\ \ Z^-(a,b)=a(x+t)-b(x-t).$$

The following result will be proved in Section 4.

Lemma 3.1 *There exists a unique pair of functions* $(a^0, b^0)\in C_n^1\times C_n^1$, $\|a^0\|_{C_n^1}\le R_0$, $\|b^0\|_{C_n^1}\le R_0$, *satisfying*

$$K(a^0, b^0)=0.$$

If for these functions a^0, b^0 we define,

$$u^0=Z^+(a^0,b^0),\ \ v^0=Z^-(a^0,b^0),$$

it is obvious that the functions u^0, v^0, a^0, b^0 satisfy (3.3) with $\varepsilon=0$.

Eventually, let us denote by H the partial Fréchet derivative of the mapping G with respect to (u,v,a,b) taken at the point $(u^0, v^0, a^0, b^0, 0)$. Written by its components, the mapping H has the form

$$H_1(u,v,a,b)=-u+Z^+(a,b), \tag{3.4}$$

$$H_2(u,v,a,b)=-v+Z^-(a,b). \tag{3.5}$$

$$H_3(u,v)=[(D_u^1G_3)\,(u^0,v^0)]u+[(D_v^1G_3)\,(u^0,v^0)]\,v. \tag{3.6}$$

$$H_4(u,v)=[(D_u^1G_4)\,(u^0,v^0)]u+[(D_v^1G_4)\,(u^0,v^0)]\,v. \tag{3.7}$$

We now show that H is a linear homeomorphism. For any $f=(f_1, f_2, f_3, f_4)\in(\mathcal{C}_n^1)^2\times(C_n^1)^2$ we find $(u,v,a,b)\in(\mathcal{C}_n^1)^2\times(C_n^1)^2$ satisfying $H(u,v,a,b)=f$ and establish an estimate of the norm of (u,v,a,b).

Using (3.4) and (3.5) we get

$$u=Z^+(a,b)-f_1,\ \ v=Z^-(a,b)-f_2. \tag{3.8}$$

These two equations inserted into (3.6) and (3.7) give

$$H_3(Z^+(a,b),\ Z^-(a,b))=f_3+H_3(f_1,f_2), \tag{3.9}$$

$$H_4(Z^+(a,b),\ Z^-(a,b))=f_4+H_4(f_1,f_2). \tag{3.10}$$

It is easily observed that, for both $\varkappa=3$ and 4.

$$\|f_\varkappa+H_\varkappa(f_1,f_2)\|_{C_n^1}\le c(\|f_\varkappa\|_{C_n^1}+\sum_{\sigma=1}^{2}\|f_\sigma\|_{\mathcal{C}_n^1}) \tag{3.11}$$

with a constant c independent of f. Further $K'(a^0, b^0)$, the Fréchet derivative of the mapping K taken at the point (a^0, b^0), has the form

$$K'(a^0,b^0)=(H_3(Z^+(a^0,b^0),\ Z^-(a^0,b^0)),\ H_4(Z^+(a^0,b^0),\ Z^-(a^0,b^0))).$$

And, finally, we use the following lemma which will be proved in Section 5.

Lemma 3.2. *The Fréchet derivative of the mapping K at the point (a^0, b^0) is a linear homeomorphism of $C_n^1\times C_n^1$ onto itself.*

Thus by this lemma, there is a unique pair $(a,b)\in C_n^1\times C_n^1$ which solve (3.9) and (3.10). Moreover, (3.11) yields

$$\|a\|_{C_n^1}+\|b\|_{C_n^1}\le c\sum_{\varkappa=1}^{2}(\|f_\varkappa\|_{\mathcal{C}_n^1}+\|f_{\varkappa+2}\|_{C_n^1})$$

with a constant c independent of f. This fact and (3.8) show that H is a homeomorphism. The proof of the Theorem 3.1 is complete.

4. Proof of Lemma 3.1. The following lemma will be useful in this and the next sections.

Lemma 4.1. *Let $h\in L_\infty(R)$, $\sigma\in C^0$ (i.e., σ is π-antiperiodic), and $M\ge\|\sigma\|_{C^0}$. Then for every $\eta\in R$, $|\eta|\le M$, and any integers $\varkappa_1, \varkappa_2$, we have*

$$\left|\int_0^{2\pi}h(\tau)\,(\eta+(-1)^{\varkappa_1}\sigma(x+(-1)^{\varkappa_2}2\tau)\,d\tau\right|\le 2\pi\|h\|_\infty M.$$

Proof. We prove the lemma for $\varkappa_1=\varkappa_2=1$. Other cases can be treated similarly. Using π-antiperiodicity of σ, we have

$$\int_0^{2\pi} h(\tau)\,(\eta+\sigma(x+2\tau))d\tau=\left(\int_0^{\pi/2}+\int_{\pi/2}^{\pi}\right)+\left(\int_{\pi}^{3\pi/2}+\int_{3\pi/2}^{2\pi}\right)=$$

$$=\int_0^{\pi/2}[h(\tau)\,(\eta+\sigma(x+2\tau))+h\left(\tau+\frac{\pi}{2}\right)(\eta-\sigma(x+2\tau))]\,d\tau+$$

$$+\int_{\pi}^{3\pi/2}[h(\tau)\,(\eta+\sigma(x+2\tau))+h\left(\tau+\frac{\pi}{2}\right)(\eta-\sigma(x+2\tau))]\,d\tau.$$

Thus, by applying the equality $|a+b|+|a-b|=2\max(|a|,|b|)$, we obtain

$$\left|\int_0^{2\pi} h(\tau)\,(\eta+\sigma(x+2\tau))\,d\tau\right|\leqq 2\,\|h\|_\infty\int_0^{\pi/2}(|\eta+\sigma(x+2\tau)|+$$

$$+|\eta-\sigma(x+2\tau)|)d\tau=4\,\|h\|_\infty\int_0^{\pi/2}\max(|\eta|,|\sigma(x+2\tau)|)\,d\tau\leqq 2\pi\,\|h\|_\infty M.$$

This completes the proof.

For simplicity, we denote the space $C_n^j\times C_n^j$ by C_{2n}^j and to an element $(a,b)\in C_{2n}^j$ we assign the norm

$$\|(a,b)\|_{C_{2n}^j}=\max(\|a\|_{C_n^j},\ \|b\|_{C_n^j}).$$

We introduce an operator $S^\delta: C_{2n}^1\to C_{2n}^1$ by

$$S^\delta(a,b)=(a,b)-\delta K(a,b),$$

where δ is a small positive parameter which will be specified later. By definition of K, the mapping S^δ has the form $S^\delta=(S_1^\delta, S_2^\delta)$ where $S_\varkappa^\delta: C_{2n}^1\to C_n^1$, $\varkappa=1,2$, and

$$(S_{1i}^\delta(a,b))\,(x)=a_i(x)-\delta\int_0^{2\pi}(f_i^1+f_i^2)\,(t,\,x-t,\,a(x)+ \tag{4.1}$$
$$+b(x-2t),\ a(x)-b(x-2t))\,dt,$$

$$(S_{2i}^\delta\,(a,b))\,(x)=b_i(x)-\delta\int_0^{2\pi}(f_i^1-f_i^2)\,(t,\,x+t,\,a(x+2t)+ \tag{4.2}$$
$$+b(x),\ a(x+2t)-b(x))dt.$$

We now introduce a number of functions which make it possible to express the components of the operator S^δ in another form. Putting

$$U_{ij}^{\varkappa-}(a,b,x\,t)=(D_{u_j}^1 f_i^\varkappa)\,(t,\,x-t,\,a(x)+b(x-2t),\ a(x)-b(x-2t)),$$
$$V_{ij}^{\varkappa-}(a,b,x,t)=(D_{v_j}^1 f_i^\varkappa)\,(t,\,x-t,\,a(x)+b(x-2t),\ a(x)-b(x-2t)),$$
$$U_{ij}^{\varkappa+}(a,b,x,t)=(D_{u_j}^1 f_i^\varkappa)\,(t,\,x+t,\,a(x+2t)+b(x),\ a(x+2t)-b(x)),$$
$$V_{ij}^{\varkappa+}(a,b,x,t)=(D_{v_j}^1 f_i^\varkappa)\,(t,\,x+t,\,a(x+2t)+b(x),\ a(x+2t)-b(x)),$$

we can write

$$(S_{1i}^\delta\,(a,b))(x)=a_i(x)-\delta\alpha_{ii}^1\int_0^{2\pi}(a_i(x)+b_i(x-2t))\,dt-$$
$$-\delta\int_0^1\int_0^{2\pi}(U_{ii}^{1-}\,(\rho a,\rho b,x,t)-\alpha_{ii}^1)\,(a_i(x)+b_i(x-2t))\,dt\,d\rho-$$
$$-\delta\sum_{\substack{j=1\\ j\neq i}}^{n}\int_0^1\int_0^{2\pi}U_{ij}^{1-}\,(\rho a,\rho b,x,t)\,(a_j(x)+b_j(x-2t))\,dt\,d\rho-$$
$$-\delta\sum_{j=1}^{n}\int_0^1\int_0^{2\pi}U_{ij}^{2-}\,(\rho a,\rho b,x,t)\,(a_j(x)+b_j(x-2t))\,dt\,d\rho-$$
$$-\delta\sum_{j=1}^{n}\int_0^1\int_0^{2\pi}V_{ij}^{1-}\,(\rho a,\rho b,x,t)\,(a_j(x)-b_j(x-2t)\,dt\,d\rho- \tag{4.3}$$
$$-\delta\alpha_{ii}^2\int_0^{2\pi}(a_i(x)-b_i(x-2t))dt-$$
$$-\delta\int_0^1\int_0^{2\pi}(V_{ii}^{2-}\,(\rho a,\rho b,x,t)-\alpha_{ii}^2)\,(a_i(x)-b_i(x-2t))\,dt\,d\rho-$$
$$-\delta\sum_{\substack{j=1\\ j\neq i}}^{n}\int_0^1\int_0^{2\pi}V_{ij}^{2-}\,(\rho a,\rho b,x,t)\,(a_j(x)-b_j(x-2t))dt\,d\rho-$$
$$-\delta\sum_{\varkappa=1}^{2}\int_0^{2\pi}f_i^\varkappa\,(t,\,x-t,\,0,\,0)dt$$

and

$$(S_{2i}^\delta\,(a,b))\,(x)=b_i(x)-\delta\alpha_{ii}^1\int_0^{2\pi}(a_i(x+2t)+b_i(x))\,dt- \tag{4.4}$$
$$-\delta\int_0^1\int_0^{2\pi}(U_{ii}^{1+}(\rho a,\rho b,x,t)-\alpha_{ii}^1)\,(a_i(x+2t)+b_i(x))\,dt\,d\delta-$$

$$
\begin{aligned}
&-\delta \sum_{\substack{j=1\\ j\neq i}}^{n} \int_0^1\int_0^{2\pi} U_{ij}^{1+}(\rho a, \rho b, x, t)\,(a_j(x+2t)+b_j(x))\,dt\,d\rho\,+\\
&+\delta \sum_{j=1}^{n} \int_0^1\int_0^{2\pi} U_{ij}^{2+}(\rho a, \rho b, x, t)\,(a_j(x+2t)+b_j(x))\,dt\,d\rho\\
&-\delta \sum_{j=1}^{n} \int_0^1\int_0^{2\pi} V_{ij}^{1+}(\rho a, b\rho, x, t)\,(a_j(x+2t)-b_j(x)\,dt\,d\rho\,+\\
&+\delta\alpha_{ii}^{2}\int_0^{2\pi}(a_i(x+2t)-b_i(x))\,dt+\delta\int_0^1\int_0^{2\pi}(V_{ii}^{2+}(\rho a, \rho b, x, t)-\alpha_{ii}^{2})
\end{aligned}
\tag{4.4}
$$

$$
(a_i(x+2t)-b_i(x))\,dt\,d\rho+\delta\sum_{\substack{j=1\\ j\neq i}}^{n}\int_0^1\int_0^{2\pi}V_{ij}^{2+}(\rho a, \rho b, x, t)\,(a_j(x+2t)-b_j(x))\,dt\,d\rho\,+
$$

$$
+\delta\int_0^{2\pi}\sum_{\varkappa=1}^{2}(-1)^{\varkappa}f_i^{\varkappa}(t, x+t, 0, 0)\,dt.
$$

Finally, we put

$$
B_0=\max\{\,|f_i^{\varkappa}(t, x, u, v)|\,\}, \quad B_1=\max\{\,|D^1 f_i^{\varkappa}(t, x, u, v)|\,\}
$$

where the maxima are taken for $\varkappa=1, 2$, $i=1, \dots, n$, $(t, x, u, v)\in M(R_0)$, the set defined in Section 3, and, in the case of B_1, over all first order derivatives D^1 of the functions $f_i^{\varkappa}$.

We now turn to the proof of Lema 3.1. We fix a number η satisfying

$$
0<\eta<R_0, \tag{4.5}
$$

$$
R_0\sum_{\varkappa=1}^{2}\Big(\alpha_{ii}^{\varkappa}-\sum_{\substack{j=1\\ j\neq i}}^{n}\alpha_{ij}^{\varkappa}-\sum_{j=1}^{n}\beta_{ij}^{\varkappa}\Big)-2B_0^2\geqq\eta\Big(2B_1+\sum_{\varkappa=1}^{2}\alpha_{ii}^{\varkappa}\Big), \tag{4.6}
$$

for every $i=1, \dots, n$ which is possible by the assumption (3.2). Further we choose $\delta_0>0$ for which

$$
4\pi\delta_0 B_0\leqq\eta \tag{4.7}
$$

In the first step we show that the following implication holds :

$$
\text{If } \|(a, b)\|_{C^0_{2n}}\leqq R_0 \text{ and } \delta\in(0, \delta_0], \text{ then } \|S^{\delta}(a, b)\|_{C^0_{2n}}\leqq R_0. \tag{4.8}
$$

or, equivalently, for every $\varkappa=1, 2$, $i=1, \dots, n$,

$$
\|S_{\varkappa i}^{\delta}(a, b)\|_{C^0}\leqq R_0. \tag{4.9}
$$

To prove this inequality, let us put $\varkappa=1$ at first and choose any $i\in\{1, \dots, n\}$. Let $x\in R$ be arbitrary but fixed. Three cases are to be distinguished.

A) $0\leqq a_i(x)\leqq R_0-\eta$, B) $R_0-\eta<a_i(x)\leqq R_0$, C) $a_i(x)<0$.

In virtue of π-antiperiodicity, the case C) can be reduced to the case A) or B), since

$$
(S_{1i}^{\delta}(a, b))(x+\pi)=-S_{1i}^{\delta}(a, b)(x)
$$

and $a_i(x+\pi)=-a_i(x)$.
Thus only the cases A) and B) remain to be discussed. In the case A), using (4.1) and (4.7), we have

$$
|S_{1i}^{\delta}(a, b)(x)|\leqq R_0-\eta+2\pi\delta\,2B_0\leqq R_0,
$$

which proves (4.9) in the case A). Two different estimates must be made in the case B). Using (4.1), (4.5) and (4.7) we get an estimate from below

$$
(S_{1i}^{\delta}(a, b))(x)\geqq R_0-\eta-2\pi\delta\,2B_0\geqq-R_0.
$$

An estimate from above will be obtained by using (4.3). To explain the arrangements of (4.3), we recall that

$$
\int_0^{2\pi}b_i(x+2t)\,dt=\int_0^{2\pi}b_i(x-2t)\,dt=0,
$$

by π-antiperiodicity of the functions b_i. All the terms standing behind the sign Σ will be estimated by Lemma 4.1, i.e., for example

$$
\Big|\int_0^{2\pi}U_{ij}^{1-}(\rho a, \rho b, x, t)\,(a_j(x)+b_i(x-2t))\,dt\Big|\leqq 2\pi\alpha_{ij}^{1}R_0.
$$

The most difficult terms to be estimated are those two containing the expressions $(U_{ii}^{1-}-\alpha_{ii}^{1})$ and $(V_{ii}^{2-}-\alpha_{ii}^{2})$ under the integral sign. We show the first of then is estimated. Let us denote

$$
\Pi^{+}=\{t\in[0, 2\pi]\,;\ a_i(x)+b_i(x-2t)\geqq 0\},
$$

$$
\Pi^{-}=\{t\in[0, 2\pi]\,;\ a_i(x)+b_i(x-2t)<0\}.
$$

Since $R_0-\eta<a_i(x)\leqq R_0$, and $\|b_i\|_{C^0}\leqq R_0$, we have $-\eta\leqq a_i(x)+b_i(x-2t)<0$ for $t\in\Pi^{-}$. By assumption, $U_{ii}^{1-}(\rho a, \rho b, x, t)-\alpha_{ii}^{1}\geqq 0$. Therefore, we get :

$$
\int_0^1\int_0^{2\pi}(U_{ii}^{1-}(\rho a, \rho b, x, t)-\alpha_{ii}^{1})\,(a_i(x)+b_i(x-2t))\,dt\,d\rho=
$$

$$
=\int_0^1\Big(\int_{\Pi^+}+\int_{\Pi^-}\Big)\dots dt\,d\rho\geqq\int_0^1\int_{\Pi^-}\dots dt\,d\rho\geqq-2\pi\eta B_1.
$$

Similarly,

$$
\int_0^1\int_0^{2\pi}(V_{ii}^{2-}(\rho a, \rho b, x, t)-\alpha_{ii}^{2})\,(a_i(x)-b_i(x-2t))\,dt\,d\rho\geqq-2\pi\eta B_1.
$$

Applying all these relations and (4.6) in estimating (4.3) from above, we obtain

$$(S^\delta_{1i}(a,b))(x) \le R_0 - 2\pi\delta\Big\{R_0 \sum_{\varkappa=1}^{2} \Big(\alpha^\varkappa_{ii} - \sum_{\substack{j=1\\ j\ne i}}^{n} \alpha^\varkappa_{ij} - \sum_{j=1}^{n} \beta^\varkappa_{ij}\Big) -$$

$$- r_i(\alpha^1_{ii}+\alpha^2_{ii}) - 2r_i B_1 - 2B^0_0\Big\} \le R_0.$$

Now (4.9) is proved for the case B) and therefore it is valid for $\varkappa=1$ and any $i=1,2,\ldots,n$. The case $\varkappa=2$ will be omitted, since using (4.4) instead of (4.3), it can be treated similarly. Hence (4.8) is proved and the first step of the proof is completed.

As preliminaries to the second step, let us choose $R_1 \ge 1$ and $\delta_1 > 0$ such that

$$R_1 \sum_{\varkappa=1}^{2} \Big(\alpha^\varkappa_{ii} - \sum_{\substack{j=1\\ j\ne i}}^{n} \alpha^\varkappa_{ij} - \sum_{j=1}^{n} \beta^\varkappa_{ij}\Big) \ge 4B_1 + \sum_{\varkappa=1}^{2} \alpha^\varkappa_{ii}, \tag{4.10}$$

for $i=1,\ldots,n$, and

$$2\pi\delta_1 B_1(8n\,R_1+2) \le 1. \tag{4.11}$$

The second step of the proof consists in showing that for $\delta \in (0, \delta_1]$ the following implication holds:

(4.12) If $(a,b) \in C^1_{2n}$, $\|(a,b)\|_{C^0_{2n}} \le R_0$, $\|(a',b')\|_{C^0_{2n}} \le R_1$,

then $\|D^1_x(S^\delta_{\varkappa i}(a,b))\|_{C^0} \le R_1$ for every $\varkappa=1,2$ and $i=1,\ldots,n$.

To prove (4.12) we use the expressions:

$$(D^1_x S^\delta_{1i}(a,b))(x) = a_i'(x) - \delta \sum_{\varkappa=1}^{2}\sum_{j=1}^{n} \int_0^{2\pi} U^{\varkappa-}_{ij}(a,b,x,t)\,(a'(x)+b'(x-2t))\,dt$$

$$- \delta \sum_{\varkappa=1}^{2}\sum_{j=1}^{n} \int_0^{2\pi} V^{\varkappa-}_{ij}(a,b,x,t)\,(a'(x)-b'(x-2t))\,dt + \tag{4.13}$$

$$+ \delta \sum_{\varkappa=1}^{2} \int_0^{2\pi} W^{\varkappa-}_{i}(a,b,x,t)\,dt,$$

$$(D^1_x S^\delta_{2i}(a,b))(x) = b_i'(x) - \delta \sum_{\varkappa=1}^{2}\sum_{j=1}^{n} (-1)^{\varkappa+1} \int_0^{2\pi} U^{\varkappa+}_{ij}(a,b,x,t)\,(a'(x+2t)+$$

$$+ b'(x))\,dt - \delta \sum_{\varkappa=1}^{2}\sum_{j=1}^{n} (-1)^{\varkappa+1} \int_0^{2\pi} V^{\varkappa+}_{ij}(a,b,x,t)\,(a'(x+2t)-b'(x))\,dt \tag{4.14}$$

$$- \delta \sum_{\varkappa=1}^{2} (-1)^{\varkappa+1} \int_0^{2\pi} W^{\varkappa+}_{i}(a,b,x,t)\,dt,$$

where

$$W^{\varkappa-}_i(a,b,x,t) = g^\varkappa_i(t, x-t, a(x)+b(x-2t), a(x)-b(x-2t)),$$

$$W^{\varkappa+}_i(a,b,x,t) = g^\varkappa_i(t, x+t, a(x+2t)+b(x), a(x+2t)-b(x)),$$

and $g^\varkappa_i(t,x,u,v) = D^1_x f^\varkappa_i(t,x,u,v)$. Let us choose $\varkappa=2$ in this step and fix an arbitrary i and $x \in R$. Again, the three cases

A) $0 \le b_i(x) \le R_1 - 1$, B) $R_1 - 1 < b_i(x) \le R_1$, C) $b_i(x) < 0$

will be distinguished. The case C) can be again reduced to one of the preceding cases. The technique which has been used in the first step is applied also here. In the case A) using (4.14), we get

$$|(D^1_x S^\delta_{2i}(a,b))(x)| \le R_1 - 1 + 2\pi\delta B_1(8nR_1+2) \le R_1,$$

In the case B) using (4.14) again, we get

$$(D^1_x S^\delta_{2i}(a,b))(x) \ge R_1 - 1 - 2\pi\delta B_1(8n\,R_1+2) \ge -1 \ge -R_1$$

and

$$(D^1_x S^\delta_{2i}(a,b)(x) \le R_0 - 2\pi\delta\Big\{R_1 \sum_{\varkappa=1}^{n}\Big(\alpha^\varkappa_{ii} - \sum_{\substack{j=1\\ j\ne i}}^{n} \alpha^\varkappa_{ii} - \sum_{j=1}^{n} \beta^\varkappa_{ij}\Big) - 4B_1 - \sum_{\varkappa=1}^{2} \alpha^\varkappa_{ii}\Big\} \le R_0.$$

In the second estimate we have used (4.10). As the case $\varkappa=1$ can be treated in the same manner, if we only start from (4.13) instead of (4.14), (4.12) is proved.

Analogously, there is $R_2 \ge 1$ and $\delta_2 > 0$ such that for every $\delta \in (0, \delta_2]$ the following implication is valid:

(4.13) If $(a,b) \in C^2_{2n}$, $\|(a,b)\|_{C^0_{2n}} \le R_0$, $\|(a',b')\|_{C^0_{2n}} \le R_1$, $\|(a'',b'')\|_{C^0_{2n}} \le R_2$,

then $\|D^2_x S^\delta(a,b)\|_{C^0_{2n}} \le R_2$.

If $\delta = \min(\delta_0, \delta_1, \delta_2)$ and $M \subset C^2_{2n}$ is given by

$$M = \{(a,b) \in C^2_{2n};\ \|(a,b)\|_{C^0_{2n}} \le R_0,\ \|(a',b')\|_{C^0_{2n}} \le R_1,\ \|(a'',b'')\|_{C^0_{2n}} \le R_2\},$$

then (4.8), (4.12), and (4.13) imply that M is mapped into itself by the mapping S^δ. If we consider M as a subset of C^1_{2n}, M is a relatively compact and convex. Hence the closure of M in C^1_{2n} is compact and convex and is mapped by the continuous mapping S^δ into itself. By Schauder's theorem there is a $(a^0, b^0) \in C^1_{2n}$, $\|(a^0,b^0)\|_{C^0_{2n}} \le R_0$, such that $S^\delta(a^0,b^0) = (a^0,b^0)$; i.e., by definition of S^δ, $K(a^0,b^0)=0$.

Now only the uniqueness of the pair (a^0,b^0) remains to be proved. This can be done by showing that if there should be another pair (a^1,b^1), $\|(a^1,b^1)\|_{C^0_{2n}} \le R_0$, $K(a^1,b^1)=0$, then by applying Lemma 4.1 and methods previously used, we would obtain a contradiction with $K(a^0,b^0) - K(a^1,b^1) = 0$. Thus the proof of Lemma 3.1 is complete.

5. Proof of Lemma 3.2. In this section we shall use the notation

$$A_{ij}^{\varkappa-}(x,t)=U_{ij}^{\varkappa+}(a^0,b^0,x,t),\qquad B_i^{\varkappa-}(x,t)=V_{ij}^{\varkappa+}(a^0,b^0,x,t),$$

$$A_{ij}^{\varkappa-}(x,t)=U_{ij}^{\varkappa+}(a^0,b^0,x,t),\qquad B_{ij}^{\varkappa+}(x,t)=V_{ij}^{\varkappa+}(a^0,b^0,x,t),$$

where the function a^0 and b^0 are those given by Lemma 3.1. The Fréchet derivative of the mapping $K: C_{2n}^1\to C_{2n}^1$ taken at the point (a^0,b^0) has the form

$$K'(a^0,b^0)=(K_1'(a^0,b^0),\ K_2'(a^0,b^0)),$$

where both $K_1'(a^0,b^0)$ and $K_2'(a^0,b^0)$ map C_{2n}^1 into C_n^1 and have the form

$$(5.1)\qquad [K_{1i}'(a^0,b^0)](a,b)(x)=\sum_{\varkappa=1}^{2}\sum_{j=1}^{n}\left\{\int_0^{2\pi}A_{ij}^{\varkappa-}(x,t)\,(a_j(x)+b_j(x-2t))dt+\right.$$
$$\left.+\int_0^{2\pi}B_{ij}^{\varkappa-}(x,t)\,(a_j(x)-b_j(x-2t))dt\right\},$$

$$[K_{2i}'(a^0,b^0)](a,b)(x)=\sum_{\varkappa=1}^{2}\sum_{j=1}^{n}(-1)^{\varkappa+1}$$
$$(5.2)\qquad \left\{\int_0^{2\pi}A_{ij}^{\varkappa+}(x,t)(a_j(x+2t)+b_j(x))dt+\int_0^{2\pi}B_{ij}^{\varkappa+}(x,t)\,(a_j(x+2t)-b_j(x))dt\right\}.$$

At first we prove

$$(5.3)\qquad K'(a^0,b^0)(a,b)=0 \text{ implies } (a,b)=0$$

which means that $K'(a^0,b^0)$ is a one-to-one mapping. Really, let us suppose that there is $(a,b)\neq 0$ satisfying $K'(a^0,b^0)(a,b)=0$. Then at least one of the following two possibilities must occur:

$$(5.4)\qquad 0<\|(a,b)\|_{C_{2n}^0}=a_i(x_0) \text{ for some } i \text{ and } x_0\in R,$$

$$(5.5)\qquad 0<\|(a,b)\|_{C_{2n}^0}=b_i(x_0) \text{ for some } i \text{ and } x_0\in R.$$

If (5.4) holds, then $a_i(x_0)\pm b_i(x_0-2t)\geqq 0$ for all $t\in[0,2\pi]$. By assumption, $A_{ii}^{\varkappa-}(x,t)-\alpha_{ii}^1\geqq 0$ and therefore

$$\int_0^{2\pi}A_{ii}^{\varkappa-}(x_0,t)\,(a_i(x_0)+b_i(x_0-2t))\,dt=\int_0^{2\pi}\alpha_{ii}^1(a_i(x_0)+b_i(x_0-2t))dt+$$
$$+\int_0^{2\pi}(A_{ii}^{\varkappa-}(x_0,t)-\alpha_{ii}^1)\,(a_i(x_0)+b_i(x_0-2t))dt\geqq 2\pi\,\alpha_{ii}^1\,a_i(x_0).$$

Similarly,

$$\int_0^{2\pi}B_{ii}^{\varkappa-}(x_0,t)\,(a_i(x_0)-b_i(x_0-2t))dt\geqq 2\pi\alpha_{ii}^2a_i(x_0).$$

Using these two inequalities in (5.1) where all other terms are estimated by Lemma 4.1, we get

$$[K_{1i}'(a^0,b^0)](a,b)(x_0)\,2\pi a_i(x_0)\sum_{\varkappa=1}^{2}\left(\alpha_{ii}^{\varkappa}-\sum_{\substack{j=1\\ j\neq i}}^{n}\alpha_{ij}^{\varkappa}-\sum_{j=1}^{n}\beta_{ij}^{\varkappa}\right).$$

By the assumption (3.2), the right hand side of this inequality is positive. This is a contradiction with $K'(a^0,b^0)(a,b)=0$. The case (5.5) can be treated similarly. Hence (5.3) is proved.

Two linear mappings $T, D: C_{2n}^1\to C_{2n}^1$ such that $K'(a^0,b^0)=T+D$ are now defined by $T=(T_1,T_2)$ and $D=(D_1,D_2)$, where

$$(T_{1i}(a,b))(x)=\sum_{j=1}^{n}\int_0^{2\pi}\sum_{\varkappa=1}^{2}(A_{ij}^{\varkappa-}(x,t)+B_{ij}^{\varkappa-}(x,t))dt\,a_j(x),$$

$$(T_{2i}(a,b))(x)=\sum_{j=1}^{n}\int_0^{2\pi}\sum_{\varkappa=1}^{2}(-1)^{\varkappa+}(A_{ij}^{\varkappa+}(x,t)-B_{ij}^{\varkappa+}(x,t))dt\,b_j(x)$$

and

$$(D_{1i}(a,b))(x)=\sum_{j=1}^{n}\int_0^{2\pi}\sum_{\varkappa=1}^{2}(A_{ij}^{\varkappa-}-B_{ij}^{\varkappa-})(x,t)\,b_j(x-2t)dt=$$

$$=\frac{1}{2}\sum_{j=1}^{n}\int_{x-4\pi}^{x}\sum_{\varkappa=1}^{2}(A_{ij}^{\varkappa-}-B_{ij}^{\varkappa-})\left(x,\frac{x-\xi}{2}\right)b_j(\xi)d\xi,$$

$$(D_{2i}(a,b))(x)=\sum_{j=1}^{n}\int_0^{2\pi}\sum_{\varkappa=1}^{2}(A_{ij}^{\varkappa+}+B_{ij}^{\varkappa+})(x,t)\,a_j(x+2t)dt=$$

$$=\frac{1}{2}\sum_{j=1}^{n}\int_{x}^{x+4\pi}\sum_{\varkappa=1}^{2}(A_{ij}^{\varkappa+}+B_{ij}^{\varkappa+})\left(x,\frac{\xi-x}{2}\right)a_j(\xi)\,d\xi.$$

The operator T is a linear homeomorphism of C_{2n}^1 onto itself since T is a matrix operator of the form

$$\operatorname{col}(T(a,b))=\begin{Vmatrix}T_{11} & 0\\ 0 & T_{22}\end{Vmatrix}\operatorname{col}(a,b)$$

where matrices $T_{11}(x)$ and $T_{22}(x)$ are regular for all $x \in R$ as it simply follows from the assumption (3.2). It is easily seen that the operator D is completely continuous. Thus, by a well-known theorem, the operator $K'(a^0, b^0)$ is a linear homeomorphism because it is a one-to-one mapping and also a sum of a linear homeomorphism and a completely continuous operator. This completes the proof of Lemma 3.2.

6. An additional result. Let the functions f_i^k satisfy, $i = 1, \dots, n$,

$$f_i^1(t, -x, -u, v) = -f_i^1(t, x, u, v), \tag{6.1}$$

$$f_i^2(t, -x, -u, v) = f_i^2(t, x, u, v). \tag{6.2}$$

Theorem 6.1. *Let the assumptions (3.1), (3.2), (6.1), and (6.2) be satisfied. Then there exists a unique* $a^0 \in C_n^1$, $\| a^0 \|_{C_n^1} \leq R_0$, *and two continuous mappings* $u = u(\varepsilon)$, $v = v(\varepsilon)$ *of an interval* $[-\varepsilon_0, \varepsilon_0]$, $\varepsilon_0 > 0$, *into* $C_n^1(R^2)$ *such that, for every* ε, *the functions* $u = u(\varepsilon)$, $v = v(\varepsilon)$ *satisfy the equations (1.1) – (1.8) and*

$$u(0)\,(t, x) = a^0(t+x) - a^0(t-x), \qquad v(0)\,(t, x) = a^0(t+x) + a^0(t-x).$$

The proof of this theorem can be performed by using the technique of the preceding sections. If for $w \in \mathscr{C}_n^1([0, 2\pi])$, two operators E and O are introduced by

$$(Ew)\,(t, x) = \frac{1}{2}(w(t, x) + w(t, -x)), \qquad (Ow)\,(t, x) = \frac{1}{2}(w(t, x) - w(t, -x)),$$

and if two functions $w \in \mathscr{C}_1^n([0, 2\pi])$ and $a \in \mathscr{C}_n^1$ satisfy $i = 1, \dots, n$,

$$-w_i(t, x) + 2a_i(t+x) + \frac{\varepsilon}{2}\int_0^t \sum_{\varkappa=1}^{2} F_i^{\varkappa}(Ow, Ew)\,(\tau, x+t-\tau)\,d\tau = 0, \tag{6.3}$$

$$t \in [0, 2\pi], \quad x \in R,$$

and

$$\sum_{\varkappa=1}^{2} \int_0^{2\pi} F_i^{\varkappa}(Ow, Ew)\,(t, x-t)\,dt = 0, \quad x \in R, \tag{6.4}$$

then $u = Ow$, $v = Ew$, a, $b(\xi) = -a(-\xi)$, and ε satisfy $G(u, v, a, b, \varepsilon) = 0$. Hence the problem is reduced to solving (6.3) and (6.4) which can be achieved by applying the methods used in the preceding sections.

REFERENCES

1. Barbu V. — *Nonlinear boundary-value problems for a class of hyperbolic systems*, Revue Roumaine de Mathématiques Pures et Appliquées, 22 (1977), 155–168.
2. Lovicarová H. — *Periodic solutions of a weakly nonlinear wave equation in one dimension*. Czechoslovak Math. J., 19 (94), 1969, 324–342.

Received 20. VI. 1980

Mathematical Institute
Czechoslovak Academy of Sciences
Žitná 25 11567 Praha 1
Czechoslovakia

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXVII, s. I a, 1981 f. 2

ON THE EXPONENTIAL STABILITY OF THE SOLUTIONS OF A CLASS OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH DELAY

BY

T. HAVÂRNEANU

1. Preliminaries. In [5] we have studied the existence, uniqueness and representation of the solutions of the following problem:

$$\frac{d}{dt}\left[x(t) - \sum_{j=1}^{\infty} B_j x(t-h_j)\right] = Ax(t) - \sum_{j=1}^{\infty} A_j x(t-h_j), \; t \geq 0, \tag{1.1a}$$

$$x(t) = \Phi(t), \quad t \in (-\infty\,; 0], \; \Phi \in C, \tag{1.1b}$$

where C is the Banach space defined as

$$C = C[R, X] = \{\Phi \mid \Phi : (-\infty\,; 0] \to X, \; \Phi - a \text{ continuous function}, \; \exists \lim_{t \to -\infty} \Phi(t) \in X\}$$

with the norm

$$\| \Phi \| = \sup_{t \in R} | \Phi(t) |,$$

X being a Banach space with the norm $|\cdot|$.

In the following, we need the space:

$AC_0 = \{\Phi \in C \mid \Phi$ *is absolutely continuous and* $\Phi' \in C_-\}$. In [5] we have also proved that, in the hypotheses:

(1.2) $A : \mathscr{D}(A) \subset X \to X$ *is the infinitesimal generator of a* (C_0)*-semigroup*

$$\{T(t),\; t \geq 0\} \subset L(X)\,;$$

$$\{B_j\}_{j=1}^{\infty} \subset L(X), \; \textit{range } B_j \subset \mathscr{D}(A), \; AB_j \in L(X), \quad \forall j \in N\,; \tag{1.3}$$

$$\sum_{j=1}^{\infty} \| B_j \| < +\infty\,; \tag{1.4}$$

$$\sum_{j=1}^{\infty} \| AB_j \| < +\infty\,; \tag{1.5}$$

$$\{A_j\}_{j=1}^{\infty} \subset L(X), \quad \sum_{j=1}^{\infty} \| A_j \| < +\infty\,; \tag{1.6}$$