($R(10)=(1\ 10)\ (2\ 11)\ (3\ 12)\ (4\ \alpha)\ (5\ \beta)\ (6\ \gamma)$, where the values of α, β and γ are given by (19).

Considering the combinations of the values given by (4), (6), (8) and (19), we obtained 54 representations. These representations were tested on the computer for the conditions of Theorem 1. The details are given below.

(i) If $c=10$, $d=12$, $e=11$, $r=11$, $s=10$, $t=12$, $u=12$, $u=10$, $v=12$, $w=11$, $\alpha=7$, $\beta=8$, $\gamma=9$, then Π is the representation of D_6.

(ii) If $c=11$, $d=10$, $e=12$, $r=12$, $s=11$, $t=10$, $u=12$, $v=11$, $w=10$, $\alpha=8$, $\beta=9$ and $\gamma=7$, then $\Pi \sim D_x$.

(iii) If $c=11$, $d=11$, $e=10$, $r=10$, $s=12$, $t=11$, $u=11$, $v=10$, $w=12$, $\alpha=9$, $\beta=7$ and $\gamma=8$, then $\Pi \sim D_6$.

Case (*e*) *Three elements of order* 2. In this case Π contains three elements order 2, two elements of order 3 and six elements of order 6. Without loss generality we assume that $R(4)$, $R(7)$ and $R(10)$ are of order 2, $R(2)$ and $R(3)$ are of order 3. and
$R(5)$, $R(6)$, $R(8)$, $R(9)$, $R(11)$ and $R(12)$ are of order 6. Thus

$R(2) = (1\ 2\ 3)\ (4\ 5\ 6)\ (7\ 8\ 9)\ (10\ 11\ 12)$

$R(3) = (1\ 3\ 2)\ (4\ 6\ 5)\ (7\ 9\ 8)\ (10\ 12\ 11)$

$R(4) = (1\ 4)\ (2\ 5)\ (3\ 6)\ (7\ p)\ (8\ q)\ (9\ f)$, where the values of p, q and r are given by (15).

$R(17)=(1\ 7)\ (2\ 8)\ (3\ 9)\ (4\ p_1)\ (5\ q_1)\ (6\ r_1)$, where the values of p_1, q_1 and r_1 are as follows:

$$\begin{aligned} p_1&=10, & q_1&=11, & r_1&=12,\\ p_1&=11, & q_1&=12, & r_1&=10,\\ p_1&=12, & q_1&=10, & r_1&=11. \end{aligned} \tag{20}$$

$R(10)=(1\ 10)\ (2\ 11)\ (3\ 12)\ (u\ \alpha)\ (5\ \beta)\ (6\ \gamma)$, where the values of α, β and γ are given by (19). Considering the combinations of the values given by (15) (19) and (20), we obtained twenty seven tables representations. These representations were tested on the computer for the conditions of Theorem 1.

(i) In case $p=10$, $q=11$, $f=12$, $p_1=10$, $q_1=11$, $r_1=12$, $\alpha=7$, $\beta=8$ and $\gamma=9$, then $\Pi \sim C_6 \times C_2$ (Abelian—group).

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ON THE THERMODYNAMICS OF NONLOCAL CONTINUA WITH MICROSTRUCTURE

BY

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In the continuum mechanics various models, in local or nonlocal description, for bodies with microstructure were elaborated. Among the papers existing in the scientific literature, concerning the bodies with micro-structure in local description, we quote [1]—[8]. Some results and references to this problem can be found in [9]—[16].

In the theory of nonlocal continua there exist many papers and many points of view too [17]—[30].

In the papers [27]—[30] a new point of view, in order to derive the equations of motion for nonlocal continua, was given.

In our description, the factors which concur to deformation are divided into external and internal factors. Both, the external and the internal factors can be modeled by body forces, body couples, surface tractions, surface couples, heat sources, magnetic fields, etc.

Making all combinations between the possible external actions and possible internal capacity of the body into reply against these actions, we can derive different models in order to describe the behaviour for some real bodies.

In this paper, we will consider that the external influences received by the body are represented by external surface forces and external surface couples, external body forces and external body couples, external heat body sources and an external surface heat flux.

As nonlocal reply, the body opposes internal body forces and internal body couples, internal surface tractions and internal surface couples, internal body heat sources and an internal surface heat flux.

We refer the motion of the body to fixed rectangular Cartesian axes, the unit vectors of axes being $\mathbf{e}_i$.

Let V be the region occupied by the body and let P be an arbitrary part of V.

In addition to the above suppositions the body has a microstructure.

For a medium with microstructure we adopt the following definition.

There exist a tensor field $J_{ik}=J_{ki}$ ($J_{ik}\ \xi_i\xi_k>0$, $\xi_k\neq 0$), called micro-inertia field and a vector field $\varphi=\varphi_k \mathbf{e}_k$, called rotation field, defined on V, such that the linear momentum $\mathbf{H}_P$, the angular momentum $\mathbf{K}_P$ and the kinetic energy E_P of the medium located in the domain $P\subset V$ are given by

$$H_P = \int_P \rho(x)v(x)dX = \int_P \rho v dX, \quad K_P = \int_P \rho(r \times v + J_{mn}\dot{\varphi}_m e_n)dX,$$

$$E_P = \frac{1}{2}\int_P \rho(v^2 + J_{mn}\dot{\varphi}_m\dot{\varphi}_n)dX,$$

where $\rho(x)$ is the density of the medium, $v(x)$ is the velocity field, r is the position vector of the point x with respect to the coordinate origin and $\dot{\varphi}_i = d\varphi_i/dt$. Here and in the following dX and dY are elements of volume.

We use the summation convention over repeated indices.

The influence of the mass contained in the domain $V - P$ on the mass contained in the domain P can be described by an internal body forces field, an internal body couples field and an internal body heat sources field acting at the points of P. On the boundary ∂P of P we consider the surface tractions, the surface couples and a heat surface flux.

The model which we intend to develop obeis to the following principles:

I. **Principle of conservation of mass**

$$\frac{d}{dt}\int_P \rho(x)dX = 0. \tag{1}$$

II. **Principle of internal body actions.** a) There exists a vector function $g(x, y)$ defined on $V \times V$, called internal body forces density, such that, the contribution $g_P(x)$ of the mass contained in a region $P \subset V$ to the total internal body forces in the point $x \in V$ is given by

$$\rho(x)g_P(x) = \int_P \rho(y)g(x, y)dY \quad x \in V, \quad y \in P. \tag{2}$$

We will denote

$$\rho(x)g(x) = \rho(x)g_V(x) = \int_V \rho(y)g(x, y)dY \tag{3}$$

and that means $g(x)$ is the total internal body forces density per unit mass in the point $x \in V$.

b) There exists a vector function $m(x, y)$ defined on $V \times V$, called internal body couples density, such that, the internal body couples $m_P(x)$ in the point $x \in V$ caused by the mass contained in P is given by

$$\rho(x)m_P(x) = \int_P \rho(y)m(x, y)dY \tag{4}$$

and

$$\rho(x)m(x) = \rho(x)m_V(x) = \int_V \rho(y)m(x, y)dY. \tag{5}$$

Thus $m(x)$ is the total internal body couples density per unit mass in the point $x \in V$.

c) The functions $\overline{g}(x, y)$ and $\overline{m}(x, y)$ have the following properties

$$\int_P\int_P \rho(y)g(x, y)dXdY = \int_P \rho(x)g_P(x)dX = 0, \tag{6}$$

$$\begin{aligned} &\int_P \left[r \times \int_P \rho(y)g(x, y)dY\right]dX + \int_P\int_P \rho(y)m(x, y)dXdY = \\ &= \int_P [r \times \rho(x)g_P(x)]dX + \int_P \rho(x)m_P(x)dX = 0. \end{aligned} \tag{7}$$

The above formulae mean that the resultant and the resultant moment of internal body forces and internal body couples of P are zero.

d) There exists a scalar function $w(x, y)$ defined on $V \times V$, called internal body heat sources density, such that, the contribution $w_P(x)$ of the mass contained in a region $P \subset V$ to the internal body heat sources at the point $x \in V$ can be obtained in the following manner

$$\rho(x)w_P(x) = \int_P \rho(y)w(x,y)dY. \tag{8}$$

We will denote

$$\rho(x)w(x) = \rho(x)w_V(x) = \int_V \rho(y)w(x, y)dY \tag{9}$$

and $w(x)$ is the total internal body heat sources density per unit mass in the point $x \in V$.

e) The internal body forces $g(x, y)$ the internal body couples $\overline{m}(x, y)$ and the internal heat sources $w(x, y)$ appear as response to deformation and they satisfy.

$$\begin{aligned} &\int_P dX \int_P \rho(y)[g(x, y)v(x) + m(x, y)\dot{\varphi}(x) + w(x, y)]dY = \\ &= \int_P \rho(x)[g_P(x)v(x) + m_P(x)\dot{\varphi}(x) + w_P(x)]dX = 0. \end{aligned} \tag{10}$$

The condition (10) has an evident meaning. Because g_P, m_P and w_P are considered as response to deformation only, this relation means that we cannot create or destroy energy by a symple deformation. The first member of (10) do not vanish if for example, the internal forces are of the Newtonian type.

From (10) we have

$$\int_V \rho(x)[\mathbf{g}(x)\mathbf{v}(x)+\mathbf{m}(x)\dot{\varphi}(x)+w(x)]dX=0. \tag{11}$$

As a particular case we may consider the media for which

$$\mathbf{g}(x)\mathbf{v}(x)+\mathbf{m}(x)\dot{\varphi}(x)+w(x)=0 \quad x\in V. \tag{12}$$

III. **Principle of linear momentum.** The derivative of the linear momentum of P with respect to the time is equal to the resultant of external (to P) forces:

$$\frac{d}{dt}\int_P \rho(x)\mathbf{v}(x)dX=\int_P \rho(x)\mathbf{f}(x)dX+\int_{\partial P}\mathbf{t}d\sigma+\int_P dX\int_{V-P}\rho(y)\mathbf{g}(x,y)dY, \tag{13}$$

where $\mathbf{f}(x)$ is the external body forces per unit mass, the surface tractions on ∂P per unit area as contact effects of the actions of the medium located in $V-P$.

IV. **Principle of angular momentum.** The derivative of the angular momentum of the mass located in P is equal to the resultant momentum of external (to P) applied actions. Thus we have

$$\begin{aligned}\frac{d}{dt}\int_P [\mathbf{r}\times\rho(x)\mathbf{v}(x)+J_{mn}\dot{\varphi}_m\mathbf{e}_n]dX=\int_P \rho(x)[\mathbf{r}\times\mathbf{f}(x)+\mathbf{M}(x)]dX+\\ +\int_{\partial P}(\mathbf{r}\times\mathbf{t}+\mathbf{c})d\sigma+\int_P\left[\mathbf{r}\times\int_{V-P}\rho(y)\mathbf{g}(x,y)dY\right]dX+\int_P dX\int_{V-P}\rho(y)\mathbf{m}(x,y)dY,\end{aligned} \tag{14}$$

where $\mathbf{M}(x)$ is the external body couples per unit mass and $\mathbf{c}$ is the surface couples vector stress per unit area across the surface ∂P.

V. **Principle of energy.** There exists a specific energy ε per unit mass defined on V, such that, the balance of energy can be stated as

$$\begin{aligned}\frac{d}{dt}\int_P\left[\varepsilon+\frac{1}{2}(\mathbf{v}^2+J_{mn}\dot{\varphi}_m\dot{\varphi}_n)\right]dX=\int_P(\mathbf{f}\mathbf{v}+\mathbf{M}\dot{\varphi}+r)dX+\int_{\partial P}(\mathbf{t}\mathbf{v}+\mathbf{c}\dot{\varphi}-h)d\sigma+\\ +\int_P dX\int_{V-P}\rho(y)[\mathbf{g}(x,y)\mathbf{v}(x)+\mathbf{m}(x,y)\dot{\varphi}(x)+w(x,y)]dY,\end{aligned} \tag{15}$$

where r is the external heat sources per unit mass and h is the surface heat flux per unit area across the surface ∂P.

VI. **Principle of entropy.** There exists a specific entropy S per unit mass, such that, for any processes of deformation we have

$$\frac{d}{dt}\int_P \rho(x)S(x)dX-\left\{\int_P\frac{\rho(x)r(x)}{T(x)}dX-\int_{\partial P}\frac{h}{T}d\sigma\right\}\geq 0, \tag{16}$$

where $T>0$ is the absolute temperature*).

*) Here the principle of entropy was stated as in [30] which is different from the formulation given in [29].

Using these principles we can derive the equations of motion for our continuum medium.

Let us first observe that according to (6), (7) and (10), we have

$$\int_P dX\int_{V-P}\rho(y)\mathbf{g}(x,y)dY=\int_P dX\int_V\rho(y)\mathbf{g}(x,y)dY=\int_P\rho(x)\mathbf{g}(x)dX,$$

$$\begin{aligned}&\int_P dX\int_{V-P}\rho(y)[\mathbf{r}(x)\times\mathbf{g}(x,y)+\mathbf{m}(x,y)]dY=\\ &=\int_P dX\int_{V-P}\rho(y)[\mathbf{r}(x)\times\mathbf{g}(x,y)+\mathbf{m}(x,y)]dY=\int_P\rho(x)[\mathbf{r}\times\mathbf{g}(x)+\mathbf{m}(x)]dX\end{aligned}$$

and

$$\begin{aligned}&\int_P dX\int_V\rho(y)[\mathbf{g}(x,y)\mathbf{v}(x)+\mathbf{m}(x,y)\dot{\varphi}(x)+w(x,y)]dY=\\ &=\int_P\rho(x)[\mathbf{g}(x)\mathbf{v}(x)+\mathbf{m}(x)\dot{\varphi}(x)+w(x)]dX.\end{aligned}$$

Now, let us examine the consequences resulting from the principle of linear momentum.

Taking into account the above results and making use of divergence theorem and of equation of continuity

$$\frac{d\rho}{dt}+\rho\operatorname{div}\mathbf{v}=0, \tag{17}$$

derived from first principle, we easily get

$$\int_P[\mathbf{t}_{i,i}+\rho(\mathbf{f}+\mathbf{g})-\rho\mathbf{a}]dX=0, \tag{18}$$

where $\mathbf{a}$ is the acceleration field and the index i after comma indicates partial differentiation with respect to x_i.

Since the integrand is supposed to be continuous and (18) must hold for an arbitrary P, it results that

$$\mathbf{t}_{i,i}+\rho(\mathbf{f}+\mathbf{g})=\rho\mathbf{a}. \tag{19}$$

In a rectangular Cartesian sistem of coordinates x_1, x_2, x_3, we obtain the well-known equations of motion

$$t_{ji,j}+\rho(f_i+g_i)=\rho a_i, \tag{20}$$

where t_{ij} are the stress tensor components

$$\mathbf{t}=t_k\mathbf{e}_k=t_{jk}n_j\mathbf{e}_k=\mathbf{t}_j n_j,\quad \mathbf{f}=f_i\mathbf{e}_i,\quad \mathbf{g}=g_i\mathbf{e}_i,\quad \mathbf{a}=a_i\mathbf{e}_i.$$

Analogously we have

$$\mathbf{c} = c_k\mathbf{e}_k = c_{ji}n_j\mathbf{e}_k = \mathbf{c}_j n_j, \quad \mathbf{M} = M_i\mathbf{e}_i, \quad \mathbf{m} = m_i\mathbf{e}_i$$

and, from the postulate of angular momentum, at last, we get

$$c_{ji,j} + \varepsilon_{ijk}t_{jk} + \rho(M_i + m_i) = \rho J_{ij}\ddot{\varphi}_j, \tag{21}$$

where c_{ij} are the couple stress tensor components and ε_{ijk} is the alterning symbol.

From the postulate of energy, by known procedure, we obtain

$$\begin{aligned}\int_P \rho(\dot{\varepsilon} + v_iv_i + J_{ij}\dot{\varphi}_i\ddot{\varphi}_j)dX = \int_P \rho(f_i + g_i)v_i + (M_i + m_i)\dot{\varphi}_i + r + w]dX + \\ + \int_P (t_{ji,j}v_i + t_{ji}v_{i,j} + c_{ji,j}\dot{\varphi}_i + c_{ji}\ddot{\varphi}_{i,j} - q_{i,i})dX,\end{aligned} \tag{22}$$

where q_i are the components of the heat flux.

Making use of (20), (21) and remembering that (22) holds for arbitrary P we obtain the equation of energy in the form

$$\rho\dot{\varepsilon} - t_{ij}v_{i,j} - c_{ji}\dot{\varphi}_{i,j} + \varepsilon_{ijk}t_{jk}\dot{\varphi}_i + q_{i,i} - \rho(r + w) = 0. \tag{23}$$

In the same way from (16) we get

$$\rho T\dot{S} - \rho r + q_{i,i} - \frac{q_iT_i}{T} \geqslant 0. \tag{24}$$

With the Helmholtz specific free energy $\psi = \varepsilon - ST$ the equation (23) and the inequality (24) become

$$\rho(\dot{\psi} + \dot{S}T + S\dot{T}) - t_{ij}v_{i,j} - c_{ji}\dot{\varphi}_{i,j} + \varepsilon_{ijk}t_{ji}\dot{\varphi}_i + q_{i,i} - \rho(r + w) = 0, \tag{25}$$

$$-\rho(\dot{\psi} + S\dot{T}) + t_{ji}v_{i,j} + c_{ji}\dot{\varphi}_{i,j} - \varepsilon_{ijk}t_{jk}\dot{\varphi}_i \geqslant 0. \tag{26}$$

If the equation (12) are valid then (25) and (26) are replaced by

$$\rho(\dot{\psi} + \dot{S}T + S\dot{T}) - t_{j,i}v_{i,j} - c_{ji}\dot{\varphi}_{i,j} + q_{i,i} - \rho r + \rho g_iv_i + (\rho m_i + \varepsilon_{ijk}\dot{t}_{jk})\dot{\varphi}_i = 0.$$

$$-\rho(\dot{\psi} + S\dot{T}) + t_{ji}v_{i,j} + c_{ji}\dot{\varphi}_{i,j} - \rho g_iv_i - (\rho m_i + \varepsilon_{ijk}t_{jk})\dot{\varphi}_i - \frac{q_iT_{,i}}{T} \geqslant 0.$$

Now, it remains to prescribe appropriate constitutive equations for $t_{ij}, c_{ij}, q_i, g_i(x)$[or $g_i(x, y)$], $m_i(x)$ [or $m_i(x, y)$], $w(x)$ [or $w(x, y)$].

As we can see, the energy equation and the entropy inequality are similar to the local classical case and as a consequence we can employ similar technics to derive the constitutive equations. Likewise, we can use the technics developed by Eringen [20], [22], Edelen [24], Green and Naghdi [26].

Some remarks. For some of nonlocal theories in the central point are the so called localization residuals in the balance laws. Here, the localization residuals are removed out by the introduction of the internal replies of the body. All of these new-entities have a precise meaning and in consequence by this procedure the nonlocal problem of deformation gets in clarity.

— It seems that even when the body is acted by mechanical external means only, the internal reply of the body is purely mechanical only in very special cases. Usually, the internal reply of the body is accompanied by an internal heat field, even in the statical case.

— The model which obeis to the equations (20), (21) is similar with the micropolar models (free rotations), but the presented model has a different behaviour and we call it model with natural rotations. Informations about micropolar models can be found in [11], [12].

— It seems that the presented model and the models of micropolar type describe two different phases for the same process of deformation. The connection between them, seems to be similar with the connection between elasticity and plasticity in the classical theory. When the internal body couples are absent the presented model becomes a model of micropolar type (see [29]).

If the couple „elasticity-plasticity" describes the deformation for the materials which can flow on the charge, the couple „model with natural rotation—model with free rotations" seems to describe the deformation of the bodies which arrive to be distroied by dislocations.

— When, in the response of the body, the surface couple stress are absent, then $c_{ij} = 0$ and the equation (21) become

$$t_{ij} - t_{ji} + \varepsilon_{ijk}\rho(M_k + m_i) = \varepsilon_{ijk}\rho J_{km}\ddot{\varphi}_m.$$

If, in addition, the external body couples and the internal body couples are absent, then

$$t_{ij} - t_{ji} = \varepsilon_{ijk}\rho J_{km}\ddot{\varphi}_m$$

If the microstructure of the medium is absent and the above suppositions are still valid, then the stress tensor is symmetrical :

$$t_{ij} = t_{ji}.$$

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BOUNDARY VALUE PROBLEMS IN MICROPOLAR THERMOELASTICITY

BY

I. CRĂCIUN

1. Introduction. There are many papers and books which study the stationary vibrations in elastic solids. This kind of problems have been initiated by V. D. Kupradze, the results of his researches and of others, especially from his school, or from elsewhere, up to 1976, being published in [1].

In our previous papers [5]—[7] some problems concerning the stationary vibrations of an isotropic, homogeneous and centrosymmetric solid in the linear theory of micropolar thermoelasticity [2], have been studied. The same theory has been discussed by Agniasvili [8].

In this paper two interior and two exterior boundary value problems of the above mentioned theory, which have been introduced in [6], are considered. For the study of these problems we have deduced in Sect. 3 some Green identities from which, by using the same method as in [1], [3]—[5], [7], [8], the integral representation formulae for the regular solution of the differential equations of the governing theory have been given. Starting from the results of Sect. 3, we introduce in Sect. 4 two micropolar thermoelastic surface potentials for which some properties are given. In Sect. 5, we reformulate the two interior and the two exterior boundary value problems by means of some boundary operators introduced in Sect. 3.

In the last section these problems are reduced to singular integral equations of Fredholm type. The study of these singular integral equations allow to prove existence theorems for the considered problems.

2. Basic Equations. We consider an isotropic, homogeneous and centrosymmetric micropolar thermoelastic solid occupying either a three-dimensional domain D^+, bounded by a closed Lyapunov surface $S \in L_1(\alpha)$, $0 < \alpha \leqslant 1$, or the infinite domain D^-, where $D^- = E_3 / D^+ \cup S$, E_3 being the three-dimensional Euclidian space.

If we consider that this solid is acting on his surface S by stresses, couple-stresses and heat flux varying harmonicaly on time, then, in the absence of the body force, couple-body force and heat source, the basic differential equations which give the amplitudes $\mathbf{u}$, φ, θ of the displacement vector, the rotation vector and the increament of temperature in the point $x(x_1, x_2, x_3)$, respectively, are [4]