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ON FINSLER SPACES WITH NEO-PSEUDO-PROJECTIVE CURVATURE TENSOR OF SCALAR CURVATURE

BY

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1. **Finsler spaces with neo-pseudo-projective curvature tensor of scalar curvature.** In an isotropic Finsler space projective the deviation tensor, W^i_j vanishes indentically (Rund, 1959). The necessary and sufficient condition for a Finsler space F_n with neo-pseudo-projective deviation tensor to be a space of scalar curvature is

$$Q^i_j = -qF^2R(\delta^i_j - l^i l_j), \tag{1.1}$$

here $l^j = \dot{x}^j/F$ and the scalar R is the Riemannian curvature. If R vanishes, $Q^j_i = 0$, which implies $Q^i_{jkh} = 0$. Thus we see that every isotropic Finsler space of vanishing Riemannian curvature is Q-flat.

Theorem 1.1. *In every Finsler space $F_n(n > 2)$ of scalar curvature,*

$$Q^r_{[s} G^m_{k]hr} = 0 \tag{1.2}$$

holds.

Proof. Transvecting (1.1) by G^m_{khi} and using the symmetric property of G^m_{jkh}, we have $Q^r_j G^m_{khr} = Q^r_k G^m_{jhr}$, which by differentiation with respect to $\dot{x}^s$ gives (1.2).

Theorem 1.2. *Every Finsler space $F_n(n > 2)$ of scalar curvature admitting*

$$Q^i_r G^r_{khm} = Q^r_m G^i_{khr}, \tag{1.4}$$

is a Riemannian space of constant Riemannian curvature R, provided $R \neq 0$.

Proof. Transvecting (1.2) by G^j_{khm}, we have

$$Q^i_j G^j_{khm} = -qF^2 R G^i_{khm} + qFRl^i y_j G^j_{khm}, \tag{1.5}$$

which due to (1.4) yields $y_j G^j_{khm} = 0$ (Landsberg space). A Landsberg space $F_n(n > 2)$ of scalar is Riemannian space of constant Riemannian curvature R, provided $R \neq 0$ (Numata's theorem, 1975).

Theorem 1.3. *Every Q-symmetric Finsler space $F_n(n > 2)$ of scalar curvature is a Riemannian space of constant Riemannian curvature R, provided $R \neq 0$.*

Proof. Differentiating $Q^i_{kh(m)} = 0$ partially with respect to $\dot{x}^j$ and using the commutation formula (Rund 1959, (6.11) b, p. 127) and the transvection by $\dot{x}^k$, we have (1.4), which due to $Q^r_j G^m_{khr} = -qF^2 R G^m_{jkh}$ yields the result.

Theorem 1.4. *A Q-symmetric Finsler space $F_n(n>2)$ of non-zero scalar curvature does not exist.*

Proof. Suppose that a Q-symmetric Finsler space of non-zero scalar curvature exist. Contracting i and j in (1.1), we get

$$T=-F^2R, \tag{1.6}$$

which implies $T\neq 0$ that is $H\neq 0$ (Sinha, 1971). Therefore recurrent vector λ_m will be independent of $\dot{x}^i$, s and $R_{(m)}=0$ (Pandey, to appear). (1.6) yields $T_{(m)}=0$. Contracting the recurrence equation $Q^i_{jkh(m)}=\lambda_m Q^i_{jkh}$ successively we have $T_{(m)}=\lambda_m T$ which implies $\lambda_m=0$.

Theorem 1.5. *A Q-bisymmetric Finsler space $F_n(n>2)$ of non-zero constant Riemannian curvature is Riemannian.*

Proof. For an isotropic Finsler space $F_n(n>2)$ of non-zero constant Riemannian curvature R, we have

$$Q^i_{jkh}=-R(\delta^i_h g_{jk}-\delta^i_k g_{hj}). \tag{1.7}$$

Differentiating the above equation covariantly twice and contracting it in i and h, we have $g_{jk(m)(l)}=0$. Using commutation formula (Rund (1959), (6.10), p. 126), we get

$$g_{rk}Q^r_{jlm}+g_{rj}Q^r_{klm}+2c_{jlr}Q^r_{lm}=0, \tag{1.9}$$

which yields

$$Q^i_{kh}=R(\delta^i_h y_k-\delta^i_k y_h). \tag{1.10}$$

Using (1.7) and (1.10) in (1.9), we have $y_l c_{jkm}-y_m c_{jkl}=0$, which implies that $c_{ijk}=0$.

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ON A TYPE OF SEMI-SYMMETRIC CONNECTION ON A RIEMANNIAN MANIFOLD

BY

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Introduction. The present paper deals with a type of semisymmetric metric connection ∇[1] on a Riemannian manifold (M^n, g) $(n>3)$ such that the curvature tensor R of ∇ satisfies the relation

$$(\bar{\nabla}_W R)(X, Y)Z=B(W)R(X, Y)Z, \tag{1}$$

where B is a 1-form and $\bar{\nabla}$ denotes the Levi-Civita connection. It is shown that if (1) is satisfied, then the manifold (M^n, g) is conformally recurrent [2].

1. **Preliminaries.** Let (M^n, g) be an n-dimensional $(n>3)$ Riemannian manifold with Levi-Civita connection $\bar{\nabla}$. A linear connection ∇ on (M^n, g) is said to be semi-symmetric if its torsion tensor T-satisfies the relation

$$T(X, Y)=\pi(Y)X-\pi(X)Y \tag{1.1}$$

where π is a 1-form.

It is known that [1] if ∇ is a *semi-symmetric metric* connection, then

$$\nabla_X Y=\bar{\nabla}_X Y+\pi(Y)X-g(X, Y)\rho \tag{1.2}$$

where

$$g(X, \rho)=\pi(X) \tag{1.3}$$

for every vector field X. Further, it is also known [1] that if R and K denote the curvature tensor of ∇ and $\bar{\nabla}$ respectively, then

$$\begin{aligned} R(X, Y)Z= {} & K(X, Y)Z-\alpha(Y, Z)X+\alpha(X, Z)Y- \\ & -g(Y, Z)AX+g(X, Z)AY \end{aligned} \tag{1.4}$$

where α is a tensor field of type (0,2) defined by

$$\alpha(X, Y)=(\bar{\nabla}_X\pi)(Y)-\pi(X)\pi(Y)+\frac{1}{2}\pi(\rho)g(X, Y) \tag{1.5}$$

and A is a tensor field of type (1, 1) defined by

$$g(AX, Y)=\alpha(X, Y) \tag{1.6}$$

for any vector fields X, Y, We shall use the above results in the next section.