Theorem 1.4. *A Q-symmetric Finsler space $F_n(n>2)$ of non-zero scalar curvature does not exist.*

Proof. Suppose that a Q-symmetric Finsler space of non-zero scalar curvature exist. Contracting i and j in (1.1), we get

$$T=-F^2R, \tag{1.6}$$

which implies $T\neq 0$ that is $H\neq 0$ (Sinha, 1971). Therefore recurrent vector λ_m will be independent of $\dot{x}^i$, s and $R_{(m)}=0$ (Pandey, to appear). (1.6) yields $T_{(m)}=0$. Contracting the recurrence equation $Q^i_{jkh(m)}=\lambda_m Q^i_{jkh}$ successively we have $T_{(m)}=\lambda_m T$ which implies $\lambda_m=0$.

Theorem 1.5. *A Q-bisymmetric Finsler space $F_n(n>2)$ of non-zero constant Riemannian curvature is Riemannian.*

Proof. For an isotropic Finsler space $F_n(n>2)$ of non-zero constant Riemannian curvature R, we have

$$Q^i_{jkh}=-R(\delta^i_h g_{jk}-\delta^i_k g_{hj}). \tag{1.7}$$

Differentiating the above equation covariantly twice and contracting it in i and h, we have $g_{jk(m)(l)}=0$. Using commutation formula (Rund (1959), (6.10), p. 126), we get

$$g_{rk}Q^r_{jlm}+g_{rj}Q^r_{klm}+2c_{jlr}Q^r_{lm}=0, \tag{1.9}$$

which yields

$$Q^i_{kh}=R(\delta^i_h y_k-\delta^i_k y_h). \tag{1.10}$$

Using (1.7) and (1.10) in (1.9), we have $y_l c_{jkm}-y_m c_{jkl}=0$, which implies that $c_{ijk}=0$.

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ON A TYPE OF SEMI-SYMMETRIC CONNECTION ON A RIEMANNIAN MANIFOLD

BY

M. C. CHAKI and GOURI KUMAR

Introduction. The present paper deals with a type of semisymmetric metric connection ∇[1] on a Riemannian manifold (M^n, g) $(n>3)$ such that the curvature tensor R of ∇ satisfies the relation

$$(\bar{\nabla}_W R)(X, Y)Z=B(W)R(X, Y)Z, \tag{1}$$

where B is a 1-form and $\bar{\nabla}$ denotes the Levi-Civita connection. It is shown that if (1) is satisfied, then the manifold (M^n, g) is conformally recurrent [2].

1. **Preliminaries.** Let (M^n, g) be an n-dimensional $(n>3)$ Riemannian manifold with Levi-Civita connection $\bar{\nabla}$. A linear connection ∇ on (M^n, g) is said to be semi-symmetric if its torsion tensor T-satisfies the relation

$$T(X, Y)=\pi(Y)X-\pi(X)Y \tag{1.1}$$

where π is a 1-form.

It is known that [1] if ∇ is a *semi-symmetric metric* connection, then

$$\nabla_X Y=\bar{\nabla}_X Y+\pi(Y)X-g(X, Y)\rho \tag{1.2}$$

where

$$g(X, \rho)=\pi(X) \tag{1.3}$$

for every vector field X. Further, it is also known [1] that if R and K denote the curvature tensor of ∇ and $\bar{\nabla}$ respectively, then

$$\begin{aligned} R(X, Y)Z= & K(X, Y)Z-\alpha(Y, Z)X+\alpha(X, Z)Y- \\ & -g(Y, Z)AX+g(X, Z)AY \end{aligned} \tag{1.4}$$

where α is a tensor field of type (0,2) defined by

$$\alpha(X, Y)=(\bar{\nabla}_X\pi)(Y)-\pi(X)\pi(Y)+\frac{1}{2}\pi(\rho)g(X, Y) \tag{1.5}$$

and A is a tensor field of type (1, 1) defined by

$$g(AX, Y)=\alpha(X, Y) \tag{1.6}$$

for any vector fields X, Y, We shall use the above results in the next section.

2. A special type of semi-symmetric connection. In this section we consider a semi-symmetric metric connection ∇ on (M^n, g) whose curvature tensor satisfies the condition

$$(\bar\nabla_W R)(X, Y)Z = B(W)R(X, Y)Z, \tag{2.1}$$

where B is a 1-form.

From (1.4) we get

$$(\bar\nabla_W R)(X, Y)Z = (\bar\nabla_W K)(X, Y)Z - [(\bar\nabla_W \alpha)(Y, Z)]X + [(\bar\nabla_W \alpha)(X, Z)]Y - g(Y, Z)(\bar\nabla_W A)(X) + g(X, Z)(\bar\nabla_W A)(Y) \tag{2.2}$$

In virtue of (2.1) we can express (2.2) as follows:

$$[(\bar\nabla_W K)(X, Y)Z - B(W)K(X, Y)Z] - [(\bar\nabla_W \alpha)(Y, Z) - B(W)\alpha(Y, Z)]X + [(\bar\nabla_W \alpha)(X, Z) - B(W)\alpha(X, Z)]Y - g(Y, Z)[(\bar\nabla_W A)(X) - B(W)AX] + g(X, Z)[(\bar\nabla_W A)(Y) - B(W)AY] = 0. \tag{2.3}$$

Let C denote Weyl's conformal curvature tensor of (M^n, g). Then

$$C(X, Y)Z = K(X, Y)Z + \lambda(Y, Z)X - \lambda(X, Z)Y - g(Y, Z)LX - g(X, Z)LY \tag{2.4}$$

where

$$\lambda(Y, Z) = -\frac{1}{n-2}S(Y, Z) + \frac{\gamma}{2(n-1)(n-2)}g(Y, Z) \tag{2.5}$$

and L is a $(1-1)$ tensor field such that

$$g(LY, Z) = \lambda(Y, Z)... \tag{2.6}$$

S and γ being the Ricci tensor and the scalar curvature of (M^n, g) respectively.

From (2.4) we get

$$(\bar\nabla_W C)(X, Y)Z = (\bar\nabla_W K)(X, Y)Z + [(\bar\nabla_W \lambda)(Y, Z)]X - [(\bar\nabla_W \lambda)(X, Z)]Y + g(Y, Z)(\bar\nabla_W L)(X) - g(X, Z)(\bar\nabla_W L)(Y) \tag{2.7}$$

Since $\bar\nabla g = \theta$ we have $(\bar\nabla_X g)(LY, Z) = 0$, from which we get in virtue of (2.6)

$$(\bar\nabla_X \lambda)(Y, Z) = g[(\bar\nabla_X L)(Y), Z] \tag{2.8}$$

Again, since $(\bar\nabla_X g)(AY, Z) = 0$ from this we get by using (1.6)

$$(\bar\nabla_X \alpha)(Y, Z) = g[(\bar\nabla_X A)(Y), Z] \tag{2.9}$$

Now, from (2.1) we get

$$(\bar\nabla_W S')(Y, Z) = B(W)S'(Y, Z) \tag{2.10}$$

and

$$\bar\nabla_W r' = B(W)r' \tag{2.11}$$

where S' and r' are the Ricci tensor and the scalar curvature of ∇ respectively.

Again, from (1.4) it follows that

$$S'(Y, Z) = S(Y, Z) - (n-2)\alpha(Y, Z) - ag(Y, Z) \tag{2.12}$$

and

$$r' = r - 2(n-1)a \tag{2.13}$$

where a is the trace of A.

From (2.12) we obtain

$$(\bar\nabla_W S')(Y, Z) = (\bar\nabla_W S)(Y, Z) - (n-2)(\bar\nabla_W \alpha)(Y, Z) - (\bar\nabla_W a)g(Y, Z) \tag{2.14}$$

Again, from (2.13) we have

$$\bar\nabla_W r' = \bar\nabla_W r - 2(n-1)\bar\nabla_W a$$

which gives in virtue of (2.11) and (2.13)

$$\bar\nabla_W a - B(W)a = \frac{1}{2(n-1)}[\bar\nabla_W r - B(W)r]. \tag{2.15}$$

Using (2.10) we can express (2.14) as

$$B(W)S'(Y, Z) = (\bar\nabla_W S)(Y, Z) - (n-2)(\bar\nabla_W \alpha)(Y, Z) - (\bar\nabla_W a)g(Y, Z)$$

which gives in virtue of (2.12) and (2.15)

$$(n-2)[(\bar\nabla_W \alpha)(Y, Z) - B(W)\alpha(Y, Z)] = [(\bar\nabla_W S)(Y, Z) - B(W)S(Y, Z)] - \frac{1}{2(n-1)}g(Y, Z)[\bar\nabla_W Y - B(W)r].$$

Now using (2.5) we obtain from this

$$(\bar\nabla_W \alpha)(Y, Z) - B(W)\alpha(Y, Z) = -(\bar\nabla_W \lambda)(Y, Z) + B(W)\lambda(Y, Z). \tag{2.16}$$

In virtue of (2.9), (2.16), (2.8) and (2.6) we obtain

$$g[g(Y, Z)(\bar\nabla_W A)(X), U] = g[-g(Y, Z)(\bar\nabla_W L)(X) + g(Y, Z)B(W)LX + g(Y, Z)B(W)AX, U]$$

from which it follows that

$$g(Y, Z)(\bar\nabla_W A)(X) - g(Y, Z)B(W)AX = -g(Y, Z)(\bar\nabla_W L)(X) + g(Y, Z)B(W)LX. \tag{2.17}$$

Using (2.16) and (2.17) we can express (2.3) as follows:

$$(\bar\nabla_W K)(X, Y)Z + [(\bar\nabla_W \lambda)(Y, Z)]X - [(\bar\nabla_W \lambda)(X, Z)]Y + g(Y, Z)(\bar\nabla_W L)(X) - g(X, Z)(\bar\nabla_W L)(Y) = B(W)[K(X, Y)Z + \lambda(Y, Z)X - \lambda(X, Z)Y + g(r, Z)LX - g(X, Z)LY]$$

which gives in virtue of (2.7) and (2.4)

$$(\bar\nabla_W C)(X, Y)Z = B(W)C(X, Y)Z. \tag{2.18}$$

From (2.18) we can state the following theorem:

Theorem 1. *If a Riemannian manifold* (M^n, g) $(n>3)$ *admits a semi-symmetric connection whose curvature tensor is a recurrent tensor with respect to the Levi-Civita connection having B as its associated 1-form, then the manifold* (M^n, g) *is conformally recurrent with B as 1-form of recurrence.*

If, in particular, B is zero, then the curvature tensor of ∇ becomes covariant constant with respect to the Levi-Civita connection and consequently (M^n, g) becomes conformally symmetric[3]. Hence we have the following corollary :

Corollary. If a Riemannian manifold (M^n, g) $(n>3)$ admits a semi-symmetric metric connection whose curvature tensor is covariant constant with respect to the Levi-Civita connection, then (M^n, g) is conformally symmetric.

The case considered in the above corollary was treated by T. Imai [4]. From (2.18) we get

$$(\text{div } C)(X, Y)Z = B(C(X, Y)Z). \tag{2.19}$$

But

$$(\text{div } C)(X, Y)Z = \frac{n-3}{n-2}\Big[(\bar{\nabla}_X S)(Y, Z) - (\bar{\nabla}_Y S)(X, Z) - \frac{1}{2(n-1)}\{g(Y, Z)\bar{\nabla}_X r - g(X, Z)\bar{\nabla}_Y r\}\Big]$$

Hence (2.19) can be written as

$$B(C(X, Y)Z) = \frac{n-2}{n-3}\Big[[(\bar{\nabla}_X S)(Y, Z) - (\bar{\nabla}_Y S)(X, Z) - \frac{1}{2(n-1)}\{g(Y, Z)\bar{\nabla}_X r - g(X, Z)\bar{\nabla}_Y r\}\Big] \tag{2.20}$$

From (2.20) it follows that r is constant if and only if

$$(\bar{\nabla}_X S)(Y, Z) - (\bar{\nabla}_Y S)(X, Z) = (n-2)/(n-3)B(C(X, Y)Z). \tag{2.21}$$

Hence we can state the following theorem :

Theorem 2. *If a Riemannian manifold* (M^n, g) $(n>3)$ *admits a semi-symmetric metric connection whouse curvature tensor is a recurrent tensor with respect to the Levi-Civita connection having B as the associated 1-form, then* (M^n, g) *is of constant scalar curvature if and only if* (2.21) *holds.*

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NEW CLASSES OF QUASICONFORMAL MAPPINGS IN R^n (I)

BY

PETRU CARAMAN

To Professor Adolf Haimovici on the occasion of his 70-th birthday

The most important tools in the theory of qc (quasiconformal mappings) in the euclidean n-space R^n are the conformal capacity, the module and the extremal length. They correspond to the p-capacity, the p-module and the p-extremal length for $p=n$. In the case $1<p<\infty (p\neq n)$, one obtains a subclass — the (K, p)-qc — of the class of qc (corresponding to $p=n$). Some years ago [4], we started the generalization of the theory of qc in the infinite dimensional spaces. When we investigated the possibility of extending to such a space the concepts of conformal capacity, module and extremal length, it appeared to us, as the only natural way, to consider something like p-capacity, p-module and p-extremal length with $p=\infty$. But, since the corresponding classes of qc have been studied neither in the finite dimensional case, we think it will be interesting to study them previously in R^n, establishing their inclusion in the class of qc and this will be the topic of the first part (§1) of our paper.

In the second part (§2)., we give four equivalent definitions of the (K, p)-qc. We show that the class (p)-qc — corresponding to a fixed $p\neq n$ and all the $K\in[1, \infty)$ — is contained in the class of the qc and the inclusion is strict. Next, we establish that, if $1<p\leq p'\leq 2$, the class of (K, p)-qc is contained in the class of (K, p') and that there is no inclusion relation between such classes if p, p' do not satisfy the preceding inequality. Finally, we obtain an analogous result in the case of a class of (K, p)-qc introduced by F. Gehring [9] by means of the p-capacity, i.e. we prove that his class of (K, p)-qc is contained in his class of (K, p')-qc if $2(n-1)\leq p\leq p'$ and otherwise, there is no inclusion relation.

§1. **(K, ∞)-quasiconformal mappings.** Let us introduce first the concepts of ∞-module, ∞-extremal length and ∞-capacity.

Let Γ be an arc family of R^n and $F(\Gamma)$ the class of admissible functions $\rho\geq 0$ Borel measurable and such that $\int_\gamma \rho\, dH^1 \geq 1\ \forall\gamma\in\Gamma$ ($\forall$ means „for every"), where H^1 is linear Hausdorff measure. The ∞-*module* of Γ is

$$M_\infty\Gamma = \inf_{\rho\in F(\Gamma)} \operatorname{ess\,sup}_{R^n} \rho(x), \quad \text{where} \quad \operatorname{ess\,sup}_{R^n} \rho(x) = \inf_E [\sup_{R^n - E} \rho(x)],$$

the infimum being taken over all the null-sets (i.e. with n-dimensional Lebesgue measure zero).