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BOL LOOPS OF ORDER 12

BY

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1. **Introduction.** Recently Burn [4] has shown that there exist exactly six non-associative Bol loops of order 8. The Object of this paper is to prove that (up to isomorphisms) there are only two Bol loops of order 12 which are not groups and Moufang loops.

For the definition of a loop, the reader should consult Bruck [3]. Albert [1] defines a right multiplication $R(a)$ $a \in L$, as a permutation of a loop $(L,.)$

(1) $$R(a) : x \to x \,.\, a. \quad \forall x \in L.$$

We call the set $\Pi = \{R(a) \mid a \in L\}$ the right regular representation of $(L,.)$ or briefly, the representation of L.

A Bol loop L is a loop in which we have

(2) $$(xy \,.\, z)y = x(yz \,.\, y) \quad \text{for all } x, y, z \in L.$$

Strictly speaking, (2) defines a right Bol loop. (For Bol loop, see [2], [6], [7], [8, 9]). The following theorems due to Burn [4] will be used in the investigations.

Theorem 1. *If Π is the representation of a loop L, then L is a Bol loop if and only if α, $\beta \in \Pi$ implies $\alpha\beta\alpha \in \Pi$.*

Theorem 2. *If Π is a representation of a finite Bol loop and $\alpha \in \Pi$, then α is a product of disjoint cycles of equal length.*

Theorem 3. *The order of an element of a finite Bol loop divides the order of the loop.*

Theorem 4. *If $R(a)$ and $R(b)$ are right multiplications of a Bol loop and $R(a)^2 = R(b)^2 = 1$, then $R(a)R(b) = R(b)R(a)$ if and only if $aR(b) = bR(a)$.*

2. In this section we show that if Π is a representation of a finite Bol loop of order 12, then Π can not contain an element of order 4.

Theorem 5. *If Π is a representation of a finite Bol loop of order 12, then Π can not contain two elements of order 4 and two element of order 3.*

Proof. We assume that α, $\beta \in \Pi$, such that $\alpha^4 = 1$ and $\beta^3 = 1$. Without loss of generality, we take

$$\alpha = R(2) = (1\ 2\ 3\ 4)\ (5\ 6\ 7\ 8)\ (9\ 10\ 11\ 12),$$

$$\alpha^2 = R(3) = (1\ 3)\ (2\ 4)\ (5\ 7)\ (6\ 8)\ (9\ 11)\ (10\ 12),$$

$$\alpha^3 = R(4) = (1\ 4\ 3\ 2)\ (5\ 8\ 7\ 6)\ (9\ 12\ 11\ 10), \text{ and}$$

$$\beta = R(5) = (1\ 5\ 9)\ (2\ x\ a)\ (3\ y\ b)\ (4\ z\ c),$$

where $\{x, y, z\} = \{6, 7, 8\}$,; $\{a, b, c\} = \{10, 11, 12\}$.

(a) Let us take $y=6$ and $b=11$. Now $R(3)\ R(5)\ R(3) \in \Pi$ by Theorem 1. But $R(3)\ R(5)\ R(3) = R(8) = (1\ 8\ 9)\ (\ \ )\ (\ \ )\ (\ \ ) \notin \Pi$. Hence b can not be equal to 11.

(b) Let us take $y=6$ and $b=10$. Now $\{x, z\} = \{7, 8\}$ and $\{a, c\} = \{11, 12\}$ and there will be four possible forms of $R(5)$.

(i) $y=6$, $b=10$, $x=7$, $a=11$, $z=8$ and $c=12$. Now $R(3)\ R(5)\ R(3) \in \Pi$ by Theorem 1, but
$R(3)\ R(5)\ R(3) = R(8) = (1\ 8\ 12)\ (2\ 6\ 10)\ (\ )\ (\ ) \notin \Pi$, which is a contradiction

(ii) $y=6$, $b=10$, $x=7$, $a=12$, $z=8$ and $c=11$. Now $R(2)\ R(5)\ R(2) \in \Pi$ by Theorem 1. But $R(2)\ R(5)\ R(2) = R(8) = (1\ 8\ 10)\ (2\ 7\ 12)\ (\ \ )\ (\ \ ) \notin \Pi$. which is a contradiction.

(iii) $y=6$, $b=10$, $x=8$, $a=11$, $z=7$ and $c=12$. Now $R(2)\ R(5)\ R(2) \in \Pi$ by Theorem 1. But $R(2)\ R(5)\ R(2) = R(5) = (1\ 5\ 11)\ (\ \ )\ (\ \ )\ (\ \ ) \in \Pi$; a contradiction.

(iv) $y=6$, $b=10$, $x=8$, $a=12$, $z=7$ and $c=11$. Now $R(2)\ R(5)\ R(2) \in \Pi$ by Theorem 1. But $R(2)\ R(5)\ R(2) = (1\ 5\ 11)\ (\ \ )\ (\ \ )\ (\ \ ) \notin \Pi$; Π contradiction.

Thus we have shown that if $y=6$ then $b \neq 10$.

(c) Let us take $y=6$ and $b=12$. Now $\{x, z\} = \{7, 8\}$ and $\{a, c\} = \{11, 12\}$ and there will be four possible forms of $R(5)$.

(i) $y=6$, $b=12$, $x=7$, $a=10$, $z=8$ and $c=11$. Now $R(4)\ R(5)\ R(4)$, $R(5)\ R(4)\ R(5) \in \Pi$ by Theorem 1. But $R(4)\ R(5)\ R(4) = R(7) = (1\ 7\ 11)\ (\ \ )\ (\ )\ (\ )$ therefore $R(11) = (1\ 11\ 7)\ (\ \ )\ (\ \ )\ (\ \ )$, also $R(5)\ R(4)\ R(5) = R(11) = (1\ 11\ 6)\ (\ \ )\ (\ \ )\ (\ \ )$; a contradiction.

(ii) $y=6$, $b=12$, $x=7$, $a=11$, $z=8$ and $c=10$. Now $R(3)\ R(5)\ R(3)$, $R(5)\ R(3)\ R(5) \in \Pi$ by Theorem 1. But $R(3)\ R(5)\ R(3) = R(8) = (1\ 8\ 10)\ (\ \ )\ (\ \ )\ (\ \ )$ and $R(5)\ R(3)\ R(5) = R(11) = (1\ 11\ 8)\ (\ \ )\ (\ \ )\ (\ \ )$ therefore $R(8) = (1\ 8\ 11)\ (\ \ )\ (\ \ )\ (\ \ )$' a contradiction.

(iii) $y=6$, $b=12$, $x=8$, $a=10$, $z=7$ and $c=11$. Now $R(3)\ R(5)\ R(3)$, $R(5)\ R(3)\ R(5) \in \Pi$ by Theorem 1. But $R(3)\ R(5)\ R(3) = R(8) = (1\ 8\ 10)$ and
$R(5)\ R(3)\ R(5) = R(11) = (1\ 11\ 8)\ (\ \ )\ (\ \ )\ (\ \ )$ therefore $R(8) = (1\ 8\ 10)\ (\ \ )\ (\ \ )\ (\ \ )$; a contradiction.

Thus we have shown that if $y=6$ then $b \neq 12, 11, 10$. Following the same argument we can easily show that if $y \in \{7, 8\}$, then $b \notin \{10, 11, 12\}$.

Hence it is not possible to find $R(5)$ which belongs to Π. This completes the proof of the theorem.

3. In this section we show that if Π is a representation of a finite Bol loop of order 12, then Π can not contain an element of order 4 and another distinct element of order 2. Before proving the theorem, we prove the following lemma.

Lemma. *If Π is a representation of a finite Bol loop of order* 12, *then Π can not contain four elements of order* 4.

Proof. We assume that $\alpha, \beta \in \Pi$, such that $\alpha^4 = 1$, $\beta^4 = 1$ $\alpha^2 \neq \beta^2$. Without loss of generality, we take

$$\alpha = R(2) = (1\ 2\ 3\ 4)\ (5\ 6\ 7\ 8)\ (9\ 10\ 11\ 12).$$

$$\alpha^2 = R(3) = (1\ 3)\ (2\ 4)\ (5\ 7)\ (6\ 8)\ (9\ 11)\ (10\ 12).$$

$$\alpha^3 = R(4) = (1\ 4\ 3\ 2)\ (5\ 8\ 7\ 6)\ (9\ 12\ 11\ 10).$$

$$\beta = R(5) = (1\ 5\ x\ y)\ (2\ r\ s\ t)\ (3\ u\ v\ w).$$

where $x \in \{9, 10, 11, 12\}$ it is impossible to replace y, r, s, t, u, v and w by 4. Hence it is impossible to obtain $R(5)$ in this case.

In the second case we assume that $\beta^2 = \alpha^2$. Now $R(5) = (1\ 5\ 3\ 7)\ (2\ x\ 4\ y)\ (p\ q\ u\ v)$, where $\{(x, y)\} = \{(6, 8)\ (9, 11)\ (10, 12)\}$.

(i) We assume that $x=6$, and $y=8$, then we fix $p=9$, but it is impossible to fix q. Thus $R(5)$ can not be obtained.

(ii) Now we assume that $x=9$ and $y=11$, then we can take $p=6$, $u=8$, $q=10$ and $v=12$ without loss of generality.

Thus

$$R(5) = (1\ 5\ 3\ 7)\ (2\ 9\ 4\ 11)\ (6\ 10\ 8\ 12). \quad \text{Also}$$

$$R(2)\ R(5)\ R(2) = R(10) = (1\ 10\ 3\ 12)\ (2\ 8\ 4\ 6)\ (5\ 11\ 7\ 9).$$

$$\text{Now } R(6) = (1\ 6\ 3\ 8)\ (2\ 10\ 4\ 12)\ (5\ u\ 7\ v).$$

It is impossible to fix u and v. In this case we can not obtain $R(6)$.

Similarly, if $x=10$ and $y=12$, then we can not obtain $R(6)$.

This completes the proof of the theorem.

Theorem 6. *If Π is a representation of a finite Bol loop of order 12, then Π can not contain two elements of order 4 and two elements of order 2.*

Proof. We assume $\alpha, \beta \in \Pi$ such that $\alpha^4 = 1$ and $\beta^2 = 1$, without loss of generality we take

$$\alpha = R(2) = (1\ 2\ 3\ 4)\ (5\ 6\ 7\ 8)\ (9\ 10\ 11\ 12)$$

$$\alpha^2 = R(3) = (1\ 3)\ (2\ 4)\ (5\ 7)\ (6\ 8)\ (9\ 11)\ (10\ 12)$$

$$\alpha^3 = R(4) = (1\ 4\ 3\ 2)\ (5\ 8\ 7\ 6)\ (9\ 12\ 11\ 10) \text{ and}$$

$$\beta = R(5) = (1\ 5)\ (2\ x)\ (3\ y)\ (4\ z)\ (p\ q)\ (u\ v).$$

where $x \in \{6, 7, 8, 9, 10, 11, 12\}$.

(a) Let us assume $x=6$, thus

$$R(5) = (1\ 5)\ (2\ 6)\ (3\ y)\ (4\ z)\ (p\ q)\ (u\ v). \text{ Now}$$

$$R(2)\ R(5) = R(5)\ R(2)$$

$$\therefore\ 2R(2)\ R(5) = 2R(5)\ R(2),$$

or

$$3R(5) = 7 \Rightarrow y = 7,$$

also

$$3R(2)\ R(5) = 3R(5)\ R(2), \quad 4R(5) = 8, \quad z = 8.$$

Hence

$$R(5) = (1\ 5)\ (2\ 6)\ (3\ 7)\ (4\ 8)\ (9\ q)\ (10\ v). \text{ But}$$

$q \notin \{12, 11\}$. Thus it is impossible to obtain $R(5)$, when $x=6$.

Following the same argument, $x \notin \{7, 8\}$.

(b) *If* $x=9$, then

$$R(5)=(1\ 5)\ (2\ 9)\ (3\ y)\ (4\ z)\ (p\ q)\ (u\ v). \text{ But}$$

$$R(2)\ R(5)\ R(2)=R(10).$$

$R(10)$ can not be of order 3 or 6 by Theorem 5. Also $R(10)$ can not be of order 4 by Lemma. Thus $R(10)$ must be of order 2, then $z=11$ and $y \neq 12$. Hence $R(5)=(1\ 5)\ (2\ 9)\ (3\ y)\ (4\ 11)\ (p\ q)\ (u\ v)$, where $y \in \{6, 7, 8, 10\}$. If $y=10$, then

$R(2)\ R(5)\ R(2)=(1\ 10)\ (2\ 11..)\ (\quad)\ (\quad) \notin \Pi$, by Theorem 2. Hence $y \neq 10$.

($R(2)\ R(5)\ R(2)=(1\ 10)\ (2\ 7..)\ (\quad)\ (\quad) \notin \Pi$, by Theorem 2. Hence $y \neq 6$.

If $y=7$, then without loss of generality, we take

$R(5)=(1\ 5)\ (2\ 9)\ (3\ 7)\ (4\ 11)\ (6\ 10)\ (8\ 12)$. Also $R(5)\ R(2)\ R(5)=(1\ 10\ 3\ 12)\ (\quad)\ (\quad) \notin \Pi$.

If $y=8$, then without loss of generality we take

$$R(5)=(1\ 5)\ (2\ 9)\ (3\ 8)\ (4\ 11)\ (6\ 10)\ (7\ 12). \text{ Also}$$

$$R(5)\ R(2)\ R(5)=(1\ 10\ 12\ 3)\ (\quad)\ (\quad) \notin \Pi. \text{ Hence } y \neq 8.$$

Thus x can not be equal to 9.

Applying the above argument it can also be shown that

$$x \in \{10, 11, 12\}$$

This completes the proof of the Theorem 6.

4. **Theorem 7.** *A Bol loop of order* 12 *which is not a group or a Moufang loop has one of the two following representations.*

Π_1	Π_2
$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$,	$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$
$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 8\ 9)\ (10\ 12\ 11)$	$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11)$.
$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 10)\ (8\ 12)\ (9\ 11)$	$R(4) = (1\ 4)\ (2\ 5)\ (3\ 6)\ (7\ 10)\ (8\ 11)\ (9\ 12)$
$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ 11)\ (8\ 10)\ (9\ 12)$.	$R(5) = (1\ 5\ 3\ 4\ 2\ 6)\ (7\ 11\ 9\ 10\ 8\ 12)$,
$R(6) = (1\ 6)\ (2\ 5)\ (3\ 4)\ (7\ 12)\ (8\ 11)\ (9\ 10)$.	$R(6) = (1\ 6\ 2\ 4\ 3\ 5)\ (7\ 12\ 8\ 10\ 9\ 11)$
$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 10)\ (5\ 12)\ (6\ 11)$,	$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 10)\ (5\ 12)\ (6\ 11)$.
$R(8) = (1\ 8)\ (2\ 7)\ (3\ 9)\ (4\ 11)\ (5\ 10)\ (6\ 12)$.	$R(8) = (1\ 8)\ (2\ 7)\ (3\ 9)\ (4\ 11)\ (5\ 10)\ (6\ 12)$.
$R(9) = (1\ 9)\ (2\ 8)\ (3\ 7)\ (4\ 12)\ (5\ 11)\ (6\ 10)$,	$R(9) = (1\ 9)\ (2\ 8)\ (3\ 7)\ (4\ 12)\ (5\ 11)\ (6\ 10)$,
$R(10) = (1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 7)\ (5\ 9)\ (6\ 8)$,	$R(10) = (1\ 10)\ (2\ 11)\ (3\ 12)\ (4\ 7)\ (5\ 8)\ (6\ 9)$.
$R(11) = (1\ 11)\ (2\ 10)\ (3\ 12)\ (4\ 8)\ (5\ 7)\ (6\ 9)$.	$R(11) = (1\ 11\ 3\ 10\ 2\ 12)\ (4\ 8\ 6\ 7\ 5\ 9)$,
$R(12) = (1\ 12)\ (2\ 11)\ (3\ 10)\ (4\ 9)\ (5\ 8)\ (6\ 7)$,	$R(12) = (1\ 12\ 2\ 10\ 3\ 11)\ (4\ 9\ 5\ 7\ 6\ 8)$.

Proof. If Π is the representation of a Bol loop of order 12, then, by Theorem 3, 5, 6 and Lemma the elements of Π have order 1, 2, 3, 6 or 12.

Case (*a*) Π *contains an element of order* 12. It is well known that a Bol loop is generated by at least two elements, in this case it is generated by one element, hence Π is the representation of the cyclic group of order 12.

For the rest of the proof we presume that Π is the representation of a Bol loop $\{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\}$ containing no elements of order 12 and 4.

Case (*b*). Π *contains nine elements of order* 2. In this case Π contains two elements of order 3 and nine elements of order 2, without loss of generality, we assume that

$$R(2)=(1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$$

$$R(3)=(1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 8\ 9)\ (10\ 12\ 11)$$

$$R(4)=(1\ 4)\ (2\ a)\ (3\ b)\ (7\ c)\ (8\ d)\ (9\ e), \text{ where}$$

$$\{a, b\}=\{5, 6\} \text{ and } \{c, d, e\}=\{10, 11, 12\}.$$

We assume that $a=5$ and $b=6$, then

$$R(4)=(1\ 4)\ (2\ 5)\ (3\ 6)\ (7\ c)\ (8\ d)\ (9\ e).$$

But $R(2)\ R(4)\ R(2)=(1\ 6\ 2\ 4\ 3\ 5)\ (\quad) \notin \Pi$, since there is no element of order 6 in Π by assumption. Hence

$$R(4)=(1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ c)\ (8\ d)\ (9\ e).$$

$R(2)\ R(4)\ R(2)=R(4) \in \Pi$. Hence $R(2)\ R(4)=R(4)\ R(3)$.

Thus $7R(2)\ R(4)=7R(4)\ R(3)$

$$cR(3)=d. \tag{3}$$

Thus from (3) we get three possible cases

$$\begin{aligned} &c=10,\ d=12,\ e=11, \\ &c=11,\ d=10,\ e=12, \\ &c=12,\ d=11,\ e=10. \end{aligned} \tag{4}$$

Thus there are three possible forms of $R(4)$.

Next we assume that $R(5)=(1\ 5)\ (2\ u)\ (3\ w)\ (7\ r)\ (8\ s)\ (9\ t)$, where $\{u, w\}=\{4, 6\}$ and $\{r, s, t\}=\{10, 11, 12\}$. u can not be equal 6, because of $R(4)$. Thus we have

$$R(5)=(1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ r)\ (8\ s)\ (9\ t)$$

But $R(2)\ R(5)=R(5)\ R(3)$ or

$$7R(2)\ R(5)=7R(5)\ R(3)$$

$$\text{or } rR(3)=s. \tag{5}$$

From (5), we obtain three possible cases

$$\begin{aligned} &r=10,\ s=12,\ t=11, \\ &r=11,\ s=10,\ t=12, \\ &r=12,\ s=11,\ t=10. \end{aligned} \tag{6}$$

Thus we get three possible forms $R(5)$. $R(6)$ can be easily obtained from $R(4)$ and $R(5)$. Next we consider

$R(7)=(1\ 7)\ (2\ p)\ (3\ q)\ (4\ u)\ (5\ v)\ (6\ w)$, where $\{p, q\}=\{8, 9\}$ and $\{u, v, w\}=\{10, 11, 12\}$. If $p=8$ and $q=9$ in $R(7)$, then we obtain $R(2)\ R(7)\ R(2)=(1\ 9\ 2\ 7\ 3\ 8)\ (\quad)\notin \Pi$. Thus $R(7)=(1\ 7)\ (2\ 8)\ (3\ 9)\ (4\ u)\ (5\ v)\ (6\ w)$. Now $R(2)\ R(7)=R(7)\ R(3)$ or $4R(2)\ R(7)=4R(7)R(3)$

$$\therefore\ 4R(3)=v. \tag{7}$$

From (7), we obtain three possible cases

$$\begin{aligned} &u=10,\ v=12,\ w=11,\\ &u=11,\ v=10,\ w=12,\\ &u=12,\ v=11,\ w=10. \end{aligned} \tag{8}$$

Thus there are three possible forms of $R(7)$.

$R(8)$ and $R(9)$ can be easily obtained (except in few cases).

In the last we consider

$R(10)=(1\ 10)\ (2\ p)\ (3\ q)\ (4\ l)\ (5\ m)\ (6\ n)$, where $\{p, q\}=\{11, 12\}$ and $\{l, m, n\}=\{7, 8, 9\}$. If $p=11$ and $q=12$, then $R(2)\ R(10)\ (R2)=(1\ 12\ 2\ 10\ 3\ 11)\ (\quad)\notin\Pi$. Thus $R(10)=(1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ l)\ (5\ m)\ (6\ n)$.

But $R(2)\ R(10)=R(10)\ R(3)$ or $4R(2)\ R(10)=4R(10)\ R(3)$

$$\therefore 1R(3)=m. \tag{9}$$

From (9), we obtain three possible cases

$$\begin{aligned} &l=7,\ m=9,\ n=8,\\ &l=8,\ m=7,\ n=9,\\ &l=9,\ m=8,\ n=7. \end{aligned} \tag{10}$$

Thus there are three possible forms of $R(10)$. $R(11)$ and $R(12)$ can be easily obtain (except in few cases). Considering the combination of the values given by (4), (6), (8) and (10) (except in few cases where $R(8)$, and $R(11)$ can not be obtained)we obtained fifty one representations. These representations were tested on the computer for the conditions of Theorem 1 and twenty seven were found to be of Bol loops. The details are given below.

(i) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$, $w=11$, $l=7$, $m=9$ and $n=8$, then Π is Π_1.

(ii) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$, $w=11$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to Π_1 under conjugation by $(1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 9)\ (5\ 8)\ (6\ 7)$.

(iii) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$, $w=11$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to Π_1 under conjugation by $(1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 8)\ (5\ 7)\ (6\ 9)$.

(iv) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=11$, $v=10$, $w=12$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to Π_1 under conjugation by $(1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 12)\ (5\ 11)\ (6\ 10)$.

(v) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=11$, $v=10$, $w=12$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to Π_1 under conjugation by $(1\ 6)\ (2\ 4)\ (3\ 5)\ (7\ 10)\ (8\ 11)\ (9\ 12)$.

(vi) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=11$, $v=10$, $w=12$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to Π_1 under conjugation by $(1\ 5)\ (2\ 6)\ (3\ 4)\ (7\ 10)\ (8\ 11)\ (9\ 12)$.

(vii) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=12$, $v=11$, $w=10$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to Π_1 under conjugation by $(1\ 7)\ (2\ 9)\ (4\ 11)\ (5\ 10)\ (6\ 12)$.

(viii) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=12$, $v=11$, $w=10$, $l=8$, $m=7$ and $n=9$ then Π is isomorphic to Π_1 under conjugation by $(1\ 6)\ (2\ 4)\ (3\ 5)\ (7\ 10)\ (8\ 11)\ (9\ 12)$.

(ix) If $c=10$, $d=12$, $e=11$, $s=10$, $t=12$, $u=12$, $v=11$, $w=10$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to Π_1 under conjugation by $(1\ 5)\ (2\ 6)\ (3\ 4)\ (7\ 10)\ (8\ 11)\ (9\ 12)$.

(x) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=10$, $v=12$, $w=11$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to Π_1 under conjugation by $(1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 12)\ (8\ 11)\ (9\ 10)$.

(xi) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=10$, $v=12$, $w=11$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to the representation given by (x) under conjugation by $(1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 9)\ (5\ 8)\ (6\ 7)$. But the representation given by $(x)\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xii) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=10$, $v=12$, $w=11$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to the representation given by (x) under conjugation by $(1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 8)\ (5\ 7)\ (6\ 9)$. But representation given by $(x)\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xiii) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=11$, $v=10$, $w=12$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to the representation given by (x) under conjugation by $(1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 12)\ (5\ 11)\ (6\ 10)$. But representation given by $(x)\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xiv) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=11$, $v=10$, $w=10$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to the representation given by (xi) under conjugation by $(1\ 6)\ (2\ 4)\ (3\ 5)\ (7\ 10)\ (8\ 11)\ (9\ 12)$. But the representation given by $(xi)\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xv) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=11$, $v=10$, $w=12$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to the representation given by (xi) under conjugation by $(1\ 6)\ (2\ 4)\ (3\ 5)\ (7\ 11)\ (8\ 12)\ (9\ 10)$. But the representation given by $(xi)\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xvi) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=10$, $v=12$, $w-11$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to Π_1 under conjugation by $(1\ 9\ 2\ 7\ 3\ 8)\ (4\ 10\ 5\ 11\ 6\ 12)$.

(xvii) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=10$, $v=12$, $w=11$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to the representation given by (xvi) under conjugation by (1 10) (2 12) (3 11) (4 7) (5 9) (6 8). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xviii) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$ $l=7$, $m=9$ and $n=8$, then Π is isomorphic to the representation given by (xvi) under conjugation by (1 6) (2 4) (3 5) (7 10) (8 11) (9 12). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xix) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to the representation given by (xvi) under conjugation by (1 7) (2 9) (3 8) (4 12) (5 11) (6 10). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xx) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to the representation given by (xvi) under conjugation by (1 6) (2 4) (3 5) (7 10) (8 11) (9 12) But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxi) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=12$, $v=11$, $w=10$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to the representation given by (xvi) under conjugation by (1 6) (2 4) (3 5) (7 10) (8 11) (9 12). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxii) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=12$, $v=11$, $w=10$, $l=8$, $m=7$ and $n=9$, teen Π is isomorphic to the representation given by (xvi) under conjugation by (1 7) (2 9) (3 8) (4 11) (5 10) (6 12). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxiii) If $c=12$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=12$, $v=11$, $w=10$, $l=9$, $m=8$ and $n=7$ is isomorphic to the representation given by (xvi) under conjugation by (1 4) (2 5) (3 6) (7 10) (8 11) (9 12). But the representation given by (xvi)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxiv) If $c=12$, $d=11$, $e=10$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$, $w=11$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to Π_1 under conjugation by (1 4) (2 6) (3 5) (7 11) (8 10) (9 12).

(xxv) If $c=12$, $d=11$, $e=10$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$, $w=11$, $l=8$, $m=7$ and $n=9$, then Π is isomorphic to the representation given by (xxiv) under conjugation by (1 10) (2 12) (3 11) (4 9) (5 8) (6 7). But the representation given by (xxiv)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxvi) If $c=12$, $d=11$, $c=10$, $r=11$, $s=10$, $t=12$, $u=10$, $v=12$ $w=11$, $l=9$, $m=8$ and $n=7$, then Π is isomorphic to the representation given by (xxiv) under conjugation by (1 10) (2 12) (3 11) (4 8) (5 7) (6 9). But the representation given by (xxiv)$\sim\Pi_1$. Hence $\Pi\sim\Pi_1$.

(xxvii) If $c=12$, $d=11$, $e=10$, $r=11$, $s=10$, $t=12$, $u=11$, $v=10$, $w=12$, $l=7$, $m=9$ and $n=8$, then Π is isomorphic to the representation given by (xxiv) under conjugation by (1 7) (2 9) (4 12) (5 11) (6 10).

Now we consider the exceptional cases not covered by the above cases. In case of $c=10$, $d=12$, $e=11$, $r=12$, $s=11$, $t=10$, $u=10$, $v=12$, $w=11$, $l=7$, $m=8$ and $n=9$, we have

$$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 10)\ (8\ 12)\ (9\ 11)$$
$$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ 12)\ (8\ 11)\ (9\ 10)$$
$$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 10)\ (5\ 12)\ (6\ 11)$$
$$R(10) = (1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 7)\ (5\ 9)\ (6\ 8).$$

It is not possible to obtain $R(8)$ and $R(11)$. Without loss of generality we assume

$$R(8)=(1\ 8)\ (2\ 7)\ (3\ 9)\ (4\ s)\ (5\ t)\ (6\ w). \quad \text{But} \quad R(2)\,R(8)=R(8)\,R(3)$$
$$\text{or} \quad 4R(2)\,R(8)=4R(8)\,R(3) \quad \text{or} \quad sR(3)=t. \tag{10}$$

From (10), we obtain two possible cases

$$s=11,\ t=10,\ w=12,$$
$$s=12,\ t=11,\ w=10. \tag{11}$$

Without loss of generality, we assume
$R(11)=(1\ 11)\ (2\ 10)\ (3\ 12)\ (4\ u)\ (5\ v)\ (6\ p)$. But

$$R(2)\,R(11)=R(11)\,R(3)$$
$$\text{or} \quad 4R(2)\,R(11)=R(11)\,R(3) \tag{12}$$
$$\text{or} \quad uR(3)=v$$

From (12), we obtain two possible cases (13)

$$u=8,\ v=7,\ p=9,$$
$$u=9,\ v=8,\ p=7. \tag{13}$$

Considering the various combination of the values given by (11) and (12), we obtained four representations. These representations were tested on the computer for the conditions of Theorem 1 and the following one was found to be of Bol loop.

Π_3 (Moufang loop)

$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$,
$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11)$,
$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 10)\ (8\ 12)\ (9\ 11)$,
$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ 12)\ (8\ 11)\ (9\ 10)$,
$R(6) = (1\ 6)\ (2\ 5)\ (3\ 4)\ (7\ 11)\ (8\ 10)\ (9\ 12)$.
$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 10)\ (5\ 12)\ (6\ 11)$,
$R(8) = (1\ 8)\ (2\ 7)\ (3\ 9)\ (4\ 12)\ (5\ 11)\ (6\ 10)$,
$R(9) = (1\ 9)\ (2\ 8)\ (3\ 7)\ (4\ 11)\ (5\ 10)\ (6\ 12)$,
$R(10) = (1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 7)\ (5\ 9)\ (6\ 8)$,
$R(11) = (1\ 11)\ (2\ 10)\ (3\ 12)\ (4\ 9)\ (5\ 8)\ (6\ 7)$.
$R(12) = (1\ 12)\ (2\ 11)\ (3\ 10)\ (4\ 8)\ (5\ 7)\ (6\ 9)$.

Π_3 satisfies disassociative property. Hence Π_3 is a Moufang loop. The representation Π_3 is due to chein [5]. In case $c=11$, $d=10$, $e=12$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$, $l=8$, $m=7$ and $n=9$, we have

$$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 11)\ (8\ 10)\ (9\ 12)$$
$$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ 10)\ (8\ 12)\ (9\ 11)$$

$$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 11)\ (5\ 10)\ (6\ 12)$$

$$R(10) = (1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 8)\ (5\ 7)\ (6\ 9).$$

By using the same analysis as above, we found that if

$$R(8) = (1\ 8)\ (2\ 7)\ (3\ 9)\ (4\ 10)\ (5\ 12)\ (6\ 11) \quad \text{and}$$

$$R(11) = (1\ 11)\ (2\ 10)\ (3\ 12)\ (4\ 7)\ (5\ 9)\ (6\ 8)$$

then the representation Π satisfies the conditions of Theorem 1. But Π is isomorphic ti Π_2 under conjugation by (1 7) (2 9) (3 8) (4 12) (5 11) (6 10).

In the last case $c=12$, $d=11$, $e=10$, $r=11$, $s=10$, $t=12$, $u=12$, $v=11$, $w=10$, $l=9$, $m=8$ and $n=7$, we have

$$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ 12)\ (8\ 11)\ (9\ 10)$$

$$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ 11)\ (8\ 10)\ (9\ 12)$$

$$R(7) = (1\ 7)\ (2\ 9)\ (3\ 8)\ (4\ 12)\ (5\ 11)\ (6\ 10)$$

$$R(10) = (1\ 10)\ (2\ 12)\ (3\ 11)\ (4\ 9)\ (5\ 8)\ (6\ 7).$$

By using the same analysis as above, we found that if

$$R(8) = (1\ 8)\ (3\ 9)\ (4\ 11)\ (5\ 10)\ (6\ 12) \quad \text{and}$$

$$R(11) = (1\ 11)\ (2\ 10)\ (3\ 12)\ (4\ 8)\ (5\ 7)\ (6\ 9),$$

then the representation Π satisfies the conditions of Theorem 1. But Π is isomorphic to Π_2 under conjugation by (1 4) (2 9) (3 8) (4 11) (5 10) (6 12).

Case (c) *Five element of order* 2. In this case Π contains four elements of order 6, two elements of order 3 and five elements of order 2. Without loss of generality we assume

$$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$$

$$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11).$$

Next we assume without loss of generality that $R(4)$ is of order 2, $R(5)$ and $R(6)$ are of order 6. Thus as in case (b)

$$R(2)\,R(4) = R(4)\,R(2) \text{ or } 7R(2)\,R(4) = 7R(4)\,R(2) \text{ or } pR(2) = q \tag{14}$$

From (14), we obtain three possible cases

$$\begin{aligned} &p=10,\ q=11,\ f=12,\\ &p=11,\ q=12,\ f=10,\\ &p=12,\ q=10,\ f=11. \end{aligned} \tag{15}$$

Further we assume without loss of generality that $R(7)$, $R(8)$ and $R(9)$ are of order 2. Thus

$R(7) = (1\ 7)\ (2\ 8)\ (3\ 9)\ (4\ u)\ (5\ v)\ (6\ w)$, where the values of u, v and w are given by (8). Further

$$R(8) = (1\ 8)\ (2\ 9)\ (3\ 7)\ (4\ i)\ (5\ j)\ (6\ k). \text{ But}$$

$$R(8)\,R(2) = R(2)\,R(8) \text{ or } 4R(8)\,R(2) = R(2)\,R(8) \text{ or } iR(2) = j. \tag{16}$$

From (16), we get three possible cases

$$\begin{aligned} &i=10,\ j=11,\ k=12,\\ &i=11,\ j=12,\ k=10,\\ &i=12,\ j=10,\ k=11. \end{aligned} \tag{17}$$

Further we assume that $R(10)$ is of order 2, $R(11)$ and $R(12)$ are of order 6. As in case (a), we have

$$\begin{aligned} &R(10) = (1\ 10)\ (2\ 11)\ (3\ 12)\ (4\ \alpha)\ (5\ \beta)\ (6\ \gamma). \text{ But } R(10)\,R(2) = R(2)\\ &\quad R(10) \text{ or } 4R(10)\,R(2) = 4R(2)\,R(10) \text{ or } \alpha R(2) = \beta. \end{aligned} \tag{18}$$

From (18), we obtain three possible cases

$$\begin{aligned} &\alpha=7,\ \beta=8,\ \gamma=9,\\ &\alpha=8,\ \beta=9,\ \gamma=7,\\ &\alpha=9,\ \beta=7,\ \gamma=8. \end{aligned} \tag{19}$$

Considering the combinations of the values given by (8), (15), (17) and (19), we obtained 54 representations. These representations were tested on the computer for the conditions of Theorem 1 and only three were found to be of Bol loops. The details are given below.

(i) If $p=10$, $q=11$, $f=12$, $u=10$, $v=12$, $w=11$, $i=11$, $j=10$, $k=12$, $\alpha=7$, $\beta=8$ and $\gamma=9$, then Π is Π_2.

(ii) If $p=11$, $q=12$, $f=10$, $u=11$, $v=10$, $w=12$, $i=12$, $j=11$, $k=10$, $\alpha=9$, $\beta=7$, $\gamma=8$, then Π is isomorphic to Π_2 under conjugation by (1 7) (2 9) (3 8) (4 12) (5 11) (6 10).

(iii) If $p=12$, $q=10$, $f=11$, $u=12$, $v=11$, $w=10$, $i=10$, $j=12$, $k=11$, $\alpha=8$, $\beta=9$, $\gamma=7$, then Π is isomorphic to Π_2 under conjugation by (1 7) (2 9) (3 8) (4 11) (5 10) (6 12).

Case (d) *Seven elements of order* 2. In this case there are two element of order 6, two elements of order 3 and seven elements of order 2. Without loss of generality we assume that $R(2)$, $R(3)$ are of order 3, $R(4)$, $R(5)$, $R(6)$, $R(7)$, $R(8)$, $R(9)$ and $R(10)$ are of order 2, $R(11)$ and $R(12)$ are of order 6. Thus

$$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$$

$$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11)$$

$R(4) = (1\ 4)\ (2\ 6)\ (3\ 5)\ (7\ c)\ (8\ d)\ (9\ e)$, where the values of c, d and e are given by (4).

$R(5) = (1\ 5)\ (2\ 4)\ (3\ 6)\ (7\ r)\ (8\ s)\ (9\ t)$, where the values of r, s and t are given by (6).

$R(7) = (1\ 7)\ (2\ 8)\ (3\ 9)\ (4\ u)\ (5\ v)\ (6\ w)$, where the values of u, v and w are given by (8).

$(R(10)=(1\ 10)\ (2\ 11)\ (3\ 12)\ (4\ \alpha)\ (5\ \beta)\ (6\ \gamma)$, where the values of α, β and γ are given by (19).

Considering the combinations of the values given by (4), (6), (8) and (19), we obtained 54 representations. These representations were tested on the computer for the conditions of Theorem 1. The details are given below.

(i) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=12$, $u=10$, $v=12$, $w=11$, $\alpha=7$, $\beta=8$, $\gamma=9$, then Π is the representation of D_6.

(ii) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=12$, $v=11$, $w=10$, $\alpha=8$, $\beta=9$ and $\gamma=7$, then $\Pi\sim D_x$.

(iii) If $c=11$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$, $\alpha=9$, $\beta=7$ and $\gamma=8$, then $\Pi\sim D_6$.

Case (*e*) *Three elements of order* 2. In this case Π contains three elements order 2, two elements of order 3 and six elements of order 6. Without loss generality we assume that $R(4)$, $R(7)$ and $R(10)$ are of order 2, $R(2)$ and $R(3)$ are of order 3. and
$R(5)$, $R(6)$, $R(8)$, $R(9)$, $R(11)$ and $R(12)$ are of order 6. Thus

$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$

$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11)$

$R(4) = (1\ 4)\ (2\ 5)\ (3\ 6)\ (7\ p)\ (8\ q)\ (9\ f)$, where the values of p,q and r are given by (15).

$R(17)=(1\ 7)\ (2\ 8)\ (3\ 9)\ (4\ p_1)\ (5\ q_1)\ (6\ r_1)$, where the values of p_1, q_1 and r_1 are as follows:

$$\begin{aligned} &p_1=10,\ \ y_1=11,\ \ r_1=12,\\ &p_1=11,\ \ q_1=12,\ \ r_1=10,\\ &p_1=12,\ \ q_1=10,\ \ r_1=11. \end{aligned} \tag{20}$$

$R(10)=(1\ 10)\ (2\ 11)\ (3\ 12)\ (u\ \alpha)\ (5\ \beta)\ (6\ \gamma)$, where the values of α, β and γ are given by (19). Considering the combinations of the values given by (15) (19) and (20), we obtained twenty seven tables representations. These representations were tested on the computer for the conditions of Theorem 1.

(i) In case $p=10$, $q=11$, $f=12$, $p_1=10$, $q_1=11$, $r_1=12$, $\alpha=7$, $\beta=8$ and $\gamma=9$, then $\Pi\sim C_6\times C_2$ (Abelian—group).

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ON THE THERMODYNAMICS OF NONLOCAL CONTINUA WITH MICROSTRUCTURE

BY

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In the continuum mechanics various models, in local or nonlocal description, for bodies with microstructure were elaborated. Among the papers existing in the scientific literature, concerning the bodies with microstructure in local description, we quote [1]—[8]. Some results and references to this problem can be found in [9]—[16].

In the theory of nonlocal continua there exist many papers and many points of view too [17]—[30].

In the papers [27]—[30] a new point of view, in order to derive the equations of motion for nonlocal continua, was given.

In our description, the factors which concur to deformation are divided into external and internal factors. Both, the external and the internal factors can be modeled by body forces, body couples, surface tractions, surface couples, heat sources, magnetic fields, etc.

Making all combinations between the possible external actions and possible internal capacity of the body into reply against these actions, we can derive different models in order to describe the behaviour for some real bodies.

In this paper, we will consider that the external influences received by the body are represented by external surface forces and external surface couples, external body forces and external body couples, external heat body sources and an external surface heat flux.

As nonlocal reply, the body opposes internal body forces and internal body couples, internal surface tractions and internal surface couples, internal body heat sources and an internal surface heat flux.

We refer the motion of the body to fixed rectangular Cartesian axes, the unit vectors of axes being $\mathbf{e}_i$.

Let V be the region occupied by the body and let P be an arbitrary part of V.

In addition to the above suppositions the body has a microstructure.

For a medium with microstructure we adopt the following definition.

There exist a tensor field $J_{ik}=J_{ki}$ ($J_{ik}\ \xi_i\xi_k>0$, $\xi_k\neq0$), called microinertia field and a vector field $\varphi=\varphi_k\mathbf{e}_k$, called rotation field, defined on V, such that the linear momentum $\mathbf{H}_P$, the angular momentum $\mathbf{K}_P$ and the kinetic energy E_P of the medium located in the domain $P\subset V$ are given by